Homework for Lecture 7 of Dr. Z.'s Dynamical Models in Biology class

Version of Sept. 26 (thanks to Anna Janik, who won a dollar) Email the answers (as a .pdf file) to

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by 8:00pm Monday, Sept. 29, 2025.

Subject: hw7

with an attachment hw7FirstLast.pdf

1. For each of the following non-linear (quadratic) first-order-linear recurrences

(i) Determine all steady-states

(ii) decide which of them are stable

(iii) test it empirically, by taking a number close to each steady state, using either a calculator of Maple, find its orbit of length 10 and verify that for stable steady-states, the orbit is attracted to it, but for the unstable ones it runs away.

a. $x(n+1) \,=\, \frac{5}{2}\,x(n)\,(1-x(n)) \quad .$

b. $x(n+1) \,=\, \frac{29}{10}\,x(n)\,(1-x(n)) \quad .$

c. $x(n+1) \,=\, \frac{31}{10}\,x(n)\,(1-x(n)) \quad .$

2.: For each of the following non-linear (cubic) first-order-linear recurrences

(i) Verify that the given points are steady-states

(ii) decide which ones ones are stable

(ii) decide which of them are stable

(iii) test it empirically, by taking a number close to each steady state, using either a calculator of Maple, find its orbit of length 10 and verify that for stable steady-states, the orbit is attracted to it, but for the unstable ones it runs away.

a. $x(n+1) \,=\, \frac{1}{5}\,x^3 - \frac{6}{5}x^2 + \frac{16}{5}x - \frac{6}{5}$

Set of steady-states : $\{1, 2, 3\}$

b. $x(n+1) \,=\, \frac{1}{8}x^3 - \frac{5}{4}x^2 + \frac{39}{8}x - \frac{15}{4}$

Set of steady-states : $\{2, 3, 5\}$

1. For each of the following non-linear (quadratic) first-order-linear recurrences

(i) Determine all steady-states

(ii) decide which of them are stable

(iii) test it empirically, by taking a number close to each steady state, using either a calculator of Maple, find its orbit of length 10 and verify that for stable steady-states, the orbit is attracted to it, but for the unstable ones it runs away.

a. $x(n+1) \;=\; \frac{5}{2}\,x(n)\,(1 - x(n))$.

b. $x(n+1) \;=\; \frac{29}{10}\,x(n)\,(1 - x(n))$.

c. $x(n+1) \;=\; \frac{31}{10}\,x(n)\,(1 - x(n))$.

a.) $X(n+1) = \frac{5}{2}X(n)(1 - X(n))$

i) $f(X) = \frac{5}{2}X(1 - X(n))$

$f(X) = X$

$X = \frac{5}{2}X(1 - X(n))$

$0 = \frac{5}{2}X - \frac{5}{2}X^2 - X$

$0 = \frac{3}{2}X - \frac{5}{2}X^2$

$0 = X\left(\frac{3}{2} - \frac{5}{2}X\right)$

steady states $\boxed{X = 0 \qquad X = 3/5}$

ii) $f(x) = \frac{5}{2}X - \frac{5}{2}X^2$

$f'(X) = \frac{5}{2} - 5X$

$f'(0) = \frac{5}{2} > 1$ $\boxed{\text{unstable}}$

$f'\left(\frac{3}{5}\right) = \frac{5}{2} - 5\left(\frac{3}{5}\right) = -.5$ $\boxed{\text{stable}}$

$|-.5| = .5 < 1$

$f := x \to (2.5 \cdot x) - (2.5 \cdot x \cdot \cdot 2)$

$f := x \mapsto 2.9\,x - 2.9\,x^2$

$f(.1)$	0.261
$f(\%)$	0.5593491
$f(\%)$	0.7147852845
$f(\%)$	0.591215117
$f(\%)$	0.700871427
$f(\%)$	0.607986942
$f(\%)$	0.691182579
$f(\%)$	0.619002742
$f(\%)$	0.683931208
$f(\%)$	0.626891001
$f(\%)$	0.678306154

