Homework for Lecture 7 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as a .pdf file) to

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by 8:00pm Monday, Sept. 29, 2025.

Subject: hw7

with an attachment hw7FirstLast.pdf

1. For each of the following non-linear (quadratic) first-order-linear recurrences

(i) Determine all steady-states

(ii) decide which of them are stable

(iii) test it empirically, by taking a number close to each steady state, using either a calculator of Maple, find its orbit of length 10 and verify that for stable steady-states, the orbit is attracted to it, but for the unstable ones it runs away.

a. $x(n+1) \,=\, \frac{5}{2}\,x(n)\,(1-x(n))$.

b. $x(n+1) \,=\, \frac{29}{10}\,x(n)\,(1-x(n))$.

c. $x(n+1) \,=\, \frac{31}{10}\,x(n)\,(1-x(n))$.

2.: For each of the following non-linear (cubic) first-order-linear recurrences

(i) Verify that the given points are steady-states

(ii) decide which ones ones are stable

(ii) decide which of them are stable

(iii) test it empirically, by taking a number close to each steady state, using either a calculator of Maple, find its orbit of length 10 and verify that for stable steady-states, the orbit is attracted to it, but for the unstable ones it runs away.

a. $x(n+1) \,=\, \frac{1}{5}\,x^3 - \frac{6}{5}x^2 + \frac{16}{5}x - \frac{6}{x}$

Set of steady-states : $\{1, 2, 3\}$

b. $x(n+1) \,=\, \frac{1}{8}x^3 - \frac{5}{4}x^2 + \frac{39}{8}x - \frac{15}{4}$

Set of steady-states : $\{2, 3, 5\}$

1. For each of the following non-linear (quadratic) first-order-linear recurrences

(i) Determine all steady-states

(ii) decide which of them are stable

(iii) test it empirically, by taking a number close to each steady state, using either a calculator of Maple, find its orbit of length 10 and verify that for stable steady-states, the orbit is attracted to it, but for the unstable ones it runs away.

a. $x(n+1) = \frac{5}{2}\, x(n)\,(1 - x(n))$

(i) $f(x) = \frac{5}{2}x(1-x)$

$x = \frac{5}{2}x(1-x)$

$x - \frac{5}{2}x(1-x) = 0$

$x\left(1 - \frac{5}{2} + \frac{5}{2}x\right) = 0$

$x\left(-\frac{3}{2} + \frac{5}{2}x\right) = 0$

$x = 0, \frac{3}{5}$

(ii) $f'(x) = \frac{5}{2} - 5x$

$f'(0) = \frac{5}{2} \rightarrow$ unstable

$f'\left(\frac{3}{5}\right) = -\frac{1}{2} \rightarrow$ stable

(iii) $Orb(f, x, 0.1, 0, 10)$

[0.1, 0.2250000000, 0.4359375000, 0.6147399902, 0.5920868365, 0.6038000362, 0.5980638812, 0.6009586880, 0.5995183582, 0.6002402410, 0.5998797352]

diverges ✓

$Orb\left(f, x, \frac{3.1}{5}, 0, 10\right)$

[0.6200000000, 0.5890000000, 0.6051975000, 0.5973337150, 0.6013153698, 0.5993379895, 0.6003299095, 0.5998347732, 0.6000825452, 0.5999587102, 0.6000206408]

converges ✓

b. $x(n+1) = \frac{29}{10}\, x(n)\,(1 - x(n))$

(i) $f(x) = \frac{29}{10}x(1-x)$

$x = \frac{29}{10}x(1-x)$

$x\left(1 - \frac{29}{10}(1-x)\right) = 0$

$x\left(1 - \frac{29}{10} + \frac{29}{10}x\right) = 0$

$x\left(-\frac{19}{10} + \frac{29}{10}x\right) = 0$

$x = 0 \qquad -\frac{19}{10} + \frac{29}{10}x = 0$

$x = \frac{19}{29}$

(ii) $f'(x) = \frac{29}{10} - \frac{29}{5}x$

$f'(0) = \frac{29}{10} \rightarrow$ unstable

$f'\left(\frac{19}{29}\right) = \frac{29}{10} - \frac{29}{5}\left(\frac{19}{29}\right) = \frac{29}{10} - \frac{38}{10} = -\frac{9}{10} \rightarrow$ stable

