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Homework 7 - Dynamic Models in Bio

① (a) $x(n+1) = \frac{5}{2}x(n)(1-x(n))$

(i) $x = \frac{5}{2}x(1-x) \Rightarrow x - \frac{5}{2}x(1-x) = 0$

$$x(1 - \frac{5}{2}(1-x)) = 0$$

$$x(1 - \frac{5}{2} + \frac{5}{2}x) = 0$$

$$x(-\frac{3}{2} + \frac{5}{2}x) = 0$$

$$x = 0 \quad \text{and} \quad x = \frac{\frac{3}{2}}{\frac{5}{2}} = \frac{3}{5}$$

$x = 0, \frac{3}{5}$ steady states

(ii) $f'(x) = \frac{5}{2} - 5x$

$$f'(0) = \frac{5}{2} > 1 \quad \text{so unstable steady state}$$

$$f'(\frac{3}{5}) = \frac{5}{2} - 5(\frac{3}{5}) = -\frac{1}{2} \Rightarrow \left| -\frac{1}{2} \right| < 1 \quad \text{so stable steady state}$$

(iii) using maple you see that 0 is unstable (runs away) and $\frac{3}{5}$ is stable (attracted to)

$$\lim x(n) = \frac{3}{5}$$

$$(b) \quad x(n+1) = \frac{29}{10} x(n) (1 - x(n))$$

$$(i) \quad x = \frac{29}{10} x (1 - x) \Rightarrow x - \frac{29}{10} x (1 - x) = 0$$

$$x \left(1 - \frac{29}{10} + \frac{29}{10} x \right) = 0$$

$$x \left(-\frac{19}{10} + \frac{29}{10} x \right) = 0$$

$$x = 0 \quad x = \frac{\frac{19}{20}}{\frac{29}{10}} = \frac{19}{29}$$

$$(ii) \quad f(x) = \frac{29}{10} - \frac{29}{10} x^2$$

$$f'(x) = \frac{29}{10} - \frac{58}{10} x$$

$$f'(0) = \frac{29}{10} > 1 \text{ so unstable}$$

$$f'\left(\frac{19}{29}\right) = \frac{29}{10} - \frac{58}{10} \left(\frac{19}{29}\right) = -\frac{9}{10} \Rightarrow \left| -\frac{9}{10} \right| < 1 \Rightarrow \text{stable}$$

(iii) USED MAPLE to see that it runs away from 0 & attracts $\frac{19}{29}$

$$(c) \quad x(n+1) = \frac{31}{10} x(n) (1 - x(n))$$

$$(i) \quad x - \frac{31}{10} x (1 - x) = 0$$

$$x \left(1 - \frac{31}{10} + \frac{31}{10} x \right) = 0$$

$$x \left(-\frac{21}{10} + \frac{31}{10} x \right) = 0$$

$$x = 0 \quad x = \frac{\frac{21}{10}}{\frac{31}{10}} = \frac{21}{31}$$

$$(ii) \quad f'(x) = \frac{31}{10} - \frac{62}{10} x$$

$$f'(0) = \frac{31}{10} > 1 \Rightarrow \text{unstable}$$

$$f'\left(\frac{21}{31}\right) = 1 \Leftarrow \text{semi-stable}$$

(iii) USED MAPLE

$$(2) (a) x(n+1) = \frac{1}{5}x^3 - \frac{6}{5}x^2 + \frac{16}{5}x - \frac{6}{5}$$

$$(i) \frac{1}{5}(1)^3 - \frac{6}{5}(1)^2 + \frac{16}{5}(1) - \frac{6}{5} = 1 \checkmark$$

$$\frac{1}{5}(2)^3 - \frac{6}{5}(2)^2 + \frac{16}{5}(2) - \frac{6}{5} = 2 \checkmark$$

$$\frac{1}{5}(3)^3 - \frac{6}{5}(3)^2 + \frac{16}{5}(3) - \frac{6}{5} = 3 \checkmark$$

$$(ii) f'(x) = \frac{3}{5}x^2 - \frac{12}{5}x + \frac{16}{5}$$

$$f'(1) = \frac{3}{5}(1)^2 - \frac{12}{5}(1) + \frac{16}{5} = \frac{7}{5} > 1 \Rightarrow \text{unstable}$$

$$f'(2) = \frac{3}{5}(2)^2 - \frac{12}{5}(2) + \frac{16}{5} = \frac{4}{5} < 1 \Rightarrow \text{stable}$$

$$f'(3) = \frac{3}{5}(3)^2 - \frac{12}{5}(3) + \frac{16}{5} = \frac{7}{5} > 1 \Rightarrow \text{unstable}$$

(iii) * USED MAPLE *

showed that 1 & 3 are unstable steady states
and 2 is a stable steady state

$$(b) \quad x(n+1) = \frac{1}{8}x^3 - \frac{5}{4}x^2 + \frac{39}{8}x - \frac{15}{4}$$

$$\frac{1}{8}(2)^3 - \frac{5}{4}(2)^2 + \frac{39}{8}(2) - \frac{15}{4} = 2 \quad \checkmark \quad \text{steady state}$$

$$\frac{1}{8}(3)^3 - \frac{5}{4}(3)^2 + \frac{39}{8}(3) - \frac{15}{4} = 3 \quad \checkmark \quad \text{steady state}$$

$$\frac{1}{8}(5)^3 - \frac{5}{4}(5)^2 + \frac{39}{8}(5) - \frac{15}{4} = 5 \quad \checkmark \quad \text{steady state}$$

$$(ii) \quad f'(x) = \frac{3}{8}x^2 - \frac{10}{4}x + \frac{39}{8}$$

$$f'(2) = \frac{3}{8}(2)^2 - \frac{10}{4}(2) + \frac{39}{8} = \frac{11}{8} > 1 \quad \text{unstable}$$

$$f'(3) = \frac{3}{8}(3)^2 - \frac{10}{4}(3) + \frac{39}{8} = \frac{3}{4} < 1 \quad \text{stable}$$

$$f'(5) = \frac{3}{8}(5)^2 - \frac{10}{4}(5) + \frac{39}{8} = \frac{7}{4} > 1 \quad \text{unstable}$$

(iii) used maple $\Rightarrow$ shows that 2 and 5 are unstable
and 3 is a stable steady state