HW4

1

$$y''(x) - 16y(x) = 0$$

$$\lambda^2 - 16 = 0$$

$$(\lambda + 4)(\lambda - 4) \qquad \lambda = \pm 4$$

$$y_1(x) = C_1 e^{4x}$$

$$y_2(x) = C_2 e^{-4x}$$

$$y_3(x) = C_1 e^{4x} + C_2 e^{-4x}$$

$$y_3'(x) = 4C_1 e^{4x} - 4C_2 e^{-4x}$$

$$y_3''(x) = 16C_1 e^{4x} + 16C_2 e^{-4x}$$

$$y_3''(x) - y(x) = 16C_1 e^{4x} + 16C_2 e^{4x} - 16C_1 e^{4x} - 16C_2 e^{-4x} = 0 \checkmark$$

$$7 = 6 - 3C_2 + 4C_2$$

$$C_2 = 1 \qquad C_1 = 1$$

$\boxed{\text{HW4}}$

$\boxed{2}$

$y_1(t) = t^2$

$y_1'(t) = 2t$

$$y'(t)^2 - 4y(t) = 0$$

$$(2t)^2 - 4(t^2) = 0$$

$$4t^2 - 4t^2 = 0 \checkmark$$

NO, B/C THE CONSTANT OF 2 WILL INTERFERE W/ THE DERIVATIVE'S SQUARING

$y_2(t) = 2t^2$

$y_2'(t) = 4t$

$$y'(t)^2 - 4y(t) = 0$$

$$(4t)^2 - 4(2t^2) = 0$$

$$16t^2 - 8t^2 = 0 \times$$

HW 4

$\boxed{3}$ $a(n) = 36\,a(n-2)$

$\lambda^2 - 36 = 0$

$\lambda = \pm 6$

$a_1(n) = c_1 6^n$

$a_2(n) = c_2 (-6)^n$

$a_3(n) = c_1 6^n + c_2 (-6)^n$

$a_3(n+1) = c_1 6^{n+1} + c_2 (-6)^{n+1}$

$= 6c_1 6^n - 6c_2 (-6)^n$

$a_3(n+2) = 6c_1 6^{(n+1)} - 6c_2 (-6)^{n+1}$

$= 36\, c_1 6^n + 36 c_2 (-6)^n$

$a(n+2) - 36\,a(n) =$

$36c_1 6^n + 36c_2(-6)^n - 36c_1 6^n - 36c_2(-6)^n$

$= 0$ ✓

$\boxed{W4}$

$\boxed{4}$

$$a(n) = a(n-1)^2 \quad , \quad n \geq 0$$

$$a_1(n) = 2^{2^n} \qquad a_2(n) = 3^{2^n}$$

$$a_1(1) = 2^2 = \underbrace{\left(2^{2^{(1-1)}}\right)^2}_{a_1(0)} = \left(2^{2^{(1-1)}}\right)^2 = 2^2 \checkmark$$

$$a_2(1) = 3^{2^{(1)}} = \underbrace{\left(3^{2^{(1-1)}}\right)^2}_{a_2(0)} = 3^2 \checkmark$$

$a_3(n)$ SHOULD BE ANOTHER SOLUTION B/C OF THE SUPERPOSITION PROPERTY AS THE EQ IS HOMOGENEOUS

$$a_3(n) = 2^{2^n} + 3^{2^n}$$

$$a_3(1) = 2^{2^1} + 3^{2^1} = \underbrace{\left(2^{2^{1-1}}\right)^2 + \left(3^{2^{1-1}}\right)^2}_{a_3(0)} = 2^2 + 3^2 \checkmark$$

$c_2 = 1, \; c_1 = 1$

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> dsolve({diff(y(x),x$2) + y(x) = 0, y(0) = 1, D(y)(0) = 1}, y(x))
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$$y(x) = \sin(x) + \cos(x) \qquad (1)$$

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> rsolve({a(n) - 3 · a(n-1) + a(n-2) = n, a(0) = 1, a(1) = 3}, a(n))
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$$\left(\frac{1}{2} - \frac{3\sqrt{5}}{10}\right)\left(-\frac{\sqrt{5}}{2} + \frac{3}{2}\right)^n + \left(\frac{3\sqrt{5}}{10} + \frac{1}{2}\right)\left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)^n - \frac{(\sqrt{5}+1)\sqrt{5}\left(-\frac{2}{-3-\sqrt{5}}\right)^n}{5(-3-\sqrt{5})} - \frac{(\sqrt{5}-1)\sqrt{5}\left(-\frac{2}{\sqrt{5}-3}\right)^n}{5(\sqrt{5}-3)} - n - 1 \qquad (2)$$

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> with(linalg):
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> A := matrix([[3, 4], [2, 4]])
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$$A := \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix} \qquad (3)$$

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> eigenvalues(A)
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$$\frac{7}{2} + \frac{\sqrt{33}}{2}, \frac{7}{2} - \frac{\sqrt{33}}{2} \qquad (4)$$

```
> eigenvectors(A)
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$$\left[\frac{7}{2} + \frac{\sqrt{33}}{2}, 1, \left\{\begin{bmatrix} 1 & \frac{1}{8} + \frac{\sqrt{33}}{8} \end{bmatrix}\right\}\right], \left[\frac{7}{2} - \frac{\sqrt{33}}{2}, 1, \left\{\begin{bmatrix} 1 & \frac{1}{8} - \frac{\sqrt{33}}{8} \end{bmatrix}\right\}\right] \qquad (5)$$

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>
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