orbit values

values diverge

$f\left(\frac{3}{5}\right)$	0.696000000
$f(.697)$	0.6124539
$f(\%)$	0.688326949
$f(\%)$	0.622145585
$f(\%)$	0.681733322
$f(\%)$	0.629221699
$f(\%)$	0.676575082
$f(\%)$	0.634581597
$f(\%)$	0.672474601
$f(\%)$	0.638732285
$f(\%)$	0.669184723
$f(\%)$	0.641991936

orbit

values converge

b.) $x(n+1) = \frac{29}{10}\, x(n)\big(1 - x(n)\big)$

i) $f(x) = \frac{29}{10} x(n)\big(1 - x(n)\big)$

$f(x) = x$

$x = \frac{29}{10} x - \frac{29}{10} x^2$

$0 = x\left(\frac{19}{10} - \frac{29}{10} x\right)$

$\boxed{x = 0, \quad x = \frac{19}{29}}$ steady states

ii)

$f(x) = \frac{29}{10} x - \frac{29}{10} x^2$

$f'(x) = \frac{29}{10} - \frac{29}{5} x$

$f'(0) = \frac{29}{10} > 1$ $\boxed{\text{unstable}}$

$f'\left(\frac{19}{29}\right) = \frac{29}{10} - \frac{29}{5}\left(\frac{19}{29}\right) = -.9$ $\quad .9 < 1$ $\boxed{\text{stable}}$

$f := x \rightarrow (2.9 \cdot x) - (2.9 \cdot x \cdot 2)$	$f := x \mapsto 2.9 \cdot x - 2.9 \cdot x^2$
$f(.1)$	0.261
$f(\%)$	0.5593491
$f(\%)$	0.7147852845
$f(\%)$	0.591215117
$f(\%)$	0.700871427
$f(\%)$	0.607986942
$f(\%)$	0.691182579
$f(\%)$	0.619002742
$f(\%)$	0.683931208
$f(\%)$	0.626891001
$f(\%)$	0.678306154

orb

diverges

$f\left(\frac{19}{29}\right)$	0.655172414
$f(.656)$	0.6544256
$f(\%)$	0.655842929
$f(\%)$	0.654567646
$f(\%)$	0.655715644
$f(\%)$	0.654682651
$f(\%)$	0.655612505
$f(\%)$	0.654775770
$f(\%)$	0.655528937
$f(\%)$	0.654851174
$f(\%)$	0.655461231
$f(\%)$	0.654912237

orb

converges

c) i) $x(n+1) = \frac{31}{10} x(n)\left(1 - x(n)\right)$

$f(x) = \frac{31}{10} x(n) - \frac{31}{10} x(n)^2$

$f(x) = x$

$x = \frac{31}{10} x - \frac{31}{10} x^2$

$0 = x\left(\frac{21}{10} - \frac{31}{10} x\right)$

$\boxed{x = 0, \quad x = \frac{21}{31}}$ steady states

ii) $f(x) = \frac{31}{10} x - \frac{31}{10} x^2$

$f'(x) = \frac{31}{10} - \frac{31}{5} x$

$f'(0) = \frac{31}{10} > 1$ $\boxed{\text{unstable}}$

$f'\left(\frac{21}{31}\right) = \frac{31}{10} - \frac{31}{5}\left(\frac{21}{31}\right)$ $\boxed{\text{unstable}}$

$|-1.1| = 1.1 > 1$

$f\left(\frac{21}{31}\right)$	
	0.677419355
$f(.678)$	
	0.6767796
$f(\%)$	0.678121816
$f(\%)$	0.676645118
$f(\%)$	0.678269157
$f(\%)$	0.676482334
$f(\%)$	0.678447356
$f(\%)$	0.676285278
$f(\%)$	0.678662853
$f(\%)$	0.676046713
$f(\%)$	0.678923420
$f(\%)$	0.675757870

orbit

values diverge

$f := x \rightarrow (3.1 \cdot x) - (3.1 \cdot x \cdot{}^{\wedge} 2)$

$f := x \mapsto 3.1 \cdot x - 3.1 \cdot x^2$

$f(0)$	0.
$f(.1)$	0.279
$f(\%)$	0.6235929
$f(\%)$	0.727646865
$f(\%)$	0.614348406
$f(\%)$	0.734465771
$f(\%)$	0.604579987
$f(\%)$	0.741095382
$f(\%)$	0.594806352
$f(\%)$	0.747136442
$f(\%)$	0.585663095
$f(\%)$	0.752251686

orbit

values diverge

2.: For each of the following non-linear (cubic) first-order-linear recurrences

(i) Verify that the given points are steady-states

(ii) decide which ones ones are stable

(ii) decide which of them are stable

(iii) test it empirically, by taking a number close to each steady state, using either a calculator of Maple, find its orbit of length 10 and verify that for stable steady-states, the orbit is attracted to it, but for the unstable ones it runs away.

