

Homework 4

1. $y'' + y = 0$

$$y_1(x) = \sin x, \quad y_2(x) = \cos x$$

$$g(x) = y_1(x) + y_2(x) = \cos x + \sin x$$

$$g'(x) = -\sin x + \cos x$$

$$g''(x) = -\cos x - \sin x$$

$$(-\sin x - \cos x) + (\cos x + \sin x) = 0$$

$$0 = 0 \quad \checkmark$$

2. $y_1(t) = t^2 \rightarrow (2t)^2 - 4(t^2) \stackrel{?}{=} 0$
 $\rightarrow 4t^2 - 4t^2 = 0 \quad \checkmark$

No, $y_2(t)$ can't be a solution.

We can see that $16t^2 - 8t^2 \neq 0$.

This is not a linear ode, no superpos.

3. $a_n = 3a_{n-1} - 2a_{n-2}$

$$z^2 - 3z + 2 = 0 \rightarrow (z-1)(z-2)$$

$$a_n(1) = 1 \quad \& \quad a_n(2) = 2^n \text{ are solutions}$$

$$\Sigma = a_n^{(1)} + a_n^{(2)} = 1 + 2^n$$

$$3(1 + 2^{n-1}) - 2(1 + 2^{n-2}) = 4 \cdot 2^{n-2} + 1 \\ = 2^n + 1 \quad \checkmark$$

$$4. a(n) = a(n-1)^2, n \geq 0$$

$$a_1(n) := 2^{2^n} :$$

$$(2^{2^{n-1}})^2 = 2^{2 \cdot 2^{n-1}} = 2^{2^n} \checkmark$$

$$a_2(n) := 3^{2^n} \Rightarrow (3^{2^{n-1}})^2 = 3^{2 \cdot 2^{n-1}} = 3^{2^n} \checkmark$$

Since the a term in recurrence is squared this is not a linear recurrence.

$\therefore$ we cannot assume sum of 2 sol is a sol.

$$\text{Try: } 2^{2^n} + 3^{2^n} \stackrel{?}{=} (2^{2^n} + 3^{2^n})^2$$

$$\text{no } a^2 + b^2 \times \text{ will be } a^2 + 2ab + b^2$$

Messes it up!

$\therefore$ Sum is not also a solution.

Commands

$$5. a. \text{dsolve}(\text{diff}(y(x), x, x) + y(x), \\ y(0) = 1, D(y)(0) = 1, y(x));$$

To get value at some x

$$\text{evalf}(\text{subs}(x=2.1, \text{op}(2, \%)))$$

$$y(x) = K \rightarrow$$

b. solve $a(n) - 3 * a(n-1) + a(n-2) = n$,
 $a(0) = 1, a(1) = 3, a(n);$

c. with (linalg);

$\geq A := \text{matrix}([[3, 4], [2, 4]]);$

$\geq \text{eigenvectors}(A);$ gives both!



$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = A$$

$$P = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad E = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Check: $P \cdot A \cdot Q = \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = P$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = P$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = P$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = P$$

