

Vector recurrence / Leslie matrix

Homework 5 on Lecture 5

1. $6a(n-1) + a(n+3) + 5a(n+1) = 0,$

→ a. Convert into standard form where $a(n+4)$ is expressed in terms of $a(n+3), a(n+2), a(n+1), a(n)$.

i. replace all n with $n+1$.

$$6a(n) + a(n+4) + 5a(n+2) = 0$$

ii. $a(n+4) = -5a(n+2) - 6a(n)$

b. $A = \begin{bmatrix} 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$$a(0) = 0, a(1) = 2, a(2) = 3, a(3) = 4$$

c. i. Use Standard Form to find

$$a(n+3) \quad (b = A)$$

6a → $a(4) = -5a(2) - 6a(0)$

$$a(4) = -5$$

$$a(5) = -5a(3) - 6a(1)$$

$$= -5(4) - 6(2) = -32$$

answer!

ii. Use matrix

$$A^2 = \begin{bmatrix} -5 & 0 & -6 & 0 \\ 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -32 \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}$$

a. Leslie $A = \begin{bmatrix} 0 & 1.5 & 0.9 & 0.5 & 0.3 \\ 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0 \end{bmatrix}$

b.

$$A^2 \cdot \begin{bmatrix} 100 \\ 90 \\ 80 \\ 70 \end{bmatrix} + \text{extract}[1,3]$$

used Maple

→ 44.1 (statistic expectation)