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Homework 4 - Dynamic Model in Bio

① $y'' - y = 0$

$z^2 - 1 = 0$

$(z-1)(z+1) = 0$

$z = \pm 1$

$y_1(t) = e^t \quad y_2(t) = e^{-t}$

$e^t + e^{-t} = 2\cosh t$

② Verify that $y_1(t) = t^2$ satisfies $y'(t)^2 - 4y(t) = 0$

$y_1'(t) = 2t$

$(y_1')^2 = 4t^2$

$4y_1 = 4t^2$

$(y't)^2 - 4y_1 = 0$

NO, $y_2(t) = 2y_1(t) = 2t^2$ would not be a solution because it is nonlinear

$y_2'(t) = 4t \quad (y_2')^2 = 16t^2$

$4y_2 = 8t^2$

$(y_2')^2 - 4y_2 = 8t^2 \neq 0$

③ $a(n) = 5a(n-1) - 6a(n-2)$

$z^2 - 5z + 6 = 0$

$(z-3)(z-2)$

$z = 3 \quad z = 2$

$a(n) = 3^n + 2^n$

④ $a(n) = a(n-1)^2 \quad n \geq 0$

$a_1(n) := 2^{2^n} \quad a_2(n) := 3^{2^n}$

$a_1(n-1) = 2^{2^{n-1}} \Rightarrow a_1(n) = (2^{2^{n-1}})^2 = 2^{2^n}$

$a_2(n-1) = 3^{2^{n-1}} \Rightarrow a_2(n) = (3^{2^{n-1}})^2 = 3^{2^n}$

$a_3(n-1)^2 = 2^{2^n} + 3^{2^n} + 2 \cdot 2^{2^{n-1}} 3^{2^{n-1}} \neq 2^{2^n} + 3^{2^n}$

So, $a_3(n)$ is not a solution

⑤ $y''(x) + y(x) = 0 \quad y(0) = 1 \quad y'(0) = 1$

(a) dsolve({diff(y(x), x$2) + y(x) = 0, y(0)=1, D(y)(0)=1}, y(x));

output: $y(x) = \sin(x) + \cos(x)$

(b) rsolve({a(n)-3*a(n-1)+a(n-2)=n, a(0)=1, a(1)=3}, a(n));

output: $a(n) = \left(1 + \frac{2}{\sqrt{5}}\right)\left(\frac{3+\sqrt{5}}{2}\right)^n + \left(1 - \frac{2}{\sqrt{5}}\right)\left(\frac{3-\sqrt{5}}{2}\right)^n - n - 1$

(c) with(LinearAlgebra);
A:= Matrix([[3,4],[2,4]]);

$$A := \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}$$

eigenvalues(A);

$$\left(2, \left[\frac{(7+\sqrt{33})}{2}, \frac{(7-\sqrt{33})}{2}\right]\right)$$

Eigenvectors(A);

⇒ maple on virtual lab was not working
(not sure why)