PROBLEM #1

> $A := RandomLeslieMatrix(5, 3)$

$$A := \begin{bmatrix} 0. & 2.470000000 & 2.390000000 & 0.06000000000 & 1. & 0.3800000000 \\ 0.6900000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2700000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9600000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1700000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9000000000 & 0 \end{bmatrix}$$

$\to 1.426 > 1 \Rightarrow$ grow

> $eigenvalues(A)$

$1.42643209122304, -1.14416383779423, 0.137144538850915 + 0.234825166413316\,I, 0.137144538850915 - 0.234825166413316\,I, -0.278278665565323 + 0.0933513832671046\,I, -0.278278665565323 - 0.0933513832671046\,I$

> $A := RandomLeslieMatrix(5, 9)$

$$A := \begin{bmatrix} 0. & 7.630000000 & 1.620000000 & 2.740000000 & 1.800000000 & 3.120000000 \\ 0.4300000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8300000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2500000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9000000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9300000000 & 0 \end{bmatrix}$$

> $eigenvalues(A)$

$1.91988406990316, -1.73992473473079, 0.288367788489631 + 0.418329564453317\,I, 0.288367788489631 - 0.418329564453317\,I, -0.378347456075818 + 0.356440200678463\,I, -0.378347456075818 - 0.356440200678463\,I$

$\to 1.919 > 1 \Rightarrow$ grow

5 year olds

> $A := RandomLeslieMatrix(8, 8)$

$$A := \begin{bmatrix} 0. & 4.770000000 & 3.980000000 & 0.5000000000 & 3.790000000 & 1.750000000 & 0.08000000000 & 3.620000000 & 1.740000000 \\ 0.4400000000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.09000000000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7700000000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5900000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1600000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.01000000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.7000000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7700000000 & 0 \end{bmatrix}$$

$\to >1 \Rightarrow$ grow

> $eigenvalues(A)$

1.49417832713211, −1.40389908640232, 0.145282246376903 + 0.276991351331257 I, 0.145282246376903 − 0.276991351331257 I, −0.317104176838881, 0.0866930838375885 + 0.110453658038783 I, 0.0866930838375885 − 0.110453658038783 I, −0.118562862159946 + 0.0835693928140673 I, −0.118562862159946 − 0.0835693928140673 I

> $A := RandomLeslieMatrix(8, 6)$

$$A := \begin{bmatrix} 0. & 0.3900000000 & 5.830000000 & 4.510000000 & 5.090000000 & 2.390000000 & 3.670000000 & 0.5100000000 & 5.650000000 \\ 0.1200000000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1900000000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6300000000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4000000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9000000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.03000000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4900000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4900000000 & 0 \end{bmatrix}$$

$\to <1 \Rightarrow$ extinct

> $eigenvalues(A)$

0.707991761300449, −0.244578156652914 + 0.416371893480738 I, −0.244578156652914 − 0.416371893480738 I, 0.0835009708424963 + 0.437272541238224 I, 0.0835009708424963 − 0.437272541238224 I, −0.336165425548460 + 0.0779612308142258 I, −0.336165425548460 − 0.0779612308142258 I, 0.143246730708653 + 0.183011176049959 I, 0.143246730708653 − 0.183011176049959 I

8 year olds

> $A := RandomLeslieMatrix(4, 7)$

$$A := \begin{bmatrix} 0. & 3.570000000 & 2.020000000 & 4.740000000 & 4.580000000 \\ 0.2700000000 & 0 & 0 & 0 & 0 \\ 0 & 0.9800000000 & 0 & 0 & 0 \\ 0 & 0 & 0.7200000000 & 0 & 0 \\ 0 & 0 & 0 & 0.02000000000 & 0 \end{bmatrix}$$

$\rightarrow > 1 \Rightarrow$ grow

> $eigenvalues(A)$

1.36066123230473, -1.10038654147885, $-0.120365850470777 + 0.762826105392818\,I$, $-0.120365850470777 - 0.762826105392818\,I$, -0.0195429898843234

