Homework for Lecture 5 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Sept. 22, 2025.

Subject: hw5

with an attachment hw5FirstLast.pdf

1. a. Convert the recurrence

$$6a(n-1) + a(n+3) + 5a(n+1) = 0$$
,

into standard form where a(n + 4) is expressed in terms of a(n + 3), a(n + 2), a(n + 1), a(n).

b.

Abbreviating

$$\mathbf{a}(n) = \begin{bmatrix} a(n+3) \\ a(n+2) \\ a(n+1) \\ a(n) \end{bmatrix} .$$

Find the 4×4 matrix, let's call it A such that

$$\mathbf{a}(n+1) = A\mathbf{a}(n)$$

- **c** Assuming that a(0) = 0, a(1) = 2, a(2) = 3, a(3) = 4 Find a(5) in two ways:
- (i) Straight from the standard form, by first finding a(4), and then a(5)
- (ii) Using the matrix version by first finding A^2 and then multiplying it by the column vector $[4,3,2,0]^T$ and extracting the first component.
- 2. In a certain species only one-year-olds, two-year-olds, three-year-olds, and four-year-olds are fertile. We have
- zero-year-olds can't have babies
- Every 1-year-old female makes 1.5 babies on average
- Every 2-year-old female makes 0.9 babies on average
- Every 3-year-old female makes 0.5 babies on average

• Every 4-year-old female makes 0.3 babies on average
We also know
\bullet The probability that a zero-year-old will survive the year is 0.8
\bullet The probability that a one-year-old will survive the year is 0.7
\bullet The probability that a two-year-old will survive the year is 0.6
\bullet The probability that a three-year-old will survive the year is 0.6
a. Set up the Leslie matrix
b. If right now there are 100 zero-year-olds, 90 one-year-olds, 80 two-year-olds, 70 three-year-olds, and 60 four-year-old, what is the expected number of 3-year-olds after two years?

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$$6a(n-1) + a(n+3) + 5a(n+1) = 0$$
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$$a(n+1) = Aa(n)
A(N+1) = A(n+1) = A(n+1)
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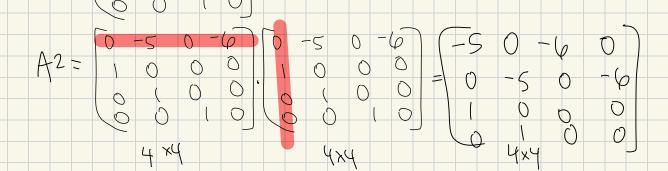
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(i)
$$\alpha(n+q) = 0 \cdot \alpha(n+3) - 5\alpha(n+2) + 0 \cdot \alpha(n+1) - 4\alpha(n+1)$$

 $\alpha(q) = -5\alpha(2) - 4\alpha(0) = -5(3) - 4\alpha(0) = -15$
 $\alpha(q) = -5\alpha(2) - 4\alpha(0) = -5(4) - 4\alpha(2) = -20 - 12 = -32$
(i) $\alpha(q) = -5\alpha(q) - 4\alpha(q) = -5(q) - 4\alpha(q) = -20 - 12 = -32$



2. In a certain species only one-year-olds, two-year-olds, three-year-olds, and four-year-olds are fertile. We have	
• zero-year-olds can't have babies $f_0 = 0$	
• Every 1-year-old female makes 1.5 babies on average $f_1 = 1.5$	
• Every 2-year-old female makes 0.9 babies on average $ + $	
• Every 3-year-old female makes 0.5 babies on average	
	f = expected # balbes
• Every 4-year-old female makes 0.3 babies on average We also know	ri= probability that i yr old new
• The probability that a zero-year-old will survive the year is 0.8	to find fa
• The probability that a one-year-old will survive the year is 0.7 $\sqrt{}$	L= 0 P1,00
• The probability that a two-year-old will survive the year is 0.6 $\varphi_2 = 0.6$	0 0 12 0
• The probability that a three-year-old will survive the year is 0.6 $\psi_{\zeta} = 0.6$	0 · PA-1 0
a. Set up the Leslie matrix	
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L=\\\\0\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\	
0.7000	
0 0 0 0	
b. If right now there are 100 zero-year-olds, 90 one-year-olds, 80 two-year-olds, 70 three-year-olds, and 60 four-year-old, what is the expected number of 3-year-olds after two years?	
and of four year ord, what is the expected number of 5 year order two years.	
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100 15 9 5 3 1100	213-3 D yr old
90 190 190	208 = 5ψ 2
80 2 W OLD 0.7 0 0 0 1 10 1	37.8
	25.8 4
(10 5 9V 010 (0 0 0 .60) [00]	
37.8 expecte	00 2 00 00
# 3 4V OLD OFFRY 2415=!	ed 3 yr olds