# Homework 4

1. $y'' + y = 0$

$y_1(x) = \sin x$ , $y_2(x) = \cos x$

$g(x) = y_1(x) + y_2(x) = \cos x + \sin x$

$g'(x) = -\sin x + \cos x$
$g''(x) = -\cos x - \sin x$

$(-\sin x - \cos x) + 1(\cos x + \sin x) = 0$
$0 = 0$ ✓

2. $y_1(t) = t^2 \rightarrow (2t)^2 - 4(t^2) \stackrel{?}{=} 0$
$\longrightarrow 4t^2 - 4t^2 = 0$ ✓

No, $y_2(t)$ cant be a solution.
we can see that $16t^2 - 8t^2 \neq 0$.
This is not a linear ode, no superpos.

3. $a_n = 3a_{n-1} - 2a_{n-2}$
$z^2 - 3z + 2 = 0 \rightarrow (z-1)(z-2)$
$a_{n(1)} = 1$ & $a_{n(2)} = 2^n$ are solutions

$\Sigma = a_n^{(1)} + a_n^{(2)} = 1 + 2^n$
$3(1 + 2^{n-1}) - 2(1 + 2^{n-2}) = 4 \cdot 2^{n-2} + 1$
$= 2^n + 1$ ✓

4. $a(n) = a(n-1)^2, n \geq 0$

$a_1(n) := 2^{2^n}$ :

$(2^{2^{n-1}})^2 = 2^{2 \cdot 2^{n-1}} = 2^{2^n}$ ✓

$a_1(n) := 3^{2^n} \Rightarrow (3^{2^{n-1}})^2 = 3^{2 \cdot 2^{n-1}} = 3^{2^n}$ ✓

Since the a term in recurrence is squared this is not a linear recurrence.
∴ we cannot assume sum of 2 sol is a sol.

Try: $2^{2^n} + 3^{2^n} \stackrel{?}{=} (2^{2^n} + 3^{2^n})^2$

no $a^2 + b^2$ X will be $a^2 + 2ab + b^2$

Messes it up!

∴ Sum is not also a solution.

Commands

5. a. dsolve(diff(y(x), x, x) + y(x),
y(0) = 1, D(y)(0) = 1, y(x));

To get value at some x
evalf(subs(x = 2.1, op(2, %)));

y(x) = K →

b. rsolve(a(n) - 3 * a(n-1) + a(n-2) = n,
a(0) = 1, a(1) = 3, a(n));

c. > with(linalg);

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> A := matrix([[3,4],[2,4]]);

...

> eigenvectors(A); gives both!