

Homework for Lecture 6 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file or .txt file) to

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by 8:00pm Monday, Sept. 29, 2025.

Subject: hw6

with an attachment hw6FirstLast.pdf and/or hw6FirstLast.txt

Also please indicate (EITHER way) whether it is OK to post *yes, can post ✓*

1. Using procedure RandomLeslieMatrix from today's Maple code, generate ten random Leslie matrices for five age groups, where By finding the eigenvalues decide which population is doomed to extinction, and which ones will grow out of hand.

2. In a mini-internet there are four websites, let's call them S_1, S_2, S_3, S_4 .

If a random surfer is currently at a given site, his or her

- Probability of staying at site S_1 is 0.5
- Probability of staying at site S_2 is 0.4
- Probability of staying at site S_3 is 0.3
- Probability of staying at site S_4 is 0.2

We also assume that, **for each of the four sites**, the probabilities of moving to another site are the same. In other words, for example, the probability of moving from site 1 to site 2, probability of moving from site 1 to site 3, probability of moving from site 1 to site 4 are the same, and similarly for the departures from each site. (Of course, it is not possible that the probability of moving from any site to a different site are all equal to each other).

a: Set up the **transition matrix $\mathbf{P}$** for this Markov chain.

b. What fraction of the surfers stay in each of the above web-sites? Use this to determine the **page-ranks** of these four web-sites in this mini-internet. In other words rank them according to "popularity".

1. Using procedure RandomLeslieMatrix from today's Maple code, generate ten random Leslie matrices for five age groups, where By finding the eigenvalues decide which population is doomed to extinction, and which ones will grow out of hand.

$L := \text{RandomLeslieMatrix}(5, 1)$

$$L := \begin{bmatrix} 0. & 0.1100000000 & 0.3700000000 & 0.7500000000 & 0.0400000000 & 0.9100000000 \\ 0.2200000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4000000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5800000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9300000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9800000000 & 0 \end{bmatrix}$$

$L := \text{Matrix}(L);$
 $E := \text{Eigenvalues}(L);$

$$L := \begin{bmatrix} 0. & 0.1100000000 & 0.3700000000 & 0.7500000000 & 0.0400000000 & 0.9100000000 \\ 0.2200000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4000000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5800000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9300000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9800000000 & 0 \end{bmatrix}$$

$$E := \begin{bmatrix} 0.645727028043661 + 0. I \\ -0.609699095311144 + 0. I \\ -0.276445454424168 + 0.517307193804216 I \\ -0.276445454424168 - 0.517307193804216 I \\ 0.258431488057909 + 0.495732166915444 I \\ 0.258431488057909 - 0.495732166915444 I \end{bmatrix}$$

$\lambda < 1$
extinct

$L := \text{RandomLeslieMatrix}(5, 2)$

$$L := \begin{bmatrix} 0. & 1.070000000 & 0.1100000000 & 0.3000000000 & 1.340000000 & 0.3200000000 \\ 0.4000000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2400000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8000000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9600000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1100000000 & 0 \end{bmatrix}$$

$L := \text{Matrix}(L);$
 $E := \text{Eigenvalues}(L);$

$$L := \begin{bmatrix} 0. & 1.070000000 & 0.1100000000 & 0.3000000000 & 1.340000000 & 0.3200000000 \\ 0.4000000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2400000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8000000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9600000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1100000000 & 0 \end{bmatrix}$$

$$E := \begin{bmatrix} 0.814918583940791 + 0. I \\ -0.598968441622930 + 0.240047295795121 I \\ -0.598968441622930 - 0.240047295795121 I \\ 0.204725010714994 + 0.497441742365054 I \\ 0.204725010714994 - 0.497441742365054 I \\ -0.0264317221249186 + 0. I \end{bmatrix}$$

extinct

$L := \text{RandomLeslieMatrix}(5, 3)$

$$L := \begin{bmatrix} 0. & 0.2300000000 & 1.690000000 & 1.280000000 & 0.5800000000 & 0.6700000000 \\ 0.8100000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6500000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6900000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0200000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3600000000 & 0 \end{bmatrix}$$

