**Homework for Lecture 5 of Dr. Z.'s Dynamical Models in Biology class**

Email the answers (as .pdf file) to

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by 8:00pm Monday, Sept. 22, 2025.

Subject: hw5

with an attachment hw5FirstLast.pdf

**1. a.** Convert the recurrence

$$6\,a(n-1) \,+\, a(n+3) \,+\, 5\,a(n+1) = 0 \quad ,$$

$a(n+4) = -6a(n) - 5a(n+2)$

into **standard form** where $a(n+4)$ is expressed in terms of $a(n+3), a(n+2), a(n+1), a(n)$.

**b.**

Abbreviating

$$\mathbf{a}(n) = \begin{bmatrix} a(n+3) \\ a(n+2) \\ a(n+1) \\ a(n) \end{bmatrix} \longrightarrow \begin{bmatrix} a(n+4) \\ a(n+3) \\ a(n+2) \\ a(n+1) \end{bmatrix} \; .$$

$a(n+4) = -6a(n) - 5a(n+2)$

$$A = \begin{bmatrix} 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Find the $4 \times 4$ matrix, let's call it $A$ such that

$$\mathbf{a}(n+1) \;=\; A\mathbf{a}(n)$$

**c** Assuming that $a(0) = 0$, $a(1) = 2$, $a(2) = 3$,$a(3) = 4$ Find $a(5)$ in two ways:

(i) Straight from the standard form, by first finding $a(4)$, and then $a(5)$

$a(4) = -6a(0) - 5a(2) = -6\cdot 0 - 5\cdot 3 = -15$

$a(5) = -6a(1) - 5a(3) = -6\cdot 2 - 5\cdot 4 = -12-20$
$= -32$

$a(5) = -32$

(ii) Using the matrix version by first finding $A^2$ and then multiplying it by the column vector $[4, 3, 2, 0]^T$ and extracting the first component.

$$A(1) = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 0+(-5)(3)+0+0 \\ 4 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} -15 \\ 4 \\ 3 \\ 2 \end{bmatrix} \quad a(2) = \begin{bmatrix} -15 \\ 4 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 0+(-5)(4)+0+(-6)(2) \\ -15 \\ 4 \\ 3 \end{bmatrix} \begin{bmatrix} -20-12 \\ -15 \\ 4 \\ 3 \end{bmatrix} \begin{bmatrix} -32 \\ -15 \\ 4 \\ 3 \end{bmatrix} \quad a(5) = -32 \checkmark$$

2. In a certain species only one-year-olds, two-year-olds, three-year-olds, and four-year-olds are fertile. We have

- zero-year-olds can't have babies
- Every 1-year-old female makes 1.5 babies on average
- Every 2-year-old female makes 0.9 babies on average
- Every 3-year-old female makes 0.5 babies on average

- Every 4-year-old female makes 0.3 babies on average

We also know

- The probability that a zero-year-old will survive the year is 0.8
- The probability that a one-year-old will survive the year is 0.7
- The probability that a two-year-old will survive the year is 0.6
- The probability that a three-year-old will survive the year is 0.6

**a.** Set up the **Leslie matrix**

$$\begin{bmatrix} 0 & 1.5 & .9 & .5 & .3 \\ .8 & 0 & 0 & 0 & 0 \\ 0 & .7 & 0 & 0 & 0 \\ 0 & 0 & .6 & 0 & 0 \\ 0 & 0 & 0 & .6 & 0 \end{bmatrix}$$

**b.** If right now there are 100 zero-year-olds, 90 one-year-olds, 80 two-year-olds, 70 three-year-olds, and 60 four-year-old, what is the expected number of 3-year-olds after two years?

$$x(0) = \begin{bmatrix} 100 \\ 90 \\ 80 \\ 70 \\ 60 \end{bmatrix}$$

$0(100) + 1.5(90) + .9(80) + .5(70) + .3(60)$

$= 135 + 72 + 35 + 18$

$= 260$

$.8(100) = .8$

$.7(90) = 63$

$.6(8) = 48$

$.6(70) = 42$

$$x(1) = \begin{bmatrix} 260 \\ 80 \\ 63 \\ 48 \\ 42 \end{bmatrix}$$

$1.5(80) + .9(63) + .5(48) + .3(42)$

$= 120 + 56.7 + 24 + 12.6$

$= 213.3$

$.8(260) = 208$

$.7(80) = 56$

$.6(63) = 37.8$

$.6(48) = 28.8$

$$x(2) = \begin{bmatrix} 213.3 \\ 208 \\ 56 \\ 37.8 \\ 28.8 \end{bmatrix} \leftarrow \boxed{37.8}$$