a. $x(n+1) = \frac{1}{5}x^3 - \frac{6}{5}x^2 + \frac{16}{5}x - \frac{6}{5}$

Set of steady-states : $\{1, 2, 3\}$

b. $x(n+1) = \frac{1}{8}x^3 - \frac{5}{4}x^2 + \frac{39}{8}x - \frac{15}{4}$

Set of steady-states : $\{2, 3, 5\}$

a) $x(n+1) = \frac{1}{5}x^3 - \frac{6}{5}x^2 + \frac{16}{5}x - \frac{6}{5}$

i.) $x(1) = \frac{1}{5} - \frac{6}{5} + \frac{16}{5} - \frac{6}{5} = \frac{5}{5} = 1$ ✓

$x(2) = \frac{1}{5}(2)^3 - \frac{6}{5}(2)^2 + \frac{16}{5}(2) - \frac{6}{5} = 2$ ✓

$x(3) = \frac{1}{5}(3)^3 - \frac{6}{5}(3)^2 + \frac{16}{5}(3) - \frac{6}{5} = 3$ ✓

ii) $f(x) = \frac{1}{5}x^3 - \frac{6}{5}x^2 + \frac{16}{5}x - \frac{6}{5}$

$f'(x) = \frac{3}{5}x^2 - \frac{12}{5}x + \frac{16}{5}$

$f'(1) = \frac{3}{5} - \frac{12}{5} + \frac{16}{5} = \frac{7}{5} > 1$ unstable

$f'(2) = \frac{3}{5}(2)^2 - \frac{12}{5}(2) + \frac{16}{5} = .8 < 1$ stable

$f'(3) = \frac{3}{5}(3)^2 - \frac{12}{5}(3) + \frac{16}{5} = 1.4 > 1$ unstable

$f := x \to \frac{1}{5} \cdot x \cdot 3 - \frac{6}{5} \cdot x \cdot 2 + \frac{16}{5} \cdot x - \frac{6}{5}$

$f := x \mapsto \frac{1}{5} x^3 - \frac{6}{5} x^2 + \frac{16}{5} x - \frac{6}{5}$

f(2)

2

f(2.1)

2.080200000

f(%)

2.064263170

f(%)

2.051463614

f(%)

2.041198151

f(%)

2.032972506

f(%)

2.026385174

f(%)

2.021111813

f(%)

2.016891333

f(%)

2.013514031

f(%)

2.010811718

f(%)

2.008649628

orbit

values converge

$f := x \to \frac{1}{5} \cdot x \cdot\cdot 3 - \frac{6}{5} \cdot x \cdot\cdot 2 + \frac{16}{5} \cdot x - \frac{6}{5}$

$f := x \mapsto \frac{1}{5} \cdot x^3 - \frac{6}{5} \cdot x^2 + \frac{16}{5} \cdot x - \frac{6}{5}$

f(1)

1

f(1.1)

1.134200000

f(%)

1.177557595

f(%)

1.230784174

f(%)

1.293599417

f(%)

1.364380503

f(%)

1.440144804

f(%)

1.517019883

f(%)

1.591082973

f(%)

1.659191120

f(%)

1.719435858

f(%)

1.771131696

orbit

values diverge

$f := x \to \frac{1}{5} \cdot x \cdot 3 - \frac{6}{5} \cdot x \cdot 2 + \frac{16}{5} \cdot x - \frac{6}{5}$

$f := x \mapsto \frac{1}{5} x^3 - \frac{6}{5} x^2 + \frac{16}{5} x - \frac{6}{5}$

f(3)

3

f(3.1)

3.146200000

f(%)

3.218129650

f(%)

3.336005586

f(%)

