2^{*}

$a(n^2-2n+1)+\beta n-\beta+y$

$an^2-2an+a+\beta n-\beta+y$

1. ① $a_1(n)=2^{2n}=a(n-1)^2$

$a_1(n-1)=2^{2^{n-1}}$

Square both side

$(a_1(n-1))^2=(2^{2^{n-1}})^2=2^{2^{n-1}}=2^{2^n}=a_1(n)$.

② For $a_2(n)=3\cdot 2^{2^n}$

$a_2(n-1)^2=(3\cdot 2^{2^{n-1}})^2=9\cdot 2^{2^n}$

$a_2(n)=3\cdot 2^{2^n}\neq 9\cdot 2^{2^n}$

2. $a(n)=3a(n-1)-2a(n-2)+n$

$r^2=3r-2$

$r^2-3r+2=0$

$(r-2)(r-1)=0$

$r=2 \quad r=1$

So $a_h(n)=A+B2^n$

$a(n)-3a(n-1)+2a(n-2)=n$

$a_p(n-1)=a(n-1)^2+\beta(n-1)+y$

$=an^2+(-2a+\beta)n+(a-\beta+y)$

$a_p(n-2)=a(n-2)^2+\beta(n-2)+y$

$=an^2+(-4a-\beta)n+(4a-2\beta+y)$

Try $a_p(n)=an^2+\beta n+y$. $(an^2+\beta n+y)-3(an^2+(-2a+\beta)n+(a-\beta+y))$

$+2(an^2+(-4a-\beta)n+(4a-2\beta+y))$

right hand side $=n$

$-2a=1 \quad 5a+\beta=0$

$=(0)n^2+n(\beta-3(-2a+\beta)+2(-4a+\beta))$

$a=-\frac{1}{2} \quad \beta=5a$

$+[y-3(a-\beta+y)+2(4a-2\beta+y)]$

the initial condition $a(0)=2$ $a(1)=3$

$n=0$ $a(0)=2=A+B$ $\rightarrow A+B=2 \rightarrow A=2-$

$n=1$ $a(1)=3=A+2B-\frac{1}{2}-\frac{5}{2}=A+2B-3=3$

$A+2B=6$

$2-B+2B=6$

$B=4$

$A=2-4=-2$

$$a(n)=4\cdot 2^n-\frac{n^2+5n}{2}-2$$

3. $a(n)=2a(n-1)+2a(n-2)-2a(n-3)+3.$ $a(0)=3$ $a(1)=2$ $a(2)=$

$a(n)-2a(n-1)-2a(n-2)+2a(n-3)=3.$

$r^3-2r^2-r+2=0.$

$\cancel{2a+2a}=3a$

$-3a$

$b(n)=C_1r_1^n+C_2r_2^n+C_3r_3^n$

$a_p(n)=R$

$R-2R-2R+2R=3$ $R=-3.$

$b(n)=a(n)-a_p(n)=a(n)+3.$

$b(n)=2b(n-1)+2b(n-2)-2b(n-3))$

$b(0)=a(0)+3=6$ $b(1)=2+3=5.$ $b(2)=6+3=9$

$\begin{cases} C_1+C_2+C_3=6 \\ C_1r_1+C_2r_2+C_3r_3=5 \\ C_1r_1^2+C_2r_2^2+C_3r_3^2=9 \end{cases}$ $r_1\approx-1.17$ $r_2\approx0.69$ $r_3\approx$

$C_1\approx0.501$ $C_2\approx4.49$ $C_3\approx1.00$

$a(n)\approx-3+0.501(-1.17)^n+4.49(0.69)^n+1.00(2.48)^n$