Homework for Lecture 6 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file or .txt file) to

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by 8:00pm Monday, Sept. 29, 2025.

Subject: hw6

with an attachment hw6FirstLast.pdf and/or hw6FirstLast.txt

Also please indicate (EITHER way) whether it is OK to post

1. Using procedure RandomLeslieMatrix from today's Maple code, generate ten random Leslie matrices for five age groups, where By finding the eigenvalues decide which population is doomed to extinction, and which ones will grow out of hand.

2. In a mini-internet there are four websites, let's call them S_1, S_2, S_3, S_4.

If a random surfer is currently at a given site, his or her

• Probability of staying at site S_1 is 0.5

• Probability of staying at site S_2 is 0.4

• Probability of staying at site S_3 is 0.3

• Probability of staying at site S_4 is 0.2

We also assume that, **for each of the four sites** , the probabilities of moving to another site are the same. In other words, for example, the probability of moving from site 1 to site 2, probability of moving from site 1 to site 3, probability of moving from site 1 to site 4 are the same, and similarly for the departures from each site. (Of course, it is not possible that the probability of moving from any site to a different site are all equal to each other).

a: Set up the **transition matrix P** for this Markov chain.

b. What fraction of the surfers stay in each of the above web-sites? Use this to determine the the **page-ranks** of these four web-sites in this mini-internet. In other words rank them according to "popularity".

Problem 1

Please don't post!

```
read "C:/Users/rlade/Downloads/DMB FILES/DMB6(1).txt" :
```

For a list of the Main procedures type: Help6(); for help with a specific procedure type: Help6 (ProcedureName); for example Help6(RandomMC);

For a list of the other procedures type: Help6a(); for help with a specific procedure type: Help6 (ProcedureName); for example Help6(RandomMC); **(1)**

```
Help6( );
```

The available procedures are: Leslie, RandomLeslieMatrix, RandomMC **(2)**

```
Help6a( );
```

The other procedures are: raFV, raSV **(3)**

```
with(LinearAlgebra);

for i from 1 to 10 do

   L := Matrix(RandomLeslieMatrix(5, 3));
   print("Matrix ", i);
   print(L);


    L := Matrix(L);

   # Compute all eigenvalues
    E := Eigenvalues(L) :
   print("Eigenvalues: ", E);


   for j from 1 to nops(E) do
      if abs(E[j]) > 1 then
         print("Eigenvalue ", E[j], ": Growth");
      elif abs(E[j]) = 1 then
         print("Eigenvalue ", E[j], ": Steady / Neutral");
      else
         print("Eigenvalue ", E[j], ": Decay / Extinction");
      end if:
   od;

   print("-------------------------------------------------");
od:
```

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm,

BilinearForm, *CARE*, *CharacteristicMatrix*, *CharacteristicPolynomial*, *Column*, *ColumnDimension*, *ColumnOperation*, *ColumnSpace*, *CompanionMatrix*, *CompressedSparseForm*, *ConditionNumber*, *ConstantMatrix*, *ConstantVector*, *Copy*, *CreatePermutation*, *CrossProduct*, *DARE*, *DeleteColumn*, *DeleteRow*, *Determinant*, *Diagonal*, *DiagonalMatrix*, *Dimension*, *Dimensions*, *DotProduct*, *EigenConditionNumbers*, *Eigenvalues*, *Eigenvectors*, *Equal*, *ForwardSubstitute*, *FrobeniusForm*, *FromCompressedSparseForm*, *FromSplitForm*, *GaussianElimination*, *GenerateEquations*, *GenerateMatrix*, *Generic*, *GetResultDataType*, *GetResultShape*, *GivensRotationMatrix*, *GramSchmidt*, *HankelMatrix*, *HermiteForm*, *HermitianTranspose*, *HessenbergForm*, *HilbertMatrix*, *HouseholderMatrix*, *IdentityMatrix*, *IntersectionBasis*, *IsDefinite*, *IsOrthogonal*, *IsSimilar*, *IsUnitary*, *JordanBlockMatrix*, *JordanForm*, *KroneckerProduct*, *LA_Main*, *LUDecomposition*, *LeastSquares*, *LinearSolve*, *LyapunovSolve*, *Map*, *Map2*, *MatrixAdd*, *MatrixExponential*, *MatrixFunction*, *MatrixInverse*, *MatrixMatrixMultiply*, *MatrixNorm*, *MatrixPower*, *MatrixScalarMultiply*, *MatrixVectorMultiply*, *MinimalPolynomial*, *Minor*, *Modular*, *Multiply*, *NoUserValue*, *Norm*, *Normalize*, *NullSpace*, *OuterProductMatrix*, *Permanent*, *Pivot*, *PopovForm*, *ProjectionMatrix*, *QRDecomposition*, *RandomMatrix*, *RandomVector*, *Rank*, *RationalCanonicalForm*, *ReducedRowEchelonForm*, *Row*, *RowDimension*, *RowOperation*, *RowSpace*, *ScalarMatrix*, *ScalarMultiply*, *ScalarVector*, *SchurForm*, *SingularValues*, *SmithForm*, *SplitForm*, *StronglyConnectedBlocks*, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *SylvesterSolve*, *ToeplitzMatrix*, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

