HW 4

1. $y'' + 2y' + y = 0$

$r^2 + 2r + 1 = 0$

$(r+1)^2 = 0$

$y = c_1 e^{-x} + c_2 x e^{-x}$

Solutions:

$y_1 = e^{-x} + -xe^{-x}$

$y_2 = 2e^{-x} + xe^{-x}$

$y_1 + y_2 = 3e^{-x}$

verify $3e^{-x}$ solves $y'' + 2y' + y = 0$

$y = 3e^{-x}$, $y' = -3e^{-x}$, $y'' = 3e^{-x}$

$y'' + 2y' + y = 3e^{-x} + 2(-3e^{-x}) + 3e^{-x} = 0$ ✓

2. $(y')^2 - 4y = 0$

$y_1 = t^2 \Rightarrow y_1' = 2t$

$(2t)^2 - 4(t^2) = 0$ ✓

y_2 is not a solution because $(y')^2 - 4y = 0$ is nonlinear.

Check: $y_2 = 2t^2 \Rightarrow y_2' = 4t$

$(4t)^2 - 4(2t^2)$

$= 16t^2 - 8t^2$

$= 8t^2 \neq 0$ ✓

3. $a(n) - a(n-2) = 0$

$r^2 - 1 = 0$

$(r+1)(r-1) = 0$

$a(n) = c_1(1)^n + c_2(-1)^n$

$a(n) = c_2(-1)^n$

Solutions:

$a_1(n) = 2(-1)^n$

$a_2(n) = 3(-1)^n$

$a_3 = a_1 + a_2 = 2(-1)^n + 3(-1)^n$

verify $a_3 = 2(-1)^n + 3(-1)^n = 5(-1)^n$ solves $a(n) - a(n-2) = 0$

$a_3(n) = 5(-1)^n$, $a_3(n-2) = 5(-1)^{n-2}$

$5(-1)^n - 5(-1)^{n-2}$

$= 5(-1)^n - 5(-1)^{-2}(-1)^n$

$= 5(-1)^n - 5(-1)^n$

$= 0$ ✓

4. $a(n) = (a(n-1))^2, \quad n \geq 0$

Check 2^{2^n}: $a(n-1)^2 =$

$$(2^{2^{n-1}})^2$$
$$= 2^{2^{n-1} \cdot 2}$$
$$= 2^{2^n} = a(n) \checkmark$$

Check 3^{2^n}: $a(n-1)^2 =$

$$(3^{2^{n-1}})^2$$
$$= 3^{2^{n-1} \cdot 2}$$
$$= 3^{2^n} = a(n) \checkmark$$

The sum of 2 solutions need not be a solution because the recurrence is nonlinear.

Verify:

$$a(n-1)^2 = (2^{2^{n-1}} + 3^{2^{n-1}})^2$$
$$= (2^{2^{n-1}})^2 + 2(2^{2^{n-1}})(3^{2^{n-1}}) + (3^{2^{n-1}})^2$$
$$= 2^{2^n} + 3^{3^n} + 2(2^{2^{n-1}})(3^{2^{n-1}})$$
$$\neq 2^{2^n} + 3^{3^n} = a(n)$$

5a. $y'' + y = 0, \quad y(0) = 1, \ y'(0) = 1$

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dsolve({diff(y(x), x$2) + y(x) = 0,
        y(0) = 1,
        D(y)(0) = 1}, y(x))
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$$y(x) = \sin(x) + \cos(x)$$

5b. $a(n) - 3a(n-1) + a(n-2) = n \qquad a(0) = 1 \quad a(1) = 3$

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rsolve({a(n) - 3*a(n-1) + a(n-2) = n,
        a(0) = 1, a(1) = 3}, a(n))
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$$\left(\frac{3\sqrt{5}}{10} + \frac{1}{2}\right)\left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)^n + \left(\frac{1}{2} - \frac{3\sqrt{5}}{10}\right)\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^n$$
$$- \frac{(\sqrt{5}-1)\sqrt{5}\left(-\frac{2}{\sqrt{5}-3}\right)^n}{5(\sqrt{5}-3)} - \frac{(\sqrt{5}+1)\sqrt{5}\left(-\frac{2}{-3-\sqrt{5}}\right)^n}{5(-3-\sqrt{5})} - n - 1$$

5c. $\begin{pmatrix} 3 & 4 \\ 2 & 4 \end{pmatrix}$

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with(linalg)
eigenvectors(matrix([3,4],[2,4]))
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$$\left[\frac{7}{2}+\frac{\sqrt{33}}{2},\ 1,\ \left\{\left[1 \quad \frac{1}{8}+\frac{\sqrt{33}}{8}\right]\right\}\right],$$

$$\left[\frac{7}{2}-\frac{\sqrt{33}}{2},\ 1,\ \left\{\left[1 \quad \frac{1}{8}-\frac{\sqrt{33}}{8}\right]\right\}\right]$$