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Homework #4

1.) $y'(t)+4y(t)=0 \Rightarrow r+4=0,\ r=-4$

$y_1=e^{-4t}$ $y_2=3e^{-4t}$ $y_3=e^{-4t}+3e^{-4t}$

$y_3'=-4e^{-4t}-12e^{-4t}$

$y_3'(t)+4y(t)=-4e^{-4t}-12e^{-4t}+4(e^{-4t}+3e^{-4t})$
$=0$

2.) $y_1(t)=t^2$ $y'(t)^2-4y(t)=0$

$y_1'(t)=2t$ $y_1'(t)^2=4t^2$

$y_1'(t)^2-4y_1(t)=4t^2-4t^2=0$

(b.) $y_2=2t^2$ $y_2'=4t$

No, y_2 should not be a sol. because the ODE is not linear

$y_2'(t)^2-4y_2(t)=16t^2-8t^2=8t^2\neq 0$

3.) $a(n)+4a(n-1)=0 \Rightarrow r=-4$

$a_1(n)=-4^n$ $a_2(n)=2(-4^n)$

$a_3(n)=-4^n+2(-4^n)=3(-4^n)$

$a_3(n)+4a_3(n-1)=3(-4^n)+12(-4^{n-1})=0$

4.) $a(n)=a(n-1)^2\ \forall n\in\mathbb{R}_{\geq 0}$

$a_1(n)=2^{2^n}$, $a_2(n)=3^{2^n}$

$a_1(n-1)^2=(2^{2^{n-1}})^2=2^{2^n}$ $a_2(n-1)^2=(3^{2^{n-1}})^2=3^{2^n}$

$a_3(n)=2^{2^n}+3^{2^n}$

$a_3(n-1)^2=(2^{2^{n-1}}+3^{2^{n-1}})^2$

$=(2^{2^{n-1}})^2+(2\cdot 2^{2^{n-1}}\cdot 3^{2^{n-1}})+(3^{2^{n-1}})^2$

$=2^{2^n}+2\cdot 2^{2^{n-1}}\cdot 3^{2^{n-1}}+3^{2^n}\neq 2^{2^n}+3^{2^n}\neq a_3(n)$

a_3 is not a sol.

5.) dsolve(diff(y(x),x,x)+y(x)=0, y(0)=1, D(y)(0)=1,
y(x));
$y(x) = \cos(x)$

b.) rsolve(a(n) - 3*a(n-1) + a(n-2) - n = 0, a(0)=1,
a(1)=3, a(n));
$a(n) = (1 + 2/\sqrt{5})*(3+\sqrt{5}/2)^n + (1 - 2/\sqrt{5})(3 - \sqrt{5}/2)^n - n - 1$

c.) with(linalg);
A:=matrix([[3,4],[2,4]]);
eigenvalues(A);
$7/2 + \sqrt{33}/2,\ 7/2 - \sqrt{33}/2$
eigenvectors(A);
$\begin{bmatrix} 1 \\ 1/8 + \sqrt{33}/8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1/8 - \sqrt{33}/8 \end{bmatrix}$