"Matrix ", 1

0.	1.570000000	2.280000000	1.080000000	0.190000 ⋯
0.8400000000	0	0	0	0 ⋯
0	0.7500000000	0	0	0 ⋯
0	0	0.6000000000	0	0 ⋯
0	0	0	0.4200000000	0 ⋯
0	0	0	0	0.530000 ⋯

"Eigenvalues: ", $\begin{bmatrix} 1.55967753554090 + 0.\,I \\ -0.582421555654570 + 0.613820796883094\,I \\ -0.582421555654570 - 0.613820796883094\,I \\ -0.681009408365963 + 0.\,I \\ 0.143087492067103 + 0.335107210856301\,I \\ 0.143087492067103 - 0.335107210856301\,I \end{bmatrix}$

"Eigenvalue ", 1.55967753554090 + 0. I, ": Growth"

"Eigenvalue ", −0.582421555654570 + 0.613820796883094 I, ": Decay / Extinction"

"Eigenvalue ", −0.582421555654570 − 0.613820796883094 I, ": Decay / Extinction"

"--"

"Matrix ", 2

$\begin{bmatrix} 0. & 2.300000000 & 1.020000000 & 2.900000000 & 2.8800\cdots \\ 0.01000000000 & 0 & 0 & 0 & 0\cdots \\ 0 & 0.3500000000 & 0 & 0 & 0\cdots \\ 0 & 0 & 0.9000000000 & 0 & 0\cdots \\ 0 & 0 & 0 & 0.3600000000 & 0\cdots \\ 0 & 0 & 0 & 0 & 0.32000\cdots \end{bmatrix}$

"Eigenvalues: ", $\begin{bmatrix} 0.402968751090705 + 0.\,I \\ 0.0902738184182320 + 0.314742469679109\,I \\ 0.0902738184182320 - 0.314742469679109\,I \\ -0.283328501156767 + 0.\,I \\ -0.150093943385201 + 0.173043806230372\,I \\ -0.150093943385201 - 0.173043806230372\,I \end{bmatrix}$

"Eigenvalue ", 0.402968751090705 + 0. I, ": Decay / Extinction"

"Eigenvalue ", 0.0902738184182320 + 0.314742469679109 I, ": Decay / Extinction"

"Eigenvalue ", 0.0902738184182320 − 0.314742469679109 I, ": Decay / Extinction"

"--"

"Matrix ", 3

$$\begin{bmatrix} 0. & 1.810000000 & 0.1900000000 & 0.08000000000 & 2.6800\cdots \\ 0.9600000000 & 0 & 0 & 0 & 0\ \cdots \\ 0 & 0.4200000000 & 0 & 0 & 0\ \cdots \\ 0 & 0 & 0.5100000000 & 0 & 0\ \cdots \\ 0 & 0 & 0 & 0.5700000000 & 0\ \cdots \\ 0 & 0 & 0 & 0 & 0.4200\cdots \end{bmatrix}$$

