Homework for Lecture 7 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Sept. 29, 2025.

Subject: hw7

with an attachment hw7FirstLast.pdf

1. For each of the following non-linear (quadratic) first-order-linear recurrences

(i) Determine all steady-states

(ii) decide which of them are stable

(iii) test it empirically, by taking a number close to each steady state, using either a calculator of Maple, find its orbit of length 10 and verify that for stable steady-states, the orbit is attracted to it, but for the unstable ones it runs away.

a. $x(n+1) \;=\; \frac{5}{2}\,x(n)\,(1-x(n))$ $s = \frac{r-1}{r} = \frac{3}{5}$ $f'(0) = r = 2.5$ - unstable $f'(s) = r(1-2s) = 2.5(1-1.2) \Rightarrow |-.5| < 1$ stable $x=0$ repelling $x = 3/5$ attracted

b. $x(n+1) \;=\; \frac{29}{10}\,x(n)\,(1-x(n))$ $s = \frac{19}{29} \approx .655$ $f'(0) = 2.9 \to$ unstable $f'(s) = 2-2.9 = |-.9| < 1 \to$ stable $x=0$ - repelling $x = \frac{19}{29}$ - attracted

c. $x(n+1) \;=\; \frac{31}{10}\,x(n)\,(1-x(n))$ $s = \frac{21}{31} \approx .677$ $f'(0) = 3.1 \to$ unstable $f'(s) = 2-3.1 = |-1.1| > 1 \to$ unstable $x=0$ - repelling $x = \frac{21}{31}$ - repelling

2.: For each of the following non-linear (cubic) first-order-linear recurrences

(i) Verify that the given points are steady-states

~~(ii) decide which ones ones are stable~~

(ii) decide which of them are stable

(iii) test it empirically, by taking a number close to each steady state, using either a calculator of Maple, find its orbit of length 10 and verify that for stable steady-states, the orbit is attracted to it, but for the unstable ones it runs away.

$f'(1) = 1+\frac{1}{5}(1-2)(1-3) \Rightarrow 1.4 \to$ unstable - repelling
$f'(2) = 1+\frac{1}{5}(2-1)(2-3) \Rightarrow .8 \to$ stable - attracted
$f'(3) = 1+1/5(3-1)(3-2) \Rightarrow 1.4 \to$ unstable - repelling

a. $x(n+1) \;=\; \frac{1}{5}\,x^3 - \frac{6}{5}x^2 + \frac{16}{5}x - \frac{6}{x}$ $\to \frac{1}{5}(x-1)(x-2)(x-3)$

Set of steady-states : $\{1, 2, 3\}$

b. $x(n+1) \;=\; \frac{1}{8}x^3 - \frac{5}{4}x^2 + \frac{39}{8}x - \frac{15}{4}$ $\frac{1}{8}(x-2)(x-3)(x-5)$

Set of steady-states : $\{2, 3, 5\}$

$f'(2) = 1+\frac{1}{8}(2-3)(2-5) = 1+\frac{3}{8} = 1.375 \to$ unstable - repelling
$f'(3) = 1+\frac{1}{8}(5-2)(5-3) = 1+\frac{6}{8} = 0.75 \to$ stable - attracted
$f'(5) = 1+\frac{1}{8}(5-2)(5-3) = 1+\frac{6}{8} = 1.75 \to$ unstable - repelling