MATH336 - Homework #6

1.

$$L1 = \begin{bmatrix} 0 & 0.72 & 1.16 & 2.39 & 1.3 \\ 0.73 & 0 & 0 & 0 & 0 \\ 0 & 0.85 & 0 & 0 & 0 \\ 0 & 0 & 0.41 & 0 & 0 \\ 0 & 0 & 0 & 0.04 & 0 \end{bmatrix}$$

|leading eigenvalue| = 1.23
population will blow up

$$L2 = \begin{bmatrix} 0 & 1.72 & 2.69 & 1.47 & 2.66 \\ 0.15 & 0 & 0 & 0 & 0 \\ 0 & 0.64 & 0 & 0 & 0 \\ 0 & 0 & 0.09 & 0 & 0 \\ 0 & 0 & 0 & 0.12 & 0 \end{bmatrix}$$

|leading eigenvalue| = 0.78
population will go extinct

$$L3 = \begin{bmatrix} 0 & 2.53 & 2.56 & 2.47 & 1.13 \\ 0.9 & 0 & 0 & 0 & 0 \\ 0 & 0.18 & 0 & 0 & 0 \\ 0 & 0 & 0.43 & 0 & 0 \\ 0 & 0 & 0 & 0.55 & 0 \end{bmatrix}$$

|leading eigenvalue| = 1.61
population will blow up

$$L4 = \begin{bmatrix} 0 & 1.09 & 0.4 & 0.17 & 2.15 \\ 0.34 & 0 & 0 & 0 & 0 \\ 0 & 0.85 & 0 & 0 & 0 \\ 0 & 0 & 0.09 & 0 & 0 \\ 0 & 0 & 0 & 0.66 & 0 \end{bmatrix}$$

|leading eigenvalue| = 0.77
population will go extinct

$$L5 = \begin{bmatrix} 0 & 1.25 & 2.92 & 1.64 & 2.66 \\ 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0.66 & 0 & 0 & 0 \\ 0 & 0 & 0.87 & 0 & 0 \\ 0 & 0 & 0 & 0.14 & 0 \end{bmatrix}$$

|leading eigenvalue| = 1.2
population will blow up

$$L6 = \begin{bmatrix} 0 & 0.61 & 0.47 & 2.84 & 0.03 \\ 0.05 & 0 & 0 & 0 & 0 \\ 0 & 0.11 & 0 & 0 & 0 \\ 0 & 0 & 0.37 & 0 & 0 \\ 0 & 0 & 0 & 0.75 & 0 \end{bmatrix}$$

|leading eigenvalue| = 0.31
population will go extinct

$$L7 = \begin{bmatrix} 0 & 2.6 & 2.14 & 1.5 & 0.4 \\ 0.58 & 0 & 0 & 0 & 0 \\ 0 & 0.93 & 0 & 0 & 0 \\ 0 & 0 & 0.98 & 0 & 0 \\ 0 & 0 & 0 & 0.11 & 0 \end{bmatrix}$$

|leading eigenvalue| = 1.6
population will blow up

$$L8 = \begin{bmatrix} 0 & 0.3 & 1.34 & 2.88 & 2.96 \\ 0.24 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0.96 & 0 & 0 \\ 0 & 0 & 0 & 0.11 & 0 \end{bmatrix}$$

|leading eigenvalue| = 0.97
population will go extinct

$$L9 = \begin{bmatrix} 0 & 0.23 & 1.69 & 1.28 & 0.58 \\ 0.67 & 0 & 0 & 0 & 0 \\ 0 & 0.81 & 0 & 0 & 0 \\ 0 & 0 & 0.65 & 0 & 0 \\ 0 & 0 & 0 & 0.69 & 0 \end{bmatrix}$$

|leading eigenvalue| = 1.16
population will blow up

$$L10 = \begin{bmatrix} 0 & 1.3 & 0.36 & 1.89 & 0.84 \\ 0.96 & 0 & 0 & 0 & 0 \\ 0 & 0.94 & 0 & 0 & 0 \\ 0 & 0 & 0.31 & 0 & 0 \\ 0 & 0 & 0 & 0.81 & 0 \end{bmatrix}$$

|leading eigenvalue| = 1.35
population will blow up

switched fractions to decimals of 5 sigfigs before computing in Maple

2. $T = \begin{bmatrix} 0.5 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{5} & 0.4 & \frac{1}{5} & \frac{1}{5} \\ \frac{7}{30} & \frac{7}{30} & 0.3 & \frac{7}{30} \\ \frac{4}{15} & \frac{4}{15} & \frac{4}{15} & 0.2 \end{bmatrix}$

Transition matrix is a stochastic matrix, so all entries within a row should add up to 1; if all outgoing probabilities are equal, then their probability is $[1 - t_{ij} \text{ (where } i=j)] / 3$.

$S = [0.311, 0.259, 0.222, 0.144]$

Stationary distribution — approx entries

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