(iii) $Orb(f, x, 0.1, 0, 10)$

[0.1, 0.2610000000, 0.5593491000, 0.7147852845, 0.5912151166, 0.7008714272, 0.6079869421, 0.6911825789, 0.6190027424, 0.6839312072, 0.6268910019]

diverges ✓

$Orb\left(f, x, \frac{19.1}{29}, 0, 10\right)$

[0.6586206897, 0.6520344827, 0.6579679967, 0.6526337248, 0.6574385437, 0.6531180045, 0.6570091424, 0.6535095747, 0.6566609504, 0.6538263051, 0.6563786569]

converges ✓

c. $x(n+1) = \frac{31}{10}\, x(n)\,(1 - x(n))$

(i) $f(x) = \frac{31}{10}x(1-x)$

$x = \frac{31}{10}x(1-x)$

$x\left(1 - \frac{31}{10} + \frac{31}{10}x\right) = 0$

$x\left(-\frac{21}{10} + \frac{31}{10}x\right) = 0$

$x = 0 \qquad -\frac{21}{10} + \frac{31}{10}x = 0$

$x = \frac{21}{31}$

(ii) $f'(x) = \frac{31}{10} - \frac{31}{5}x$

$f'(0) = \frac{31}{10} \rightarrow$ unstable

$f'\left(\frac{21}{31}\right) = \frac{31}{10} - \frac{42}{10} = -\frac{11}{10} \rightarrow$ unstable

(iii) $Orb(f, x, 0.1, 0, 10)$

[0.1, 0.1170000000, 0.1343043000, 0.1511466515, 0.1667917436, 0.1806639353, 0.1924318211, 0.2020223599, 0.2095721238, 0.2153471433, 0.2196645766]

diverges ✓

$Orb\left(f, x, \frac{21.1}{31}, 0, 10\right)$

[0.6806451613, 0.2825775234, 0.2635457067, 0.2523161774, 0.2452485412, 0.2406322025, 0.2375468493, 0.2354538468, 0.2340199326, 0.2330309848, 0.2323458084]

diverges ✓

2.: For each of the following non-linear (cubic) first-order-linear recurrences

(i) Verify that the given points are steady-states

(ii) decide which ones ones are stable

(ii) decide which of them are stable

(iii) test it empirically, by taking a number close to each steady state, using either a calculator of Maple, find its orbit of length 10 and verify that for stable steady-states, the orbit is attracted to it, but for the unstable ones it runs away.

a. $x(n+1) = \frac{1}{5}x^3 - \frac{6}{5}x^2 + \frac{16}{5}x - \frac{6}{x}$ $\to 5$

Set of steady-states : $\{1, 2, 3\}$

(i) ~~$\frac{1}{5}(1)^3 - \frac{6}{5}(1)^2 + \frac{16}{5}(1) - \frac{6}{1} = -\frac{19}{5} \neq 1$~~

$\frac{1}{5}(1)^3 - \frac{6}{5}(1)^2 + \frac{16}{5}(1) - \frac{6}{5} = 1$ ✓

~~$\frac{1}{5}(2)^3 - \frac{6}{5}(2)^2 + \frac{16}{5}(2) - \frac{6}{2} = \frac{8}{5} - \frac{24}{5} + \frac{32}{5} - 3 = \frac{1}{5} \neq 2$~~

$\frac{1}{5}(2)^3 - \frac{6}{5}(2)^2 + \frac{16}{5}(2) - \frac{6}{5} = \frac{8}{5} - \frac{24}{5} + \frac{32}{5} - \frac{6}{5} = 2$ ✓

~~$\frac{1}{5}(3)^3 - \frac{6}{5}(3)^2 + \frac{16}{5}(3) - \frac{6}{3} = \frac{27}{5} - \frac{54}{5} + \frac{48}{5} - 2 = \frac{11}{5} \neq 3$~~