> $A := RandomLeslieMatrix(4, 3)$

$$A := \begin{bmatrix} 0. & 0.7300000000 & 2.970000000 & 2.310000000 & 2.600000000 \\ 0.4400000000 & 0 & 0 & 0 & 0 \\ 0 & 0.1300000000 & 0 & 0 & 0 \\ 0 & 0 & 0.1900000000 & 0 & 0 \\ 0 & 0 & 0 & 0.1000000000 & 0 \end{bmatrix}$$

$\rightarrow < 1 \Rightarrow$ extinct

> $eigenvalues(A)$

0.768704635559919, $-0.311164447457194 + 0.240947387740656\,I$, $-0.311164447457194 - 0.240947387740656\,I$, $-0.0731878703227664 + 0.135563794688442\,I$, $-0.0731878703227664 - 0.135563794688442\,I$

4 year olds

> $A := RandomLeslieMatrix(6, 7)$

$$A := \begin{bmatrix} 0. & 1.430000000 & 5.210000000 & 3.080000000 & 2.560000000 & 4.090000000 & 3.610000000 \\ 0.9000000000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1800000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4300000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5500000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4000000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1700000000 & 0 \end{bmatrix}$$

$>1 \Rightarrow$ grow

> $eigenvalues(A)$

1.42642192918418, −0.632425420332096, −0.480584769477574 + 0.291163823401511 I, −0.480584769477574 − 0.291163823401511 I, 0.184013220957379 + 0.361300396860029 I, 0.184013220957379 − 0.361300396860029 I, −0.200853411811697

> $A := RandomLeslieMatrix(6, 9)$

$$A := \begin{bmatrix} 0. & 4.540000000 & 3.080000000 & 3.370000000 & 7.270000000 & 5.050000000 & 2.130000000 \\ 0.09000000000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6800000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8300000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6300000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7000000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3600000000 & 0 \end{bmatrix}$$

$>1 \Rightarrow$ grow

> $eigenvalues(A)$

1.03560339600821, 0.262552878835872 + 0.628392787497260 I, 0.262552878835872 − 0.628392787497260 I, −0.435552415069117 + 0.331298514981135 I, −0.435552415069117 − 0.331298514981135 I, −0.344802161770862 + 0.0230097093417849 I, −0.344802161770862 − 0.0230097093417849 I

6 year olds

> $A := RandomLeslieMatrix(9, 2)$

$$A := \begin{bmatrix} 0. & 1.640000000 & 0.1000000000 & 0.4000000000 & 0.6600000000 & 0.8700000000 & 1.440000000 & 0.9800000000 & 1.710000000 & 0.5300000000 \\ 0.6100000000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4700000000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2800000000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7500000000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.03000000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.05000000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1100000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3700000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7500000000 & 0 \end{bmatrix}$$

$<1 \Rightarrow$ extinct

> $eigenvalues(A)$

-0.981877457268863, 1.04564563558620, -0.316880063957532, $0.144300448138403 + 0.312565187090414\,I$, $0.144300448138403 - 0.312565187090414\,I$, $-0.0558534496896269 + 0.118966413614958\,I$, $-0.0558534496896269 - 0.118966413614958\,I$, $0.0993939897248477 + 0.0895469586908151\,I$, $0.0993939897248477 - 0.0895469586908151\,I$, -0.122570090707052

> $A := RandomLeslieMatrix(9, 8)$

$$A := \begin{bmatrix} 0. & 7.720000000 & 2.190000000 & 1.500000000 & 5.520000000 & 3.140000000 & 4.770000000 & 2.260000000 & 0.1100000000 & 0.3000000000 \\ 0.06000000000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3200000000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4000000000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2400000000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8000000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.9600000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1100000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2300000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4100000000 & 0 \end{bmatrix}$$