"Eigenvalues: ",

$$\begin{bmatrix} 1.39369542281273 + 0.\ I \\ -1.25119020527254 + 0.\ I \\ 0.313462686145368 + 0.505971833796784\ I \\ 0.313462686145368 - 0.505971833796784\ I \\ -0.384715294915461 + 0.0811735660251069\ I \\ -0.384715294915461 - 0.0811735660251069\ I \end{bmatrix}$$

"Eigenvalue ", 1.39369542281273 + 0. I, ": Growth"

"Eigenvalue ", −1.25119020527254 + 0. I, ": Growth"

"Eigenvalue ", 0.313462686145368 + 0.505971833796784 I, ": Decay / Extinction"

"--"

"Matrix ", 4

$$\begin{bmatrix} 0. & 2.600000000 & 1.810000000 & 1.160000000 & 2.4200\cdots \\ 0.5500000000 & 0 & 0 & 0 & 0\ \cdots \\ 0 & 0.6600000000 & 0 & 0 & 0\ \cdots \\ 0 & 0 & 0.4300000000 & 0 & 0\ \cdots \\ 0 & 0 & 0 & 0.1900000000 & 0\ \cdots \\ 0 & 0 & 0 & 0 & 0.1600\cdots \end{bmatrix}$$

"Eigenvalues: ", $\begin{bmatrix} 1.41743679810204 + 0.\,I \\ -0.914051296755387 + 0.\,I \\ -0.484811052683438 + 0.\,I \\ -0.00138639077932019 + 0.331596247988476\,I \\ -0.00138639077932019 - 0.331596247988476\,I \\ -0.0158016671045799 + 0.\,I \end{bmatrix}$

"Eigenvalue ", 1.41743679810204 + 0. I, ": Growth"

"Eigenvalue ", −0.914051296755387 + 0. I, ": Decay / Extinction"

"Eigenvalue ", −0.484811052683438 + 0. I, ": Decay / Extinction"

"--"

"Matrix ", 5

$$\begin{bmatrix} 0. & 1.200000000 & 0.1100000000 & 0.1000000000 & 2.19000\cdots \\ 0.9700000000 & 0 & 0 & 0 & 0\ \cdots \\ 0 & 0.9500000000 & 0 & 0 & 0\ \cdots \\ 0 & 0 & 0.3800000000 & 0 & 0\ \cdots \\ 0 & 0 & 0 & 0.2600000000 & 0\ \cdots \\ 0 & 0 & 0 & 0 & 0.020000\cdots \end{bmatrix}$$

"Eigenvalues: ", $\begin{bmatrix} 1.18168280915308 + 0.\,I \\ -0.914412362057826 + 0.\,I \\ -0.679741334057977 + 0.\,I \\ 0.209572618775422 + 0.476685652498352\,I \\ 0.209572618775422 - 0.476685652498352\,I \\ -0.00667435058812334 + 0.\,I \end{bmatrix}$

"Eigenvalue ", 1.18168280915308 + 0. I, ": Growth"

"Eigenvalue ", −0.914412362057826 + 0. I, ": Decay / Extinction"

"Eigenvalue ", −0.679741334057977 + 0. I, ": Decay / Extinction"

"--"

"Matrix ", 6

$$\begin{bmatrix} 0. & 0.7000000000 & 2.230000000 & 0.1400000000 & 0.510000\cdots \\ 0.7100000000 & 0 & 0 & 0 & 0\ \cdots \\ 0 & 0.4900000000 & 0 & 0 & 0\ \cdots \\ 0 & 0 & 0.1600000000 & 0 & 0\ \cdots \\ 0 & 0 & 0 & 0.1400000000 & 0\ \cdots \\ 0 & 0 & 0 & 0 & 0.700000\cdots \end{bmatrix}$$

"Eigenvalues: ",

$$\begin{bmatrix} 1.10076806890485 + 0.\ I \\ -0.546822675791172 + 0.635090454281919\ I \\ -0.546822675791172 - 0.635090454281919\ I \\ 0.0382949330850970 + 0.0974203501182237\ I \\ 0.0382949330850970 - 0.0974203501182237\ I \\ -0.0837125834926963 + 0.\ I \end{bmatrix}$$

"Eigenvalue ", 1.10076806890485 + 0. I, ": Growth"

"Eigenvalue ", −0.546822675791172 + 0.635090454281919 I, ": Decay / Extinction"