3.545734670

f(%)

3.975231162

f(%)

5.121472870

f(%)

10.58005045

f(%)

135.1920112

f(%)

472676.7887

f(%)

$2.112113789 \times 10^{16}$

f(%)

$1.884438339 \times 10^{48}$

orbit

values diverge

b.) i) $x(n+1) = \frac{1}{8}x^3 - \frac{5}{4}x^2 + \frac{39}{8}x - \frac{15}{4}$

$x(2) = \frac{1}{8}(2)^3 - \frac{5}{4}(2)^2 + \frac{39}{8}(2) - \frac{15}{4} = 2$ ✓

$x(3) = \frac{1}{8}(3)^3 - \frac{5}{4}(3)^2 + \frac{39}{8}(3) - \frac{15}{4} = 3$ ✓

$x(5) = \frac{1}{8}(5)^3 - \frac{5}{4}(5)^2 + \frac{39}{8}(5) - \frac{15}{4} = 5$ ✓

ii) $f(x) = \frac{1}{8}x^3 - \frac{5}{4}x^2 + \frac{39}{8}x - \frac{15}{4}$

$f'(x) = \frac{3}{8}x^2 - \frac{5}{2}x + \frac{39}{8}$

$f'(2) = \frac{3}{8}(2)^2 - \frac{5}{2}(2) + \frac{39}{8} = 1.375 > 1$ unstable

$f'(3) = \frac{3}{8}(3)^2 - \frac{5}{2}(3) + \frac{39}{8} = .75 < 1$ stable

$f'(5) = \frac{3}{8}(5)^2 - \frac{5}{2}(5) + \frac{39}{8} = 1.75 > 1$ unstable

$f := x \to \frac{1}{8} \cdot x \cdot 3 - \frac{5}{4} \cdot x \cdot 2 + \frac{39}{8} \cdot x - \frac{15}{4}$

$$f := x \mapsto \frac{1}{8} \cdot x^3 - \frac{5}{4} \cdot x^2 + \frac{39}{8} \cdot x - \frac{15}{4}$$

$f(2)$	2
$f(2.1)$	2.132625000
$f(\%)$	2.173856283
$f(\%)$	2.224596258
$f(\%)$	2.285014295
$f(\%)$	2.354172160
$f(\%)$	2.429821087
$f(\%)$	2.508556886
$f(\%)$	2.586391682
$f(\%)$	2.659565189
$f(\%)$	2.725255038
$f(\%)$	2.781913296

orbit

values diverge

$f := x \to \frac{1}{8} \cdot x \cdot 3 - \frac{5}{4} \cdot x \cdot 2 + \frac{39}{8} \cdot x - \frac{15}{4}$

$$f := x \mapsto \frac{1}{8} \cdot x^3 - \frac{5}{4} \cdot x^2 + \frac{39}{8} \cdot x - \frac{15}{4}$$

$f(3)$	3
$f(3.1)$	3.073875000
$f(\%)$	3.054774446
$f(\%)$	3.040726348
$f(\%)$	3.030345882
$f(\%)$	3.022647786
$f(\%)$	3.016923178
$f(\%)$	3.012657185
$f(\%)$	3.009473122
$f(\%)$	3.007093730
$f(\%)$	3.005314045
$f(\%)$	3.003982020

orbit

values converge

$f := x \to \frac{1}{8} \cdot x \cdot 3 - \frac{5}{4} \cdot x \cdot 2 + \frac{39}{8} \cdot x - \frac{15}{4}$

$$f := x \mapsto \frac{1}{8} \cdot x^3 - \frac{5}{4} \cdot x^2 + \frac{39}{8} \cdot x - \frac{15}{4}$$

$f(5)$	5
$f(5.1)$	5.181375000
$f(\%)$	5.338712630
$f(\%)$	5.669308400
$f(\%)$	6.488752370
$f(\%)$	9.403012200
$f(\%)$	35.49172768
$f(\%)$	4183.144593
$f(\%)$	$9.128095365 \times 10^{9}$
$f(\%)$	$9.507153776 \times 10^{28}$
$f(\%)$	$1.074141680 \times 10^{86}$
$f(\%)$	$1.549154452 \times 10^{257}$

orbit

values diverge