

Problem 1

$$a) x(n+1) = \frac{5}{2} x(n) (1 - x(n))$$

$$c = \frac{5}{2} c (1 - c)$$

$$c = \frac{5}{2} c - \frac{5}{2} c^2$$

$$\frac{5}{2} c^2 - \frac{5}{2} c = 0$$

$$c(5c - 5) = 0$$

$$c = 0, c = \frac{5}{5} = 1$$

$$f'(x) = \frac{5}{2} - 5x$$

$$|f'(0)| = \left| \frac{5}{2} \right| > 1 \Rightarrow$$

0 is unstable

$$|f'(1)| = \left| -\frac{5}{2} \right| < 1 \Rightarrow$$

1 is stable

In general

$$x(n+1) = k x(n) (1 - x(n))$$

$$c = k c (1 - c)$$

$$c = k c - k c^2$$

$$k c^2 + (1 - k) c = 0$$

$$c(k c + 1 - k) = 0$$

$$c = 0, c = \frac{k-1}{k}$$

$$f'(x) = k - 2kx$$

$$b) k = \frac{29}{10}$$

$$c = 0, c = \frac{19}{29}$$

$$|f'(0)| = \left| \frac{29}{10} \right| > 1 \Rightarrow$$

0 is unstable

$$|f'(\frac{19}{29})| = \left| \frac{29}{10} - 2 \cdot \frac{29}{10} \cdot \frac{19}{29} \right|$$

$$= \left| \frac{29}{10} - \frac{38}{10} \right|$$

$$= \left| -\frac{9}{10} \right| < 1 \Rightarrow$$

$\frac{19}{29}$ is stable

$$c) k = \frac{31}{10}$$

$$c = 0, c = \frac{21}{31}$$

$$|f'(0)| = \left| \frac{31}{10} \right| > 1 \Rightarrow$$

0 is unstable

$$|f'(\frac{21}{31})| = \left| \frac{31}{10} - 2 \cdot \frac{31}{10} \cdot \frac{21}{31} \right|$$

$$= \left| \frac{31}{10} - \frac{42}{10} \right|$$

$$= \left| -\frac{11}{10} \right| > 1 \Rightarrow$$

$\frac{21}{31}$ is unstable

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> read `DMBS.txt`
      For a list of the Main procedures type: Help8(); for help with a specific procedure type: Help6(ProcedureName); for example Help6(Random.MC);
      For a list of the other procedures type: Help8a(); for help with a specific procedure type: Help6(ProcedureName); for example Help6(Random.MC);
=
> Orb( $\frac{31}{10} \cdot x \cdot (1 - x)$ , x, 0.1, 1, 10)
      [0.2790000000, 0.6235929000, 0.7276468648, 0.6143484052, 0.7344657708, 0.6045799871, 0.7410953815, 0.5948063527, 0.7471364420, 0.5856630949]
=
> Orb( $\frac{31}{10} \cdot x \cdot (1 - x)$ , x,  $\frac{21}{31} + 0.1$ , 1, 10)
      [0.5364193550, 0.7708882549, 0.5475206155, 0.7679995524, 0.5523463437, 0.7665055669, 0.5548218267, 0.7656831586, 0.5561786235, 0.7652162831]
=
> Orb( $\frac{29}{10} \cdot x \cdot (1 - x)$ , x, 0.1, 1, 10)
      [0.2610000000, 0.5593491000, 0.7147852845, 0.5912151166, 0.7008714272, 0.6079869421, 0.6911825789, 0.6190027424, 0.6839312072, 0.6268910019]
=
> Orb( $\frac{29}{10} \cdot x \cdot (1 - x)$ , x,  $\frac{19}{29} + 0.1$ , 1, 10)
      [0.5361724137, 0.7212055138, 0.5830975500, 0.7049749119, 0.6031573280, 0.6941398405, 0.6156981947, 0.6861803903, 0.6244769007, 0.6800659535]
=
>
=
> Orb( $\frac{5}{2} \cdot x \cdot (1 - x)$ , x, 0.1, 1, 10)
      [0.2250000000, 0.4359375000, 0.6147399902, 0.5920868365, 0.6038000362, 0.5980638812, 0.6009586880, 0.5995183582, 0.6002402410, 0.5998797352]
=
> Orb( $\frac{5}{2} \cdot x \cdot (1 - x)$ , x,  $\frac{3}{5} + 0.1$ , 1, 10)
      [0.5250000000, 0.6234375000, 0.5869079590, 0.6061175168, 0.5968476815, 0.6015513165, 0.5992183252, 0.6003893098, 0.5998049662, 0.6000974218]
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->

