Homework 7 : DMD Ezra Chechik

1)

a. $x = \frac{5}{2}x(1-x)$

$\rightarrow x = \frac{5}{2}x - \frac{5}{2}x^2 \rightarrow \frac{5}{2}x^2 - \frac{3}{2}x = 0$

$x_1 = 0, x_2 = 3/5 \rightarrow x(5x-3) = 0$

$f'(x) = \frac{5}{2} - 5x$

$f'(0) = \frac{5}{2} \rightarrow$ unstable s-s

$f'(3/5) = \frac{5}{2} - \frac{15}{10} = \frac{+1}{2} \rightarrow$ Stable s-s

orbits:

$x(0) = 0.001$ ✓ runs away

[0.01, 0.0247, 0.06, 0.141, 0.304, 0.529, 0.62, 0.58, 0.6
0.59, 0.6]

$x(0) = 0.601$ (summarized for space)

[0.601, 0.599, 0.6, .59, .6, .59, .6, .59 ...]

attracted ✓

b. $x = \frac{29}{10}x(1-x) \rightarrow x = \frac{29}{10}x - \frac{29}{10}x^2$

$\frac{29}{10}x^2 - \frac{19}{10}x = 0, x_1 = 0, x_2 = \frac{19}{29}$

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$f'(x) = \frac{29}{10} - \frac{58}{10}x$

$f'(0) = \left|\frac{29}{10}\right| \rightarrow$ unstable

$f'(19/29) = |-0.9| < 1 \rightarrow$ stable

Orbits:

$x(0) = 0.001$

[0.002, 0.008, 0.02, 0.06, 0.18, 0.43, 0.71, 0.59, 0.7, 0.609] → runs away

$x(0) = 29/10 + .001$

[0.65, 0.65, 0.65 ...] → attracted

c. $x = \frac{31}{10}x(1-x)$

$x = \frac{31}{10}x - \frac{31}{10}x^2$

$\frac{31}{10}x^2 - \frac{21}{10}x = 0 \rightarrow x_1 = 0,\ x_2 = 21/31$

$f'(x) = \frac{31}{10} - \frac{62}{10}x$

$f'(0) = |31/10| > 1 \rightarrow$ unstable

$f'(21/31) = |-1.1| > 1 \rightarrow$ unstable

orbits:

$x(0) = 0.001$ runs away ✓

[0.001, 0.003, 0.009, 0.02, 0.08, 0.25, 0.58, 0.75 ...]

$x(0) = (21/31) + 0.01$

[0.77, 0.5, 0.8, 0.912 ...] runs away ✓

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a. $x(n+1) = \frac{1}{5}x^3 - \frac{6}{5}x^2 + \frac{16}{5}x - \frac{6}{5}$

2) $1 \stackrel{?}{=} \frac{1}{5} - \frac{6}{5} + \frac{16}{5} - \frac{6}{5} = \frac{5}{5}$ ✓

$2 \stackrel{?}{=} \frac{1}{5}(2)^3 - \frac{6}{5}(2)^2 + \frac{16}{5}(2) - \frac{6}{5}$

$\rightarrow 2 \stackrel{?}{=} \frac{8 - 24 + 32 - 6}{5} = \frac{10}{5} = 2$ ✓

$3 \stackrel{?}{=} \frac{1}{5}(3)^3 - \frac{6}{5}(3)^2 + \frac{16}{5}(3) - \frac{6}{5}$

$\rightarrow 3 \stackrel{?}{=} \frac{27 - 54 + 48 - 6}{5} = \frac{15}{5} = 3$ ✓

Stability $\rightarrow \frac{3}{5}x^2 - \frac{12}{5}x + \frac{16}{5}$

1: $f(1) = |1.4| > 1$ → unstable s-s

2: $f(2) = |0.8| < 1$ → Stable s-s

3: $f(3) = |1.4| > 1$ → unstable s-s

Orbits: around 1.1 → runs away

[1.1, 1.27, 1.37, 1.51, 1.7, 1.95, 2.26, 2.65, 3.16]

around 2.1 → attracted

[2.1, 2.036, 2.022, 2.013, 2.008, 2.005, 2.003, 2.002, 2.001]

around 3.1 → runs away

[3.1, 3.19, 3.27, 3.37, 3.51, 3.7, 3.95, 4.26, 4.65, 5.16]

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b. $x(n+1) = \frac{1}{8}x^3 - \frac{5}{4}x^2 + \frac{39}{8}x - \frac{15}{4}$

2)

$2 \stackrel{?}{=} \frac{1}{8}(2)^3 - \frac{5}{4}(2)^2 + \frac{39}{8}(2) - \frac{15}{4}$

$= \frac{8}{8} - \frac{20}{4} + \frac{78}{8} - \frac{30}{8} = 2$ ✓

$3 \stackrel{?}{=} \frac{1}{8}(3^3) - \frac{5}{4}(3)^2 + \frac{39}{8}(3) - \frac{15}{4}$

$= \frac{27 - 90 + 117 - 30}{8} = \frac{24}{8} = 3$ ✓

$5 \stackrel{?}{=} \frac{1}{8}(5)^3 - \frac{5}{4}(5^2) + \frac{39}{8}(5) - \frac{15}{4}$

$= \frac{125 - 250 + 195 - 30}{8} = \frac{40}{8} = 5$ ✓

Derivative is: $\frac{3}{8}x^2 - \frac{5}{2}x + \frac{39}{8}$

$f'(2) = \left|\frac{31}{8}\right| > 1 \rightarrow$ unstable

$f'(3) = \left|\frac{3}{4}\right| < 1 \rightarrow$ stable

$f'(5) = \left|\frac{17}{4}\right| > 1 \rightarrow$ unstable

Orbits:

around 2.1 runs away ✓

[2.1, 2.18, 2.25, 2.34, 2.47, 2.63, 2.84, 3.1, 3.4, 3.8]

around 3.1 attracted ✓

[3.1, 3.02, 3.006, 3.001, 3.0002 ...]

around 5.1 runs away ✓

[5.1, 5.28, 5.6, 6.06, 7.54, 8.6, 10.08, 11.9 ...]

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