$L := \text{Matrix}(L);$
 $E := \text{Eigenvalues}(L);$

$$L := \begin{bmatrix} 0. & 0.2300000000 & 1.690000000 & 1.280000000 & 0.5800000000 & 0.6700000000 \\ 0.8100000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6500000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6900000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0200000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3600000000 & 0 \end{bmatrix}$$

$$E := \begin{bmatrix} 1.14802808409301 + 0. I \\ -0.321963630620282 + 0.835881784472724 I \\ -0.321963630620282 - 0.835881784472724 I \\ -0.502274263454169 + 0. I \\ -0.000913279699137605 + 0.0615387252552747 I \\ -0.000913279699137605 - 0.0615387252552747 I \end{bmatrix}$$

$\lambda > 1$
grow

$L := \text{RandomLeslieMatrix}(5, 4)$

$$L := \begin{bmatrix} 0. & 1.890000000 & 0.8400000000 & 2.240000000 & 2.090000000 & 2.870000000 \\ 0.5400000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6700000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5900000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6600000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1200000000 & 0 \end{bmatrix}$$

$L := \text{Matrix}(L);$
 $E := \text{Eigenvalues}(L);$

$$L := \begin{bmatrix} 0. & 1.890000000 & 0.8400000000 & 2.240000000 & 2.090000000 & 2.870000000 \\ 0.5400000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6700000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5900000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6600000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1200000000 & 0 \end{bmatrix}$$

$$E := \begin{bmatrix} 1.29953093007819 + 0. I \\ -0.973564060158794 + 0. I \\ 0.133493900527636 + 0.631982939621355 I \\ 0.133493900527636 - 0.631982939621355 I \\ -0.296477335487336 + 0.0634098154443766 I \\ -0.296477335487336 - 0.0634098154443766 I \end{bmatrix}$$

grow

$L := \text{RandomLeslieMatrix}(5, 6)$

$$L := \begin{bmatrix} 0. & 4.940000000 & 5.850000000 & 4.970000000 & 1.600000000 & 2.260000000 \\ 0.09000000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5300000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.03000000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9800000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6900000000 & 0 \end{bmatrix}$$

$L := \text{Matrix}(L);$
 $E := \text{Eigenvalues}(L);$

$$E := \begin{bmatrix} 0.881576107048634 + 0. I \\ -0.424485264125576 + 0.359683803112545 I \\ -0.424485264125576 - 0.359683803112545 I \\ -0.219760553881877 + 0. I \\ 0.0935774875421876 + 0.166456042827316 I \\ 0.0935774875421876 - 0.166456042827316 I \end{bmatrix}$$

extinct

$L := \text{RandomLeslieMatrix}(5, 7)$

$$L := \begin{bmatrix} 0. & 3.810000000 & 1.310000000 & 0.7300000000 & 2.160000000 & 6.770000000 \\ 0.6000000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9400000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5200000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1600000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2900000000 & 0 \end{bmatrix}$$

$L := \text{Matrix}(L);$
 $E := \text{Eigenvalues}(L);$

$$E := \begin{bmatrix} 1.68322288423183 + 0. I \\ -1.35819408588296 + 0. I \\ -0.366264471239819 + 0.302928198731050 I \\ -0.366264471239819 - 0.302928198731050 I \\ 0.203750072065384 + 0.369950655679068 I \\ 0.203750072065384 - 0.369950655679068 I \end{bmatrix}$$

grow

$L := \text{RandomLeslieMatrix}(5, 8)$

$$L := \begin{bmatrix} 0. & 5.630000000 & 7.450000000 & 4.510000000 & 4.240000000 & 0.4900000000 \\ 0.6000000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6900000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3300000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3000000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.01000000000 & 0 \end{bmatrix}$$

