Homework for Lecture 3 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Sept. 15, 2025.

Subject: hw3

with an attachment hw3FirstLast.pdf and/or hw3FirstLast.txt

1. (a) Prove that $a_1(n) = 2^{2^n}$ satisfies the non-linear recurrence equation

$$a(n) = a(n-1)^2 \quad .$$

$(a_1(n-1))^2 = (2^{2^{n-1}})^2 = 2^{2^n}$

$a_2(n) = 3 \cdot 2^{2^n}$

$(a_2(n-1))^2 = (3 \cdot 2^{2^{n-1}})^2 = 9 \cdot 2^{2^n}$] not a solution

Is the following constant multiple of the seqeuncec $a_1(n)$, given by $a_2(n) = 3 \cdot 2^{2^n}$, also a solution? Why? $a_1(n) = 2^{2^n}$ works because squaring the term gives the same term back but $a_2(n) = 3 \cdot 2^{2^n}$ doesn't work because the 3^2 became 9 so it doesn't follow the same rule

2. Solve the following recurrence with the given initial conditions

$r^2 - 3r + 2 = 0$

$(r-1)(r-2) \to r = 1, 2$

$a(n) = 1^n C_1 + 2^n C_2$

$a(n) = Dn^2 + En$

$= 2D + 1 = 0 \Rightarrow D = -1/2$

$-5D + E = 0 \Rightarrow E = -5/2$

$a(n) = -1/2 n^2 - 5/2 n$

$a(n) = C_1 + C_2 2^n - 1/2 n^2 - 5/2 n$

$$a(n) = 3a(n-1) - 2a(n-2) + n \quad ; \quad a(0) = 2 \quad , \quad a(1) = 3 \quad .$$

$a(0) = 2 \to C_1 + C_2 = 2$

$a(1) = 3 \to C_1 + 2C_2 - 1/2 - 5/2 = 3 \Rightarrow C_1 + 2C_2 = 6$

$C_1 = -2$, $C_2 = 4$ $\Rightarrow \boxed{a(n) = -2 + 4(2^n) - \frac{1}{2} n^2 - 5/2 n}$

3. Solve the following recurrence with the given initial conditions

$$a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3) + 3 \quad ; \quad a(0) = 3 \quad , \quad a(1) = 2 \quad , \quad a(2) = 6 \quad .$$

$r^3 - 2r^2 - 2r + 2 = 0$

$r_1 \approx -1.17 \quad r_2 \approx .69 \quad r_3 \approx 2.48$

$a(n) = Ar_1^n + Br_2^n + Cr_3^n - 3$

$k = 2k + 2k - 2k + 3 \Rightarrow k = -3 \quad a(n) = -3$

$a(0) = 3 \Rightarrow A + B + C - 3 = 3 \Rightarrow A + B + C = 6$

$a(1) = 2 \Rightarrow Ar_1 + Br_2 + Cr_3 - 3 = 2 \Rightarrow Ar_1 + Br_2 + Cr_3 = 5$

$a(2) = 6 \Rightarrow Ar_1^2 + Br_2^2 + Cr_3^2 - 3 = 6 \Rightarrow Ar_1^2 + Br_2^2 + Cr_3^2 = 9$

$A \approx .5 \quad B \approx 4.49 \quad C \approx 1$

$\boxed{a(n) \approx .5(-1.17)^n + 4.49(.69)^n + 1(2.48)^n - 3}$