Dyn Mod in Bio

1. a. Convert the recurrence

$$6\,a(n-1)\;+\;a(n+3)\;+\;5\,a(n+1)=0\quad,$$

into **standard form** where $a(n+4)$ is expressed in terms of $a(n+3), a(n+2), a(n+1), a(n)$.

b.

Abbreviating

$$\mathbf{a}(n)=\begin{bmatrix} a(n+3)\\ a(n+2)\\ a(n+1)\\ a(n)\end{bmatrix}\quad.$$

Find the 4×4 matrix, let's call it A such that

$$\mathbf{a}(n+1)\;=\;A\mathbf{a}(n)$$

c Assuming that $a(0)=0,\ a(1)=2,\ a(2)=3, a(3)=4$ Find $a(5)$ in two ways:

(i) Straight from the standard form, by first finding $a(4)$, and then $a(5)$

(ii) Using the matrix version by first finding A^2 and then multiplying it by the column vector $[4,3,2,0]^T$ and extracting the first component.

1.

a. largest i is 1. So, $n\to n+1$

$$6a(n)+a(n+4)+5a(n+2)=0$$

$$\boxed{a(n+4)=0\cdot(n+3)-5a(n+2)+0\cdot(n-1)-6a(n)}$$

b. $$a(n+1)=\begin{bmatrix} a(n+4)\\ a(n+3)\\ a(n+2)\end{bmatrix}\qquad A=\begin{bmatrix} 0 & -5 & 0 & -6\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

i. straight from the standard form

if $n=0\to\ a(4)=-5a(2)-6a(0)=-5\cdot 3-6\cdot 0=-15$

if $n=1\to\ a(5)=-5a(3)-6a(1)=-5\cdot 4-6\cdot 2=-32$

$$\boxed{a(4)=-15\ ,\ a(5)=-32}$$

ii. matrix method

$$X_0=\begin{bmatrix} a(3)\\ a(2)\\ a(1)\\ a(0)\end{bmatrix}=\begin{bmatrix} 4\\ 3\\ 2\\ 0\end{bmatrix}\qquad A^2=\begin{bmatrix} -5 & 0 & -6 & 0\\ 0 & -5 & 0 & -6\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}$$

$$X_2=A^2X_0=\begin{bmatrix} -5 & 0 & -6 & 0\\ 0 & -5 & 0 & -6\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix} 4\\ 3\\ 2\\ 0\end{bmatrix}=\begin{bmatrix} -32\\ -15\\ 4\\ 3\end{bmatrix}$$

$$\boxed{\begin{array}{l} a(5)=-32\\ a(4)=-15\end{array}}$$

2. In a certain species only one-year-olds, two-year-olds, three-year-olds, and four-year-olds are fertile. We have

- zero-year-olds can't have babies
- Every 1-year-old female makes 1.5 babies on average
- Every 2-year-old female makes 0.9 babies on average
- Every 3-year-old female makes 0.5 babies on average
- Every 4-year-old female makes 0.3 babies on average

$$\boxed{L = \begin{bmatrix} 0 & 1.5 & 0.9 & 0.5 & 0.3 \\ 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0 \end{bmatrix}}$$

We also know

- The probability that a zero-year-old will survive the year is 0.8
- The probability that a one-year-old will survive the year is 0.7
- The probability that a two-year-old will survive the year is 0.6
- The probability that a three-year-old will survive the year is 0.6

a. Set up the **Leslie matrix**

b. If right now there are 100 zero-year-olds, 90 one-year-olds, 80 two-year-olds, 70 three-year-olds, and 60 four-year-old, what is the expected number of 3-year-olds after two years?

at $t=0$ $\vec{n}(0) = \begin{bmatrix} 100 \\ 90 \\ 80 \\ 70 \\ 60 \end{bmatrix}$

$$L^2 = \begin{bmatrix} 1.2 & 0.63 & 0.3 & 0.18 & 0 \\ 0 & 1.2 & 0.72 & 0.4 & 0.24 \\ 0.56 & 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 0 & 0 \\ 0 & 0 & 0.36 & 0 & 0 \end{bmatrix}$$

$$\vec{n}(2) = L^2 \begin{bmatrix} 100 \\ 90 \\ 80 \\ 70 \\ 60 \end{bmatrix} = \begin{bmatrix} 1.2 & 0.63 & 0.3 & 0.18 & 0 \\ 0 & 1.2 & 0.72 & 0.4 & 0.24 \\ 0.56 & 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 0 & 0 \\ 0 & 0 & 0.36 & 0 & 0 \end{bmatrix} \begin{bmatrix} 100 \\ 90 \\ 80 \\ 70 \\ 60 \end{bmatrix} = \begin{bmatrix} 213.3 \\ 208 \\ 56 \\ 37.8 \\ 28.8 \end{bmatrix}$$

$$\boxed{\text{So, the number of 3 yr-olds is } 37.8}$$