$<1 \Rightarrow$ extinct

> $eigenvalues(A)$

0.773259086128470, $-0.542959219269472 + 0.0378772422484758\,I$, $-0.542959219269472 - 0.0378772422484758\,I$, $0.300601583274160 + 0.306292460514251\,I$, $0.300601583274160 - 0.306292460514251\,I$, $-0.117845585278564 + 0.375217605666856\,I$, $-0.117845585278564 - 0.375217605666856\,I$, -0.108530703184990, $0.0278390298021375 + 0.0737893939740629\,I$, $0.0278390298021375 - 0.0737893939740629\,I$

9 year olds

2. In a mini-internet there are four websites, let's call them S_1, S_2, S_3, S_4.

If a random surfer is currently at a given site, his or her

- Probability of staying at site S_1 is 0.5
- Probability of staying at site S_2 is 0.4
- Probability of staying at site S_3 is 0.3
- Probability of staying at site S_4 is 0.2

We also assume that, **for each of the four sites** , the probabilities of moving to another site are the same. In other words, for example, the probability of moving from site 1 to site 2, probability of moving from site 1 to site 3, probability of moving from site 1 to site 4 are the same, and similarly for the departures from each site. (Of course, it is not possible that the probability of moving from any site to a different site are all equal to each other).

a: Set up the **transition matrix P** for this Markov chain.

b. What fraction of the surfers stay in each of the above web-sites? Use this to determine the the **page-ranks** of these four web-sites in this mini-internet. In other words rank them according to "popularity".

a) $P = \begin{bmatrix} 0.5 & \frac{0.5}{3} & \frac{0.5}{3} & \frac{0.5}{3} \\ 0.2 & 0.4 & 0.2 & 0.2 \\ \frac{0.7}{3} & \frac{0.7}{3} & 0.3 & \frac{0.7}{3} \\ \frac{0.8}{3} & \frac{0.8}{3} & \frac{0.8}{3} & 0.2 \end{bmatrix}$

```
> P := Matrix([[0.5, 0.5/3, 0.5/3, 0.5/3], [0.2, 0.4, 0.2, 0.2], [0.7/3, 0.7/3, 0.3, 0.7/3], [0.8/3, 0.8/3, 0.8/3, 0.2]]);
```

$$P := \begin{bmatrix} 0.5 & 0.1666666667 & 0.1666666667 & 0.1666666667 \\ 0.2 & 0.4 & 0.2 & 0.2 \\ 0.2333333333 & 0.2333333333 & 0.3 & 0.2333333333 \\ 0.2666666667 & 0.2666666667 & 0.2666666667 & 0.2 \end{bmatrix} \quad (9)$$

```
> add(P[1,j], j = 1..4);
```

1.000000000 (10)

```
> add(P[2,j], j = 1..4);
```

1.0 (11)

```
> add(P[3,j], j = 1..4);
```

0.9999999999 (12)

```
> add(P[4,j], j = 1..4);
```

1.000000000 (13)

```
> read(`DMB6.txt`);
```

For a list of the Main procedures type: Help6(); for help with a specific procedure type: Help6(ProcedureName); for example Help6(RandomMC);
For a list of the other procedures type: Help6a(); for help with a specific procedure type: Help6(ProcedureName); for example Help6(RandomMC); (19)

```
> with(LinearAlgebra) :

A := matrix([
 [0.5, 0.1667, 0.1667, 0.1667],
 [0.2, 0.4, 0.2, 0.2],
 [0.2333, 0.2333, 0.3, 0.2333],
 [0.2667, 0.2667, 0.2667, 0.2]
 ]) :

Page_Rank := PageRank(A, 4) :
print("S1: ", Page_Rank[1]);
print("S2:", Page_Rank[2]);
print("S3:", Page_Rank[3]);
print("S4: ", Page_Rank[4]);
```

"S1: ", 0.3243932851
"S2:", 0.2703431932
"S3:", 0.2317322007
"S4: ", 0.2027528776 (20)