$L := \text{Matrix}(L);$
 $E := \text{Eigenvalues}(L);$

$$E := \begin{bmatrix} 2.21612074702156 + 0. I \\ -1.03304348262387 + 0.297098313150988 I \\ -1.03304348262387 - 0.297098313150988 I \\ -0.0744366873946027 + 0.249095374719526 I \\ -0.0744366873946027 - 0.249095374719526 I \\ -0.00116040698461280 + 0. I \end{bmatrix}$$

grow

$L := \text{RandomLeslieMatrix}(5, 9)$

$$L := \begin{bmatrix} 0. & 3.610000000 & 2.510000000 & 0.6400000000 & 5.550000000 & 1.850000000 \\ 0.5200000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6200000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4600000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7600000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.09000000000 & 0 \end{bmatrix}$$

$L := \text{Matrix}(L);$
 $E := \text{Eigenvalues}(L);$

$$E := \begin{bmatrix} 1.60405070362430 + 0. I \\ -0.986301739033048 + 0.315614255593735 I \\ -0.986301739033048 - 0.315614255593735 I \\ 0.199328727750280 + 0.568025685688416 I \\ 0.199328727750280 - 0.568025685688416 I \\ -0.0301046810587590 + 0. I \end{bmatrix}$$

grow

$L := \text{RandomLeslieMatrix}(5, 10)$

$$L := \begin{bmatrix} 0. & 4.980000000 & 4.370000000 & 4.210000000 & 8.560000000 & 7.530000000 \\ 0.5000000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3700000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7600000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9500000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.08000000000 & 0 \end{bmatrix}$$

$L := \text{Matrix}(L);$
 $E := \text{Eigenvalues}(L);$

$$E := \begin{bmatrix} 1.81943153170144 + 0. I \\ -1.32257418951176 + 0. I \\ -0.873517262535158 + 0. I \\ 0.224786706555885 + 0.688818008466289 I \\ 0.224786706555885 - 0.688818008466289 I \\ -0.0729134927662959 + 0. I \end{bmatrix}$$

grow

2. In a mini-internet there are four websites, let's call them S_1, S_2, S_3, S_4 .

If a random surfer is currently at a given site, his or her

- Probability of staying at site S_1 is 0.5
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We also assume that, **for each of the four sites**, the probabilities of moving to another site are the same. In other words, for example, the probability of moving from site 1 to site 2, probability of moving from site 1 to site 3, probability of moving from site 1 to site 4 are the same, and similarly for the departures from each site. (Of course, it is not possible that the probability of moving from any site to a different site are all equal to each other).

a: Set up the **transition matrix P** for this Markov chain.

b. What fraction of the surfers stay in each of the above web-sites? Use this to determine the **page-ranks** of these four web-sites in this mini-internet. In other words rank them according to "popularity".

a)
$$P = \begin{bmatrix} .5 & .5/3 & .5/3 & .5/3 \\ .2 & .4 & .2 & .2 \\ .7/3 & .1/3 & .3 & .7/3 \\ .8/3 & .8/3 & .8/3 & .2 \end{bmatrix} \approx \begin{bmatrix} .5 & .17 & .17 & .17 \\ .2 & .4 & .2 & .2 \\ .23 & .23 & .3 & .23 \\ .27 & .27 & .27 & .2 \end{bmatrix}$$

b)
$$Z := \text{matrix}\left(\left[\left[.5, \frac{.5}{3}, \frac{.5}{3}, \frac{.5}{3}\right], \left[.2, .4, .2, .2\right], \left[\frac{.7}{3}, \frac{.7}{3}, .3, \frac{.7}{3}\right], \left[\frac{.8}{3}, \frac{.8}{3}, \frac{.8}{3}, .2\right]\right]\right):$$

$$Z := \begin{bmatrix} 0.5 & 0.1666666667 & 0.1666666667 & 0.1666666667 \\ 0.2 & 0.4 & 0.2 & 0.2 \\ 0.2333333333 & 0.2333333333 & 0.3 & 0.2333333333 \\ 0.2666666667 & 0.2666666667 & 0.2666666667 & 0.2 \end{bmatrix}$$

```
PR := PageRank(Z, 4):
print("S1", PR[1]);
print("S2", PR[2]);
print("S3", PR[3]);
print("S4", PR[4])
```

"S1", 0.3151969980

"S2", 0.2626641649

"S3", 0.2251407128

"S4", 0.1969981237