1. (a) Prove that $a_1(n) = 2^{2^n}$ satisfies the non-linear recurrence equation

$$a(n) = a(n-1)^2 \quad .$$

Is the following constant multiple of the seqeuncec $a_1(n)$, given by $a_2(n) = 3 \cdot 2^{2^n}$, also a solution? Why?

a) $a_1(n) = (2^{2(n-1)})^2$

$= 2^2 \cdot 2^{2^n} \cdot 2^{-2}$

$= 2^{2^n} = 2^{2^n}$ ✓

b) $a_2(n) = [3(2^{2(n-1)})]^2$

$= 3^2 \cdot 2^{2(n-1)2}$

$= 9 \cdot 2^{2^n} \neq 3 \cdot 2^{2^n}$ x

2. Solve the following recurrence with the given initial conditions

non-homog.

$$a(n) = 3a(n-1) - 2a(n-2) + n \quad ; \quad a(0) = 2 \quad , \quad a(1) = 3$$

$a(n) - 3a(n-1) + 2a(n-2) = n$

$a(n) - 3a(n-1) + 2a(n-2) = 0$

$a(n) = 3a(n-1) - 2a(n-2)$

$z^2 = 3z - 2$

$z^2 - 3z + 2 = 0$

$(z-2)(z-1) = 0$

$\left.\begin{matrix} z_1 = 1 \\ z_2 = 2 \end{matrix}\right\} a(n) = C_1(1)^n + C_2(2)^n$

$a(n) = \alpha n^2 + \beta n \longrightarrow$ used this b/c $a(n) = \alpha n + \beta$ doesn't work b/c all n's will cancel out

$\alpha n^2 + \beta n - 3[\alpha(n-1)^2 + \beta(n-1)] + 2[\alpha(n-2)^2 + \beta(n-2)] = n$

$\alpha n^2 + \beta n - 3[\alpha(n^2 - 2n + 1) + \beta n - \beta] + 2[\alpha(n^2 - 4n + 4) + \beta n - 2\beta] = n$

$\alpha n^2 + \beta n - 3[\alpha n^2 - 2\alpha n + \alpha + \beta n - \beta] + 2[\alpha n^2 - \alpha 4n + 4\alpha + \beta n - 2\beta] = n$

$\cancel{\alpha n^2} + \cancel{\beta n} - \cancel{3\alpha n^2} + 6\alpha n - 3\alpha - \cancel{3\beta n} + 3\beta + \cancel{2\alpha n^2} - 8\alpha n + 8\alpha + \cancel{2\beta n} - 4\beta = n$

$-2\alpha n + 5\alpha - \beta = n$

$-2\alpha n = n \qquad 5\alpha - \beta = 0$

$\alpha = -1/2 \qquad 5(-\tfrac{1}{2}) = \beta$

$\beta = -5/2$

$a_{\alpha\beta}(n) = -\frac{1}{2}n^2 - \frac{5}{2}n$

$a(n) = C_1 + C_2(2)^n - \frac{1}{2}n^2 - \frac{5}{2}n$

$a(0) = C_1 + C_2(2)^0 - \frac{1}{2}(0)^2 - \frac{5}{2}(0) = 2$

$C_1 + C_2 = 2$

$C_1 = 2 - C_2 \Rightarrow C_1 = 2 - 4$

$C_1 = -2$

$a(1) = C_1 + C_2(2)^1 - \frac{1}{2}(1)^2 - \frac{5}{2}(1) = 3$

$C_1 + 2C_2 - \frac{1}{2} - \frac{5}{2} = 3$

$(2 - C_2) + 2C_2 - 3 = 3$

$2 + C_2 = 6$

$C_2 = 4$

$a(n) = -2(1)^n + 4(2)^n - \frac{1}{2}n^2 - \frac{5}{2}n$

3. Solve the following recurrence with the given initial conditions

$$a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3) \underline{+ 3} \quad ; \quad a(0) = 3 \quad , \quad a(1) = 2 \quad , \quad a(2) = 6 \quad .$$

non-homog

1. Solve homog eq.

$a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3)$

$z^3 = 2z^2 + 2z - 2$

$z^3 - 2z^2 - 2z + 2 = 0 \longrightarrow$ plugged in calc for the zeros

$z_1 = -1.17$

$z_2 = 0.69$

$z_3 = 2.48$

$\left.\begin{matrix} z_1 = -1.17 \\ z_2 = 0.69 \\ z_3 = 2.48 \end{matrix}\right\} a(n) = C_1(-1.17)^n + C_2(0.69)^n + C_3(2.48)^n$

$a(n) = \alpha n + \beta$

$a(n) - 2a(n-1) - 2a(n-2) + 2a(n-3) = 3$

$\alpha n + \beta - 2[\alpha(n-1) + \beta] - 2[\alpha(n-2) + \beta] + 2[\alpha(n-3) + \beta] = 3$

$\alpha n + \beta - 2[\alpha n - \alpha + \beta] - 2[\alpha n - 2\alpha + \beta] + 2[\alpha n - 3\alpha + \beta] = 3$

$\alpha n + \beta - \cancel{2\alpha n} + \cancel{2\alpha} - 2\beta - 2\alpha n + \cancel{4\alpha} - \cancel{2\beta} + \cancel{2\alpha n} - \cancel{6\alpha} + \cancel{2\beta} = 3$

$-\alpha n - \beta = 3$

$\underbrace{-\alpha n = 0 \qquad \beta = -3}_{a_{\alpha\beta}(n) = -3}$

$a(n) = C_1(-1.17)^n + C_2(0.69)^n + C_3(2.48)^n - 3$

$a(0) = C_1(-1.17)^0 + C_2(0.69)^0 + C_3(2.48)^0 - 3 = 3$

$C_1 + C_2 + C_3 = 6$

$a(1) = C_1(-1.17)^1 + C_2(0.69)^1 + C_3(2.48)^1 - 3 = 2$

$C_1(-1.17)^1 + C_2(0.69)^1 + C_3(2.48)^1 = 1$

$a(2) = C_1(-1.17)^1 + C_2(0.69)^1 + C_3(2.48)^1 - 3 = 6$

$C_1(-1.17)^1 + C_2(0.69)^1 + C_3(2.48)^1 = 9$

$a(n) = 0.502(-1.17)^n + 4.49(6.89)^n + (2.48)^n - 3$