HW 4

① $y''(x) - y(x) = 0$ $\quad y_1(x) = e^x$, $y_2(x) = e^{-x}$

$y_1 = e^x \;||\; y_1'' = e^x \rightarrow e^x - e^x = 0$ ✓

$y_2 = e^{-x} \;||\; y_2'' = e^{-x} \rightarrow e^{-x} - e^{-x} = 0$ ✓

$y_1 + y_2 \;||\; (y_1 + y_2)'' = (e^x + e^{-x}) - (e^x + e^{-x}) = 0$ ✓

② $y'(t) = 2t \rightsquigarrow 4t^2 - 4t^2 = 0$ ✓

Since the ODE is not linear, a constant multiple of the soln will not be a solution.

$16t^2 - 8t^2 \neq 0$

③ $3a(n-1) - 2a(n-2) = a(n)$ $\quad a(n) = 1^n$, $a_2(n) = 2^n$, $a_3 = 1^n + 2^n$

$a_1(n) = 1$, $a_1(n-1) = 1$, $a_1(n-2) = 1$ $\Rightarrow 3(1) - 2(1) = 1$ ✓

$a_2(3) = 8$, $a_2(2) = 4$, $a_2(1) = 2$ $\Rightarrow 3(4) - 2(2) = 8$ ✓

$a_3(1) = 3$, $a_3(2) = 5$, $a_3(3) = 9$ $\Rightarrow 3(5) - 2(3) = 9$ ✓

④ $a(1) = 4$ $\quad a_1(2) = 16$ $\quad 4^2 = 16$ ✓

$a_2(1) = 9$ $\quad a_2(2) = 81$ $\quad 9^2 = 81$ ✓

$a_3(n) = 2^{2^n} + 3^{2^n}$ $\quad a_3(1) = 15$ $\quad a_3(2) = 96$ $\quad 15^2 \neq 96$ ∴

* This is not a linear recurrence

⑤ (a) dsolve({diff(y(x), x, x) + y(x) = 0, y(0) = 1, D(y)(0) = 1}, y(x));

↳ $y(x) = \cos(x) + \sin(x)$

(b) rsolve({a(n) - 3*a(n-1) + a(n-2) = n, a(0) = 1, a(1) = 3}, a(n));

↳ $a(n) = 1/2 * (1+n) + 2^n$

(c) A := Matrix([3, 4], [2, 4]);

Eigenvalues(A); $\rightarrow \lambda_1 = \frac{7+\sqrt{33}}{2}$ $\quad \lambda_2 = \frac{7-\sqrt{33}}{2}$

Eigenvectors(A); $\rightarrow \vec{V_1} = \begin{bmatrix} \frac{1+\sqrt{33}/4}{2} \\ 1 \end{bmatrix}$ $\quad \vec{V_2} = \begin{bmatrix} \frac{1-\sqrt{33}}{2} \\ 1 \end{bmatrix}$