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Problem 2

$$> f(x) := \frac{1}{5}x^3 - \frac{6}{5}x^2 + \frac{16}{5}x - \frac{6}{5}$$

$$f := x \mapsto \frac{1}{5} \cdot x^3 - \frac{6}{5} \cdot x^2 + \frac{16}{5} \cdot x - \frac{6}{5}$$

$$> f(1)$$

1

$$> f(2)$$

2

$$> f(3)$$

3

> # g is the derivative of f

$$> g(x) := \frac{3}{5}x^2 - \frac{12}{5}x + \frac{16}{5}$$

$$g := x \mapsto \frac{3}{5} \cdot x^2 - \frac{12}{5} \cdot x + \frac{16}{5}$$

$$> g(1) \# \text{Unstable}$$

$\frac{7}{5}$

$$> g(2) \# \text{Stable}$$

$\frac{4}{5}$

$$> g(3) \# \text{Unstable}$$

$\frac{7}{5}$

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> read 'DMB8.txt'
For a list of the Main procedures type: Help8(); for help with a specific procedure type: Help6(ProcedureName); for example Help6(RandomMC);
For a list of the other procedures type: Help8a(); for help with a specific procedure type: Help6(ProcedureName); for example Help6(RandomMC);
> Orb(f(x), x, 1.1, 1, 10)
[1.134200000, 1.177557595, 1.230784174, 1.293599417, 1.364380503, 1.440144804, 1.517019883, 1.591082973, 1.659191120, 1.719435858]
> Orb(f(x), x, 2.1, 1, 10)
[2.080200000, 2.064263170, 2.051463614, 2.041198151, 2.032972506, 2.026385174, 2.021111813, 2.016891333, 2.013514031, 2.010811718]
> Orb(f(x), x, 3.1, 1, 10)
[3.146200000, 3.218129650, 3.336005586, 3.545734670, 3.975231162, 5.121472870, 10.58005045, 135.1920112, 472676.7887, 2.112113789 × 1016]

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$$f := x \mapsto \frac{1}{8} \cdot x^3 - \frac{5}{4} \cdot x^2 + \frac{39}{8} \cdot x - \frac{15}{4} \quad (2)$$

$$> f(2) \quad 2 \quad (3)$$

$$> f(3) \quad 3 \quad (4)$$

$$> f(5) \quad 5 \quad (5)$$

> # g is the derivative of f

$$> g(x) := \frac{3}{8}x^2 - \frac{10}{4}x + \frac{39}{8}$$

$$g := x \mapsto \frac{3}{8} \cdot x^2 - \frac{5}{2} \cdot x + \frac{39}{8} \quad (6)$$

$$> g(2) \text{ #Unstable} \quad \frac{11}{8} \quad (7)$$

$$> g(3) \text{ #Stable} \quad \frac{3}{4} \quad (8)$$

$$> g(5) \text{ #Unstable} \quad \frac{7}{4} \quad (9)$$

> Orb($f(x)$, x , 2.1, 1, 10)
[2.132625000, 2.173856283, 2.224596258, 2.285014295, 2.354172160, 2.429821087, 2.508556886, 2.586391682,
2.659565189, 2.725255038] **(10)**

> Orb($f(x)$, x , 3.1, 1, 10)
[3.073875000, 3.054774446, 3.040726348, 3.030345882, 3.022647786, 3.016923178, 3.012657185, 3.009473122,
3.007093730, 3.005314045] **(11)**

> Orb($f(x)$, x , 5.1, 1, 10);
[5.181375000, 5.338712630, 5.669308400, 6.488752370, 9.403012200, 35.49172768, 4183.144593, 9.128095365×10^9 ,
 $9.507153776 \times 10^{28}$, $1.074141680 \times 10^{86}$] **(12)**