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Homework #7

1.)

$$(a) x(n+1) = 5/2 x(n) (1 - x(n))$$

$$\Rightarrow f(x) = 5/2 x (1 - x)$$

For steady states:

$$x = 5/2 x (1 - x) \Rightarrow 5/2 x^2 - 3/2 x = 0$$

$$1/2 x (5x - 3) = 0 \Rightarrow x = 0, 3/5$$

→ steady states

$$f'(x) = -5x + 5/2$$

$$f'(0) = |5/2| > 1 \Rightarrow 0 \text{ is unstable}$$

$$f'(3/5) = -3 + 5/2 = |-1/2| < 1 \Rightarrow 3/5 \text{ is stable}$$

$$\text{Orb}(f(x), x, 0.01, 0, 9)$$

$$[0.1, 0.025, 0.061, 0.304, 0.529, 0.620, 0.59, 0.61, 0.59]$$

$$\text{Orb}(f(x), x, 0.59, 1000, 1009)$$

$$[0.59, 0.61, 0.59, 0.61, 0.59, 0.61, 0.59, 0.61, 0.59, 0.61]$$

$$(b) x(n+1) = 29/10 x(n) (1 - x(n))$$

$$\Rightarrow f(x) = (29/10) x (1 - x)$$

For steady state:

$$x = (29/10) x (1 - x) \Rightarrow 29/10 x^2 - 19/10 x = 0$$

$$(1/10) x (29x - 19) = 0, \text{ S.S. : } x = 0, 19/29$$

$$f'(x) = -29/5 x + 29/10$$

$$f'(0) = 29/10 > 1 \Rightarrow 0 \text{ is unstable}$$

$$f'(19/29) = |-19/5 + 29/10| < 1 \Rightarrow 19/29 \text{ is stable}$$

$$\text{Orb}(f(x), x, 0.01, 1, 10)$$

$$[0.029, 0.081, 0.216, 0.490, 0.725, 0.579, 0.707, 0.601, 0.696, 0.614]$$

$$\text{Orb}(f(x), x, 0.5, 1, 10)$$

$$[0.725, 0.57, 0.708, 0.6, 0.7, 0.61, 0.69, 0.62, 0.69, 0.62]$$

(C) $x(n+1) = 3/10 x(n)(1 - x(n))$

$$f(x) = -3/10 x^2 + 3/10 x$$

Steady State: $3/10 x^2 - 2/10 x = 0$

$$1/10 x(3x - 2) = 0 \Rightarrow x = 0, 2/3$$

→ steady state

$$f'(x) = -3/5 x + 3/10$$

$$f'(0) = 3/10 > 1 \Rightarrow 0 \text{ is unstable}$$

$$|f'(2/3)| = |-2/5 + 3/10| = 1/10 < 1$$

$$\Rightarrow 2/3 \text{ is stable}$$

$$\text{Orb}(f(x), x, 0.01, 1, 10)$$

$$[0.031, 0.092, 0.259, 0.596, 0.747, 0.586, 0.752, 0.578, 0.756, 0.572]$$

$$\text{Orb}(f(x), x, 0.69, 1001, 1010)$$

$$[0.765, 0.558, 0.765, 0.558, 0.765, 0.558, 0.765, 0.558, 0.765, 0.558]$$

$$2.) \textcircled{a} x(n+1) = \frac{1}{5}x^3 - \frac{6}{5}x^2 + \frac{16}{5}x - \frac{6}{5}$$

$$x = 1, 2, 3$$

$$f'(x) = \frac{3}{5}x^2 - \frac{12}{5}x + \frac{16}{5}$$

$$f'(1) = \frac{3}{5} - \frac{12}{5} + \frac{16}{5} = \frac{7}{5} > 1$$

1 is unstable

$$f'(2) = \frac{12}{5} - \frac{24}{5} + \frac{16}{5} = \frac{4}{5} < 1$$

2 is stable

$$f'(3) = \frac{27}{5} - \frac{36}{5} + \frac{16}{5} = \frac{7}{5} > 1$$

3 is unstable

$$\text{Orb}(f(x), x, 1.01, 1, 10)$$

$$[1.014, 1.019, 1.027, 1.037, 1.051, 1.070, 1.096, 1.129, 1.170, 1.222]$$

$$\text{Orb}(f(x), x, 2.5, 1, 10)$$

$$[2.424, 2.354, 2.292, 2.238, 2.193, 2.157, 2.126, 2.101, 2.081, 2.064]$$

$$\text{Orb}(f(x), x, 3.01, 1, 10)$$

$$[3.016, 3.017, 3.026, 3.035, 3.054, 3.079, 3.111, 3.167, 3.242, 3.376]$$

$$(b) \quad x(n+1) = \frac{1}{8}x^3 - \frac{5}{4}x^2 + \frac{39}{8}x - \frac{15}{4}$$

$$S.S.: x = 2, 3, 5$$

$$f'(x) = \frac{3}{8}x^2 - \frac{5}{2}x + \frac{39}{8}$$

$$f'(2) = \frac{12}{8} - 5 + \frac{39}{8} = \frac{11}{8} > 1$$

2 is unstable

$$f'(3) = \frac{27}{8} - \frac{15}{2} + \frac{39}{8} = \frac{6}{8} < 1$$

3 is stable

$$f'(5) = \frac{75}{8} - \frac{25}{2} + \frac{39}{8} = \frac{14}{8} > 1$$

5 is unstable

$$\text{Orb}(f(x), x, 1.99, 1, 10)$$

$$[1.986, 1.981, 1.974, 1.963, 1.950, 1.931, \\ 1.903, 1.863, 1.801, 1.705]$$

$$\text{Orb}(f(x), x, 2.6, 1, 10)$$

$$[2.678, 2.746, 2.803, 2.841, 2.876, \\ 2.904, 2.931, 2.948, 2.962, 2.969]$$

$$\text{Orb}(f(x), x, 4.9, 1, 10)$$

$$[4.83, 4.73, 4.57, 4.36, 4.11, 3.86, 3.64, \\ 3.46, 3.338, 3.249]$$