S M T W T F S ___ / ___ / ___

HW4 Dyn

hw4 Ali Zachariah · pdf

1. 2nd order diff eq: $y''(x) + 3y'(x) + 2y(x) = 0$

Solutions: $y_1(x) = e^{-x}$ $y_2(x) = e^{-2x}$

$$y(x) = y_1(x) + y_2(x) = e^{-x} + e^{-2x}$$

$$y'(x) = -e^{-x} - 2e^{-2x}$$

$$y''(x) = e^{-x} + 4e^{-2x}$$

$$\begin{aligned} & y''(x) + 3y'(x) + 2y(x) \\ &= (e^{-x} + 4e^{-2x}) + 3(-e^{-x} - 2e^{-2x}) + 2(e^{-x} + e^{-2x}) \\ &= e^{-x} + 4e^{-2x} - 3e^{-x} - 6e^{-2x} + 2e^{-x} + 2e^{-2x} \\ &= e^{-x} - 3e^{-x} + 2e^{-x} + 4e^{-2x} - 6e^{-2x} + 2e^{-2x} \\ &= 0 \end{aligned}$$

Thus we have verified that $y_1(x) + y_2(x)$ satisfies the ODE.

2. $y_1(t) = t^2$ $y_1'(t) = 2t$

$$y'(t)^2 - 4y(t) = (2t)^2 - 4t^2 = 4t^2 - 4t^2 = 0$$

Verified.

No as $(y'(t))^2 - 4y(t) = 0$ is nonlinear, thus we cannot expect that linear combinations of solutions are solutions.

$y_2'(t) = 4t$, $(4t)^2 - 4(2t^2) = 16t^2 - 8t^2 = 8t^2$

$8t^2$ is not necessarily equal to 0.

So $y_2(t)$ is not a solution

3. Recurrence equation $a(n) - 3a(n-1) - 40a(n-2) = 0$

$$a_1(n) = 8^n \quad a_2(n) = (-5)^n$$

$$a(n) = a_1(n) + a_2(n) = 8^n + (-5)^n$$

$$\begin{aligned} & a(n) - 3a(n-1) - 40a(n-2) \\ &= 8^n + (-5)^n - 3(8)^{n-1} - 3(-5)^{n-1} - 40(8)^{n-2} - 40(-5)^{n-2} \\ &= 8^n - \frac{3}{8}(8)^n - \frac{5}{8}(8)^n + (-5)^n + \frac{3}{5}(-5)^n - \frac{8}{5}(-5)^n = 0. \end{aligned}$$

Verified.

4. $\cancel{a(n) = a(n-1)^2}$

$\cancel{a_1(n) = 2^{2n}}$

$\cancel{a_1(n-1) = 2^{2(n-1)}}$

$\cancel{(a_1(n-1))^2 = (2^{2(n-1)})^2 = 2^2}$

$a(n) = a(n-1)^2$

$a_1(n) = 2^{2^n}$

$a_1(n-1) = 2^{2^{n-1}}$

$(a_1(n-1))^2 = (2^{2^{n-1}})^2 = 2^{2(2^{n-1})} = 2^{2^n}$

$2^{2^n} = a(n)$

So $a_1(n) = a_1(n-1)^2$

$a_2(n) = 3^{2^n}$

$a_2(n-1) = 3^{2^{n-1}}$

$(a_2(n-1))^2 = (3^{2^{n-1}})^2 = 3^{2\times 2^{n-1}} = 3^{2^n} = a_2(n)$

So $a_2(n) = a_2(n-1)^2$

As $a(n-1)^2$ makes the recurrence equation nonlinear, the superposition principle does not hold. So we can't assume that the sum of 2 solutions is a solution.

$a_3(n) = 2^{2^n} + 3^{2^n}$ $\quad a_3(n-1) = 2^{2^{n-1}} + 3^{2^{n-1}}$

$(a_3(n-1))^2 = (2^{2^{n-1}} + 3^{2^{n-1}}) = (2^{2^{n-1}})^2 + (3^{2^{n-1}})^2 + 2(2^{2^{n-1}})(3^{2^{n-1}})$

$= 2^{2^n} + 3^{2^n} + 2(6^{2^{n-1}})$

$(a_3(n-1))^2 = a_3(n) + 2(6^{2^{n-1}})$

So $a_3(n)$ is not a solution to $a_3(n) = (a_3(n-1))^2$.

S M T W T F S ___/___/___

5. a) ~~As the recurrence equation is non-linear due to the $a(n-1)^2$ term, the superp.~~

```
dsolve({diff(y(x), x$2) + y(x) = 0, y(0) = 1, D(y)(0) = 1},
    y(x));
```

$y(x) = \sin x + \cos x$

b) $a(n) - 3a(n-1) + a(n-2) = n, \; a(0) = 1, \; a(1) = 3$

```
rsolve({a(n) - 3a(n-1) + a(n-2) = n, a(0) = 1, a(1) = 3}, a(n));
```

$$a(n) = \left(\frac{3\sqrt{5}}{10} + \frac{1}{2}\right)\left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)^n + \left(\frac{1}{2} - \frac{3\sqrt{5}}{10}\right)\left(-\frac{\sqrt{5}}{2} + \frac{3}{2}\right)^n$$

$$- \frac{(\sqrt{5}+1)\sqrt{5}\left(-\frac{2}{-3-\sqrt{5}}\right)^n}{5(-3-\sqrt{5})} - \frac{(\sqrt{5}-1)\sqrt{5}\left(-\frac{2}{\sqrt{5}-3}\right)^n}{5(\sqrt{5}-3)} - n - 1$$

c)
```
with(linalg);
A := matrix([[3,4],[2,4]]);
eigenvectors(A);
```

eigenvalues are $\frac{7}{2} \pm \frac{\sqrt{33}}{2}$

eigenvectors are $\begin{pmatrix} 1 \\ \frac{1+\sqrt{33}}{8} \end{pmatrix}$ and $\begin{pmatrix} 1 \\ \frac{1-\sqrt{33}}{8} \end{pmatrix}$