Homework for Lecture 5 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Sept. 22, 2025.

Subject: hw5

with an attachment hw5FirstLast.pdf

1. a. Convert the recurrence

$$6\,a(n-1) \,+\, a(n+3) \,+\, 5\,a(n+1) = 0 \quad ,$$

into **standard form** where $a(n+4)$ is expressed in terms of $a(n+3), a(n+2), a(n+1), a(n)$.

b.

Abbreviating

$$\mathbf{a}(n) = \begin{bmatrix} a(n+3) \\ a(n+2) \\ a(n+1) \\ a(n) \end{bmatrix} \quad .$$

Find the 4×4 matrix, let's call it A such that

$$\mathbf{a}(n+1) \,=\, A\mathbf{a}(n)$$

c Assuming that $a(0) = 0$, $a(1) = 2$, $a(2) = 3$,$a(3) = 4$ Find $a(5)$ in two ways:

(i) Straight from the standard form, by first finding $a(4)$, and then $a(5)$

(ii) Using the matrix version by first finding A^2 and then multiplying it by the column vector $[4, 3, 2, 0]^T$ and extracting the first component.

2. In a certain species only one-year-olds, two-year-olds, three-year-olds, and four-year-olds are fertile. We have

- zero-year-olds can't have babies
- Every 1-year-old female makes 1.5 babies on average
- Every 2-year-old female makes 0.9 babies on average
- Every 3-year-old female makes 0.5 babies on average

- Every 4-year-old female makes 0.3 babies on average

We also know

- The probability that a zero-year-old will survive the year is 0.8
- The probability that a one-year-old will survive the year is 0.7
- The probability that a two-year-old will survive the year is 0.6
- The probability that a three-year-old will survive the year is 0.6

a. Set up the **Leslie matrix**

b. If right now there are 100 zero-year-olds, 90 one-year-olds, 80 two-year-olds, 70 three-year-olds, and 60 four-year-old, what is the expected number of 3-year-olds after two years?

1. a. Convert the recurrence

$$6\,a(n-1) + a(n+3) + 5\,a(n+1) = 0 \quad ,$$

into **standard form** where $a(n+4)$ is expressed in terms of $a(n+3), a(n+2), a(n+1), a(n)$.

$$6a(n) + a(n+4) + 5a(n+2) = 0$$

$$a(n+4) = 0a(n+3) - 5a(n+2) + 0a(n+1) - 6a(n)$$

b.

Abbreviating

$$\mathbf{a}(n) = \begin{bmatrix} a(n+3) \\ a(n+2) \\ a(n+1) \\ a(n) \end{bmatrix} \quad .$$

Find the 4×4 matrix, let's call it A such that

$$\mathbf{a}(n+1) = A\mathbf{a}(n)$$

$$A = \begin{bmatrix} 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

c Assuming that $a(0) = 0$, $a(1) = 2$, $a(2) = 3$, $a(3) = 4$ Find $a(5)$ in two ways:

(i) Straight from the standard form, by first finding $a(4)$, and then $a(5)$

(ii) Using the matrix version by first finding A^2 and then multiplying it by the column vector $[4, 3, 2, 0]^T$ and extracting the first component.

(i) $a(4) = 0a(3) - 5a(2) + 0a(1) - 6a(0)$

$a(4) = -15$

$a(5) = 0a(4) - 5a(3) + 0a(2) - 6a(1)$

$a(5) = -20 - 12 = -32$

(ii)

$$A^2 = \begin{bmatrix} 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -6 & 0 \\ 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a(5) \\ a(4) \\ a(3) \\ a(2) \end{bmatrix} = \begin{bmatrix} -5 & 0 & -6 & 0 \\ 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 0 \end{bmatrix}$$

$a(5) = -20 - 12 = -32$

2. In a certain species only one-year-olds, two-year-olds, three-year-olds, and four-year-olds are fertile. We have

- zero-year-olds can't have babies $f_0 = 0$
- Every 1-year-old female makes 1.5 babies on average $f_1 = 1.5$
- Every 2-year-old female makes 0.9 babies on average $f_2 = 0.9$
- Every 3-year-old female makes 0.5 babies on average $f_3 = 0.5$
- Every 4-year-old female makes 0.3 babies on average $f_4 = 0.3$

We also know

- The probability that a zero-year-old will survive the year is 0.8 $P_0 = 0.8$
- The probability that a one-year-old will survive the year is 0.7 $P_1 = 0.7$
- The probability that a two-year-old will survive the year is 0.6 $P_2 = 0.6$
- The probability that a three-year-old will survive the year is 0.6 $P_3 = 0.6$

a. Set up the **Leslie matrix**

$$L = \begin{bmatrix} 0 & 1.5 & 0.9 & 0.5 & 0.3 \\ 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0 \end{bmatrix}$$

b. If right now there are 100 zero-year-olds, 90 one-year-olds, 80 two-year-olds, 70 three-year-olds, and 60 four-year-old, what is the expected number of 3-year-olds after two years?

$$\begin{bmatrix} a(6) \\ a(5) \\ a(4) \\ a(3) \\ a(2) \end{bmatrix} = L^2 \begin{bmatrix} 100 \\ 90 \\ 80 \\ 70 \\ 60 \end{bmatrix}$$