"Eigenvalue ", −0.546822675791172 − 0.635090454281919 I, ": Decay / Extinction"

"--"

"Matrix ", 7

$$\begin{bmatrix} 0. & 0.2900000000 & 2.480000000 & 0.2300000000 & 0.030000\cdots \\ 0.1200000000 & 0 & 0 & 0 & 0\ \cdots \\ 0 & 0.5400000000 & 0 & 0 & 0\ \cdots \\ 0 & 0 & 0.3700000000 & 0 & 0\ \cdots \\ 0 & 0 & 0 & 0.6300000000 & 0\ \cdots \\ 0 & 0 & 0 & 0 & 0.360000\cdots \end{bmatrix}$$

"Eigenvalues: ", $\begin{bmatrix} 0.628558903414246 + 0.\,I \\ -0.304303762636784 + 0.511347690743400\,I \\ -0.304303762636784 - 0.511347690743400\,I \\ -0.408269630311977 + 0.\,I \\ 0.194159126085649 + 0.338979761072601\,I \\ 0.194159126085649 - 0.338979761072601\,I \end{bmatrix}$

"Eigenvalue ", 0.628558903414246 + 0. I, ": Decay / Extinction"

"Eigenvalue ", −0.304303762636784 + 0.511347690743400 I, ": Decay / Extinction"

"Eigenvalue ", −0.304303762636784 − 0.511347690743400 I, ": Decay / Extinction"

"--"

"Matrix ", 8

$\begin{bmatrix} 0. & 2.030000000 & 2.880000000 & 2.270000000 & 1.18000(\cdots \\ 0.2500000000 & 0 & 0 & 0 & 0 \cdots \\ 0 & 0.3200000000 & 0 & 0 & 0 \cdots \\ 0 & 0 & 0.2300000000 & 0 & 0 \cdots \\ 0 & 0 & 0 & 0.5800000000 & 0 \cdots \\ 0 & 0 & 0 & 0 & 0.230000(\cdots \end{bmatrix}$

"Eigenvalues: ", $\begin{bmatrix} 0.910435711407931 + 0.\,I \\ -0.422869114283382 + 0.189542761660198\,I \\ -0.422869114283382 - 0.189542761660198\,I \\ 0.00880877443505050 + 0.230876370272772\,I \\ 0.00880877443505050 - 0.230876370272772\,I \\ -0.0823150317112680 + 0.\,I \end{bmatrix}$

"Eigenvalue ", 0.910435711407931 + 0. I, ": Decay / Extinction"

"Eigenvalue ", −0.422869114283382 + 0.189542761660198 I, ": Decay / Extinction"

"Eigenvalue ", −0.422869114283382 − 0.189542761660198 I, ": Decay / Extinction"

"--"

"Matrix ", 9

$$\begin{bmatrix} 0. & 2.910000000 & 0.4300000000 & 0.1600000000 & 2.820000 \cdots \\ 0.2400000000 & 0 & 0 & 0 & 0 \cdots \\ 0 & 0.4100000000 & 0 & 0 & 0 \cdots \\ 0 & 0 & 0.8400000000 & 0 & 0 \cdots \\ 0 & 0 & 0 & 0.1000000000 & 0 \cdots \\ 0 & 0 & 0 & 0 & 0.450000 \cdots \end{bmatrix}$$

"Eigenvalues: ",

$$\begin{bmatrix} 0.897887985661773 + 0.\,I \\ -0.799873708702995 + 0.\,I \\ 0.191913337479253 + 0.306556132578338\,I \\ 0.191913337479253 - 0.306556132578338\,I \\ -0.240920475958642 + 0.159678222535815\,I \\ -0.240920475958642 - 0.159678222535815\,I \end{bmatrix}$$

"Eigenvalue ", 0.897887985661773 + 0. I, ": Decay / Extinction"

"Eigenvalue ", −0.799873708702995 + 0. I, ": Decay / Extinction"

"Eigenvalue ", 0.191913337479253 + 0.306556132578338 I, ": Decay / Extinction"

"--"

