MATH336 - Homework #4

1. Ex: $y''(x) - 3y'(x) - 10y(x) = 0$

sol. 1 = C_1e^{5x}

sol 2 = C_2e^{-2x}

sum = $C_1e^{5x} + C_2e^{-2x}$

$$\left.\begin{aligned} y(x) &= C_1e^{5x} + C_2e^{-2x} \\ y'(x) &= 5C_1e^{5x} - 2C_2e^{-2x} \\ y''(x) &= 25C_1e^{5x} + 4C_2e^{-2x} \end{aligned}\right\} \begin{aligned} &(25C_1e^{5x} + 4C_2e^{-2x}) - 3(5C_1e^{5x} - 2C_2e^{-2x}) - \\ &10(C_1e^{5x} + C_2e^{-2x}) = 0 \\ &0 = 0 \checkmark \end{aligned}$$

2. $y_1(t) = t^2$

$y_1'(t) = 2t$

$y'(t)^2 - 4y(t) = (2t^2) - 4t^2 = 4t^2 - 4t^2 = 0 \checkmark$

No; differential equation is nonlinear, so superposition principle will not apply

$y_2(t) = 2t^2$

$y_2'(t) = 4t$

$y_2'(t)^2 - 4y_2(t) = (4t)^2 - 4(4t) = 16t^2 - 16t \neq 0$ X

3. $a(n) - 3a(n-1) - 10a(n-2) = 0$

sol. 1 = $C_1(5)^n$

sol. 2 = $C_2(-2)^n$

sum = $C_1(5)^n + C_2(-2)^n$

$$\left.\begin{aligned} a(n) &= C_1(5)^n + C_2(-2)^n \\ a(n-1) &= C_1(5)^{n-1} + C_2(-2)^{n-1} \\ a(n-2) &= C_1(5)^{n-2} + C_2(-2)^{n-2} \end{aligned}\right\} \begin{aligned} &(C_1(5)^n + C_2(-2)^n) - 3(C_1(5)^{n-1} + C_2(-2)^{n-1}) \\ &- 10(C_1(5)^{n-2} + C_2(-2)^{n-2}) = 0 \checkmark \end{aligned}$$

4. $a(n) := a(n-1)^2, n \geq 0$; $a_1(n) := 2^{2^n}$, $a_2(n) := 3^{2^n}$

$2^{2^n} = a(n-1)^2$

$2^{2^n} = \left(2^{2^{(n-1)}}\right)^2$

$2^{2(n-1) \cdot 2}$; $2^{(n-1) \cdot 2} = 2^{(n-1)} \cdot 2^1 = 2^{(n-1)+1} = 2n \Rightarrow 2^{2^n}$

$2^{2^n} = 2^{2^n}$ ✓

$3^{2^n} = \left(3^{2^{(n-1)}}\right)^2$

$= 3^{2(n-1) \cdot 2}$; $2^{(n-1) \cdot 2} = 2^{(n-1)} \cdot 2^1 = 2^{(n-1)+1} = 2n \Rightarrow 3^{2^n}$

$3^{2^n} = 3^{2^n}$ ✓

NO, it does not follow that $a_3(n) := 2^{2^n} + 3^{2^n}$ is automatically a solution; recursion is nonlinear, so Superposition Principle will not apply.

5.

a. ~~dsolve({diff(y(x),2)+y(x)=0},y(x),y(0)=1,y~~

dsolve({D2y + y = 0}, y(x), y(0)=1, Dy(0)=1);

Output: $y(x) = \cos x + \sin x$

b. rsolve({a(n) - 3*a(n-1) + a(n-2) = n}, a(0)=1, a(1)=3, a(n));

Output: $a(n) := \left(1 + \frac{2}{\sqrt{5}}\right)\left(\frac{3+\sqrt{5}}{2}\right)^n + \left(1 - \frac{2}{\sqrt{5}}\right)\left(\frac{3-\sqrt{5}}{2}\right)^n - n - 1$

c. A := matrix([[3,4],[2,4]]):

eigenvalues(A);

eigenvectors(A);

Outputs: $\lambda_1 = \frac{7+\sqrt{33}}{2}$, $\lambda_2 = \frac{7-\sqrt{33}}{2}$

$e_{\lambda_1} = \left[1 + \frac{8}{\sqrt{33}}\right]$, $e_{\lambda_2} = \left[1 - \frac{8}{\sqrt{33}}\right]$