$\frac{1}{5}(3)^3 - \frac{6}{5}(3)^2 + \frac{16}{5}(3) - \frac{6}{5} = \frac{27}{5} - \frac{54}{5} + \frac{48}{5} - \frac{6}{5} = 3$ ✓

(ii) $f'(x) = \frac{3}{5}x^2 - \frac{12}{5}x + \frac{16}{5}$

$f'(1) = \frac{3}{5} - \frac{12}{5} + \frac{16}{5} = \frac{7}{5} \to$ unstable

$f'(2) = \frac{12}{5} - \frac{24}{5} + \frac{16}{5} = \frac{4}{5} \to$ stable

$f'(3) = \frac{27}{5} - \frac{36}{5} + \frac{16}{5} = \frac{7}{5} \to$ unstable

(iii)

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Orb(f, x, 1.1, 0, 10)
[1.1, 1.134200000, 1.177557595, 1.230784174, 1.293599417, 1.364380503, 1.440144804, 1.517019883, 1.591082973, 1.659191120, 1.719435858]
```

diverges ✓

```
Orb(f, x, 2.1, 0, 10)
[2.1, 2.080200000, 2.064263170, 2.051463614, 2.041198151, 2.032972506, 2.026385174, 2.021111813, 2.016891333, 2.013514031, 2.010811718]
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converges ✓

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Orb(f, x, 3.1, 0, 10)
[3.1, 3.146200000, 3.218129650, 3.336005586, 3.545734670, 3.975231162, 5.121472870, 10.58005045, 135.1920112, 472676.7887, 2.112113789 × 10^16]
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diverges ✓

b. $x(n+1) = \frac{1}{8}x^3 - \frac{5}{4}x^2 + \frac{39}{8}x - \frac{15}{4}$

Set of steady-states : $\{2, 3, 5\}$

(i) $\frac{1}{8}(2)^3 - \frac{5}{4}(2)^2 + \frac{39}{8}(2) - \frac{15}{4} = 1 - 5 + \frac{39}{4} - \frac{15}{4} = 2$ ✓

$\frac{1}{8}(3)^3 - \frac{5}{4}(3)^2 + \frac{39}{8}(3) - \frac{15}{4} = \frac{27}{8} - \frac{45}{4} + \frac{117}{8} - \frac{15}{4} = 3$ ✓

$\frac{1}{8}(5)^3 - \frac{5}{4}(5)^2 + \frac{39}{8}(5) - \frac{15}{4} = \frac{125}{8} - \frac{125}{4} + \frac{195}{8} - \frac{15}{4} = 5$ ✓

(ii) $f'(x) = \frac{3}{8}x^2 - \frac{5}{2}x + \frac{39}{8}$

$f'(2) = \frac{12}{8} - 5 + \frac{39}{8} = \frac{10}{8} \rightarrow$ unstable

$f'(3) = \frac{27}{8} - \frac{15}{2} + \frac{39}{8} = \frac{6}{8} \rightarrow$ stable

$f'(5) = \frac{75}{8} - \frac{25}{2} + \frac{39}{8} = \frac{14}{8} \rightarrow$ unstable

(iii)

Orb(f, x, 2.1, 0, 10)

[2.1, 2.132625000, 2.173856283, 2.224596258, 2.285014295, 2.354172160, 2.429821087, 2.508556886, 2.586391682, 2.659565189, 2.725255038]

diverges ✓

Orb(f, x, 3.1, 0, 10)

[3.1, 3.073875000, 3.054774446, 3.040726348, 3.030345882, 3.022647786, 3.016923178, 3.012657185, 3.009473122, 3.007093730, 3.005314045]

converges ✓

Orb(f, x, 5.1, 0, 10)

[5.1, 5.181375000, 5.338712630, 5.669308400, 6.488752370, 9.403012200, 35.49172768, 4183.144593, $9.128095365 \times 10^{9}$, $9.507153776 \times 10^{28}$, $1.074141680 \times 10^{86}$]

diverges ✓