

Katryn Keating

Homework #5

1.) $a(n+4) = 0 \cdot a(n+3) - 5a(n+2) + 0 \cdot a(n+1) - 6a(n)$
original form: $6a(n-1) + a(n+3) + 5a(n+1) = 0$

(C) $a(0)=0, a(1)=2, a(2)=3, a(3)=4$

i) $a(4) = -5a(2) - 6a(0)$
 $= -5(3) - 6(0) = -15$

$a(4) = -15$

$a(5) = -5a(3) - 6a(1)$
 $= -5(4) - 6(2) = -32$

$a(5) = -32$

ii) $a(n+1) = A \cdot \vec{a}(n) = A^n \cdot \vec{a}(0)$

$A = \begin{bmatrix} 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \vec{a}(0) = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -6 & 0 \\ 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

$A^2 \cdot \vec{a}(0) = \begin{bmatrix} -5 & 0 & -6 & 0 \\ 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -32 \\ -15 \\ 4 \\ 3 \end{bmatrix} = \vec{a}(5)$

$\vec{a}(5) = \begin{bmatrix} a(5) \\ a(4) \\ a(3) \\ a(2) \end{bmatrix} = \begin{bmatrix} -32 \\ -15 \\ 4 \\ 3 \end{bmatrix} \Rightarrow a(5) = -32, a(4) = -15$

$$2) L = \begin{bmatrix} 0.5 & 1.5 & 0.9 & 0.5 & 0.3 \\ 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0 \end{bmatrix}$$

6. $\vec{n}(0) = \begin{bmatrix} 60 \\ 76 \\ 86 \\ 90 \\ 100 \end{bmatrix}$ There will be approx.
202-3 year olds after 2 years.

$$L^2 = \begin{bmatrix} 0.5 & 1.5 & 0.9 & 0.5 & 0.3 \\ 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0 \end{bmatrix} \begin{bmatrix} 0.5 & 1.5 & 0.9 & 0.5 & 0.3 \\ 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1.2 & 0.63 & 0.3 & 0.18 & 0 \\ 0 & 1.2 & 0.72 & 0.4 & 0.24 \\ 0.42 & 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 0 & 0 \\ 0 & 0 & 0.32 & 0 & 0 \end{bmatrix}$$

$$L^2 \vec{n}(0) = \begin{bmatrix} 1.2 & 0.63 & 0.3 & 0.18 & 0 \\ 0 & 1.2 & 0.72 & 0.4 & 0.24 \\ 0.42 & 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 0 & 0 \\ 0 & 0 & 0.32 & 0 & 0 \end{bmatrix} \begin{bmatrix} 60 \\ 76 \\ 86 \\ 90 \\ 100 \end{bmatrix} = \begin{bmatrix} 156.3 \\ 201.6 \\ 25.2 \\ 29.4 \\ 25.6 \end{bmatrix} = \begin{bmatrix} n(4) \\ n(3) \\ n(2) \\ n(1) \\ n(0) \end{bmatrix} (2)$$