HW5

1a) $6a(n) + a(n+4) + 5a(n+2) = 0$

$a(n+4) = 0 \cdot a(n+3) - 5 \cdot a(n+2) + 0 \cdot a(n+1) - 6 \cdot a(n)$

1b) $A = \begin{bmatrix} 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

1c) i) $a(4) = -5\,a(2) + -6\,a(0) = -5(3) + -6(0) = -15$

$a(5) = -5\,a(3) + -6\,a(1) = -5(4) + -6(2) = -32$

ii) $A^2 = \begin{bmatrix} -5 & 0 & -6 & 0 \\ 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ $\vec{a}(0) = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 0 \end{bmatrix}$ $\vec{a}(1) = \begin{bmatrix} -5 \cdot 4 - 6 \cdot 2 \\ -5 \cdot 3 - 6 \cdot 0 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} -32 \\ -15 \\ 4 \\ 3 \end{bmatrix}$

$a(5) = -32$

2)a) $L = \begin{bmatrix} 0 & 1.5 & 0.9 & 0.5 & 0.3 \\ 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0 \end{bmatrix}$

b) $\vec{n}(2) = L^2 \vec{n}(0)$

$= \begin{bmatrix} 0.8(1.5) & 0.7(0.9) & 0.5(0.6) & 0.6(0.3) & 0 \\ 0 & 0.8(1.5) & 0.8(0.9) & 0.8(0.5) & 0.8(0.3) \\ 0.8(0.7) & 0 & 0 & 0 & 0 \\ 0 & 0.7(0.6) & 0 & 0 & 0 \\ 0 & 0 & 0.6(0.6) & 0 & 0 \end{bmatrix} \begin{bmatrix} 100 \\ 90 \\ 80 \\ 70 \\ 60 \end{bmatrix}$

$$= \begin{bmatrix} 1.2 & 0.63 & 0.3 & 0.18 & 0 \\ 0 & 1.2 & 0.72 & 0.4 & 0.24 \\ 0.56 & 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 0 & 0 \\ 0 & 0 & 0.36 & 0 & 0 \end{bmatrix} \begin{bmatrix} 100 \\ 90 \\ 80 \\ 70 \\ 60 \end{bmatrix}$$

$$= \begin{bmatrix} 120 + 56.7 + 24 + 12.6 + 0 \\ 0 + 108 + 57.6 + 28 + 14.4 \\ 56 \\ 37.8 \\ 28.8 \end{bmatrix} = \begin{bmatrix} 213.3 \\ 208 \\ 56 \\ 37.8 \\ 28.8 \end{bmatrix} = \begin{bmatrix} n_0(2) \\ n_1(2) \\ n_2(2) \\ n_3(2) \\ n_4(2) \end{bmatrix}$$

$n_3(2) = 37.8$ three-year-olds expected after two years