Homework for Lecture 4 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Sept. 22, 2025.

Subject: hw4

with an attachment hw4FirstLast.pdf and/or hw4FirstLast.txt

- 1. Give an example of a second-order linear differential equation, with two specific functions that are solutions and verify that their sum also satisfies that same differential equation.
- **2.** Verify that $y_1(t) = t^2$ satisfies the differential equation

$$y'(t)^2 - 4y(t) = 0 .$$

Would you expect the function $y_2(t) = 2y_1(t) = 2t^2$ to also be a solution? (after all it is a constant multiple of $y_1(t)$). Explain. Verify that indeed $y_2(t)$ is **not** a solution.

- **3.** Give an example of a second-order linear recurrence equation, with two specific sequences that are solutions and verify that their sum also satisfies that same recurrence equation.
- 4. Consider the non-linear recurrence

$$a(n) = a(n-1)^2 \quad , n \ge 0 \quad .$$

Check that both sequences

$$a_1(n) := 2^{2^n}$$
 , $a_2(n) := 3^{2^n}$,

are solutions. Does it follow that the new sequence

$$a_3(n) := a_1(n) + a_2(n) = 2^{2^n} + 3^{2^n}$$

is automatically yet-another-solution? Explain why or why not. By directly plugging-in into the recurrence find out whether it is true.

- 5. Write the Maple commands to solve each of the following problems, and give the Maple output.
- a. Solve the Initial Value Problem Differential Equation

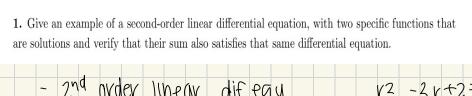
$$y''(x) + y(x) = 0$$
 , $y(0) = 1$, $y'(0) = 1$.

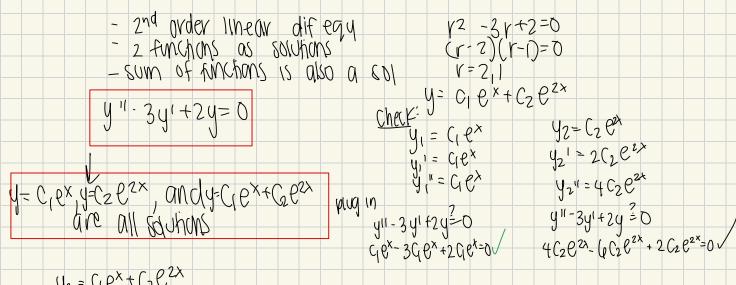
. ${\bf b}$ Solve the Initial Value Problem Difference Equation

$$a(n) - 3a(n-1) + a(n-2) = n$$
 $a(0) = 1, a(1) = 3$.

c. Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}$$





$$y_3 = \zeta_1 e^{x} + \zeta_2 e^{2x}$$

$$y_3' = \zeta_1 e^{x} + 2\zeta_2 e^{2x}$$

$$y_3'' = \zeta_1 e^{x} + 2\zeta_2 e^{2x}$$

$$y_3'' = \zeta_1 e^{x} + \zeta_2 e^{2x}$$

$$\zeta_1 e^{x} + \zeta_2 e^{2x} + \zeta_2 e^{2x}$$

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2. Verify that
$$y_1(t) = t^2$$
 satisfies the differential equation

Would you expect the function $y_2(t) = 2y_1(t) = 2t^2$ to also be a solution? (after all it is a constant multiple of $y_1(t)$). Explain. Verify that indeed $y_2(t)$ is **not** a solution.

$$y'(t)^{2} - 4y(t) = 0$$

$$y(t)$$
 isnt necessarily a sol
 $y(t) = 4t$ $(4t)^2 - 4(2t^2) \stackrel{?}{=} 0$
 $1(4t^2 - 8t^2 \neq 0)$ $y(2(t))$ is

ar(n), which is the sun of azand as

3. Give an example of a second-order linear recurrence equation, with two specific sequences that are solutions and verify that their sum also satisfies that same recurrence equation.

4. Consider the non-linear recurrence

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$$Q_{1}(N) := 2^{2n}$$

$$Q_{2}(N) := 3^{2n}$$

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$$Q_{2}(N) := 3^{2n}$$

$$Q_{2}(N$$

the sum isnf necessarily still a solution— $\frac{1}{2}$ need to check if still linear recurrence $0.3(m) = 2^{2^n} + 3^{2^n}$ $\Omega(N) \stackrel{?}{=} \Omega(N-1)^2$

$$2^{2^{n}} + 3^{2^{n}} \stackrel{?}{=} \left(2^{2^{n-1}} + 3^{2^{n-1}}\right)^{2} + 2\left(2^{2^{n-1}} + 2^{2^{n-1}}\right)^{2} + 2\left(2^{2^{n-1}} + 3^{2^{n-1}}\right)^{2}$$

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another solution

a. Solve the Initial Value Problem Differential Equation

$$y''(x) + y(x) = 0$$
 , $y(0) = 1$, $y'(0) = 1$.

$$y(x) = cos(x) + sin(x)$$

. b Solve the Initial Value Problem Difference Equation

$$a(n) - 3a(n-1) + a(n-2) = n$$
 $a(0) = 1, a(1) = 3$

$$rsolve(\{q(n)-3*\alpha(n-1)*\alpha(n-2)+n=0,\alpha(0)=1,\alpha(1)=3\}$$
 $\alpha(n));$

rsolve($\{a(n) - 3 \cdot a(n-1) + a(n-2) - n = 0, a(0) = 1, a(1) = 3\}, a(n)\};$

$$\left(\frac{1}{2} - \frac{3\sqrt{5}}{10}\right) \left(-\frac{\sqrt{5}}{2} + \frac{3}{2}\right)^{n} + \left(\frac{3\sqrt{5}}{10} + \frac{1}{2}\right) \left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)^{n} - \frac{\left(\sqrt{5} + 1\right)\sqrt{5}\left(-\frac{2}{-3 - \sqrt{5}}\right)^{n}}{5\left(-3 - \sqrt{5}\right)} - \frac{\left(\sqrt{5} - 1\right)\sqrt{5}\left(-\frac{2}{\sqrt{5} - 3}\right)^{n}}{5\left(\sqrt{5} - 3\right)} - n - 1$$
(2)

c. Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}$$

with (linaig);

$$A := Matnx([[3,4],[2,4]]);$$

eigenval
$$S(A)$$
; $\frac{7+\sqrt{33}}{2}$, $\frac{7-\sqrt{33}}{2}$

$$\text{eigenvects(A)'}, \qquad \left[\frac{7}{2} + \frac{\sqrt{33}}{2}, 1, \left\{ \left[1 \ \frac{1}{8} + \frac{\sqrt{33}}{8} \ \right] \right\} \right] \left[\frac{7}{2} - \frac{\sqrt{33}}{2}, 1, \left\{ \left[1 \ \frac{1}{8} - \frac{\sqrt{33}}{8} \ \right] \right\} \right]$$