

HW3

ABZ account the 3 play

hw3 by Zuhairah Ali, pd 5

1. (a)

Investigating
weight

$$a_1(n-1)^2 \quad \text{then } (2^{n-1})^2 = 2^{2(n-1)} \times 2 = 2^{2n-2} \times 2 = 2^{2n-1}$$

$$2^{2n} = a_1(n)$$

Thus $a_1(n) = 2^{2n}$ satisfies $a(n) = a(n-1)^2$ ~~from~~ _{by 1a}

$$\text{Now for } a_2(n) = 3 \cdot 2^{2n}$$

$$a_2(n-1)^2 = (3 \times 2^{2(n-1)})^2 = 9 \cdot 2^{2(n-1)} = 9 \cdot 2^{2n-2}$$

So

$$a_2(n) = a_2(n-1)^2$$

$$\Rightarrow 3 \times 2^{2n} = 9 \times 2^{2n-2}$$

$$\Rightarrow 3 = 9 \quad \forall n \in \mathbb{N}$$

This is clearly false so

 $a_2(n)$ is not a solution.2. Let's begin with $a_1(n)$, the homogeneous solutionSuppose $a(n) = z^n$. Then $a(n) = 3a(n-1) - 2a(n-2)$ ~~then~~

$$\Rightarrow z^n = 3z^{n-1} - 2z^{n-2}$$

$$\Rightarrow z^2 = 3z - 2$$

$$\Rightarrow z^2 - 3z + 2 = 0$$

$$\Rightarrow (z-2)(z-1) = 0$$

$$\Rightarrow z=2 \quad \text{or } z=1.$$

$$\text{So } a_h(n) = A(2^n) + B(1)^n = A2^n + B$$

for some constants A and B.

Now let's investigate the particular.

Assume $a_p(n) = c_n + d$.

$$\text{Then } c_n + d = 3(c_{n-1} + d) - 2(c_{n-2} + d) + n$$

$$\Rightarrow c_n + d = 3c_{n-1} - 3c_{n-2} + 3d - 2c_{n-2} + 4c - 2d + n$$

$$\Rightarrow c_n + d = c_n + c + d + n$$

$$\Rightarrow c = c + 1 \Rightarrow 0 = 1 \text{ which is a contradiction,}$$

$$a(n) = 3a(n-1) - 7a(n-2) + n$$

So we must adjust our ansatz.
Let's multiply out initial guess by n .

$$a_p(n) = Cn^2 + Dn$$

$$Cn^2 + Dn = 3(C(n-1)^2 + D(n-1)) - 7(C(n-2)^2 + D(n-2)) + n$$

$$= 3(Cn^2 - 6Cn + 3C + 3Dn - 3D) - 7(Cn^2 + 8Cn - 8C - 2Dn + 4D) + n$$

$$= Cn^2 + n(-6C + 3D + 8C - 2D + 1) + 3C - 3D - 8C + 4D$$

$$Cn^2 + Dn = Cn^2 + n(2C - D + 1) - 5C + D$$

$$\begin{cases} C = C \\ D = 2C - D + 1 \\ 0 = -5C + D \end{cases} \Rightarrow D = 5C \text{ and } C = -\frac{1}{2}$$

$$\text{So } D = -\frac{5}{2}$$

$$a_p(n) = -\frac{1}{2}n^2 - \frac{5}{2}n$$

$$\text{So } a(n) = A(2)^n + B - \frac{1}{2}n^2 - \frac{5}{2}n$$

$$a(0) = A + B - 0 - 0 = 2 \quad A + B = 2$$

$$a(1) = 2A + B - \frac{1}{2} - \frac{5}{2} = 3 \quad 2A + B - 3 = 3$$

$$\Rightarrow 2A + B = 6$$

$$2A + B - (A + B) = 6 - 2$$

$$\begin{matrix} A = 4 \\ B + 4 = 2 \\ B = -2 \end{matrix}$$

$$\text{So } a(n) = 4(2)^n - 2 - \frac{1}{2}n^2 - \frac{5}{2}n \quad \text{checked with Maple}$$

3.

Let's begin with the particular solution.
Suppose $a_p(n) = C$

$$\text{Then } C = 2C + 2C - 2C + 3$$

$$\Rightarrow C = -3 \quad \text{So } a_p(n) = -3$$

Now homogeneous, suppose $a_h(n) = z^n$

$$z^n = 3z^{n-1} - 7z^{n-2} + z^n \quad z^n = 2z^{n-1} + 2z^{n-2} - 2z^{n-3}$$

$$\Rightarrow z^3 - 2z^2 - 2z + 2 = 0$$

This gives 3 roots z_1, z_2, z_3
 $z_1 \approx -1.17, z_2 \approx 0.684, z_3 \approx 1.48$

$$\text{So } a_h(n) = A z_1^n + B z_2^n + C z_3^n$$

$$a(n) = A z_1^n + B z_2^n + C z_3^n - 3$$

$$\begin{aligned}
 a(0) &= A+B+C-3=3 \\
 a(1) &= A z_1 + B z_2 + C z_3 \text{ not } -3=2 \\
 a(2) &= A z_1^2 + B z_2^2 + C z_3^2 - 3=6
 \end{aligned}$$

$\Rightarrow$

$$\begin{aligned}
 A+B+C &= 6 \\
 A z_1 + B z_2 + C z_3 &= 5 \\
 A z_1^2 + B z_2^2 + C z_3^2 &= 4
 \end{aligned}$$

So we have

$$\begin{pmatrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ z_1^2 & z_2^2 & z_3^2 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \\ 4 \end{pmatrix}$$

Using my calculator I am not able to solve after multiplying by the inverse matrix

at ~~$A = 2.502$~~ $B = 2$ $C = 1.004$

So $a(n) \approx A z_1^n + B z_2^n + C z_3^n - 3$

where A, B, C are defined right above and $z_1 \approx -1.17$, $z_2 \approx 0.689$, $z_3 \approx 2.48$

Maybe did not give a helpful solution