"Matrix ", 10

$$\begin{bmatrix} 0. & 1.310000000 & 1.370000000 & 1.220000000 & 1.600000 \cdots \\ 0.4000000000 & 0 & 0 & 0 & 0 \cdots \\ 0 & 0.8600000000 & 0 & 0 & 0 \cdots \\ 0 & 0 & 0.1300000000 & 0 & 0 \cdots \\ 0 & 0 & 0 & 0.1200000000 & 0 \cdots \\ 0 & 0 & 0 & 0 & 0.420000 \cdots \end{bmatrix}$$

$$\text{"Eigenvalues: "},\ \begin{bmatrix} 1.02421374693565 + 0.\,I \\ -0.452189060962356 + 0.455322750513699\,I \\ -0.452189060962356 - 0.455322750513699\,I \\ -0.272884438190063 + 0.\,I \\ 0.0765244065895598 + 0.201871202829131\,I \\ 0.0765244065895598 - 0.201871202829131\,I \end{bmatrix}$$

"Eigenvalue ", 1.02421374693565 + 0. I, ": Growth"

"Eigenvalue ", −0.452189060962356 + 0.455322750513699 I, ": Decay / Extinction"

"Eigenvalue ", −0.452189060962356 − 0.455322750513699 I, ": Decay / Extinction"

"--" **(4)**

Problem 2

```
> read "C:/Users/rlade/Downloads/DMB FILES/DMB6(1).txt" :

  with(LinearAlgebra);
```

For a list of the Main procedures type: Help6(); for help with a specific procedure type: Help6 (ProcedureName); for example Help6(RandomMC);

For a list of the other procedures type: Help6a(); for help with a specific procedure type: Help6 (ProcedureName); for example Help6(RandomMC);

(1)

[*&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip*]

```
> #Prob that they stay on that webiste
> stay := [0.5, 0.4, 0.3, 0.2];
```

$$stay := [0.5, 0.4, 0.3, 0.2] \quad \textbf{(2)}$$

```
> #number of websites
> n := 4;
```

```
n := nops(stay) :
```

$$n := 4 \tag{3}$$

```
> P := matrix(n, n, 0) :
>
for i from 1 to n do
  for j from 1 to n do
    if i = j then
      P[i, j] := stay[i] :
    else
      P[i, j] := (1 - stay[i]) / (n - 1) :
    end if:
  od:
od:

> # Print the transition matrix
print("Transition Matrix P:");
print(P);
```

"Transition Matrix P:"

$$\begin{bmatrix} 0.5 & 0.1666666667 & 0.1666666667 & 0.1666666667 \\ 0.2000000000 & 0.4 & 0.2000000000 & 0.2000000000 \\ 0.2333333333 & 0.2333333333 & 0.3 & 0.2333333333 \\ 0.2666666667 & 0.2666666667 & 0.2666666667 & 0.2 \end{bmatrix} \tag{4}$$

```
> #PageRank stuff
> PageRank := proc(A, n)
    local B, i;
    B := evalm(A^1000) :          # Raise matrix to large power
    [seq(B[1, i], i = 1 ..n)];    # Return first row as steady-state vector
  end:

  ranks := PageRank(P, n);

  print("Steady-state distribution (Page-Ranks)");
  print(ranks);
```

$$ranks := [0.3151969980, 0.2626641649, 0.2251407128, 0.1969981237]$$

"Steady-state distribution (Page-Ranks)"

$$[0.3151969980, 0.2626641649, 0.2251407128, 0.1969981237] \tag{5}$$

```
> # Pair each website with its page-rank
  ranking := [seq([cat("S", i), ranks[i]], i = 1 ..n)];
```

```
# Sort descending by page-rank
ranking_sorted := sort(ranking, (a, b) → b[2] < a[2]);

print("Websites ranked by popularity:");
print(ranking_sorted);
```

```
ranking := [["S1", 0.3151969980], ["S2", 0.2626641649], ["S3", 0.2251407128], ["S4",
    0.1969981237]]
ranking_sorted := [["S1", 0.3151969980], ["S2", 0.2626641649], ["S3", 0.2251407128], ["S4",
    0.1969981237]]
                        "Websites ranked by popularity:"
  [["S1", 0.3151969980], ["S2", 0.2626641649], ["S3", 0.2251407128], ["S4", 0.1969981237]]   (6)
>
```