HW 3

1. Prove $a_1(n) = 2^{2^n}$ satisfies $a(n) = a(n-1)^2$?

$$a(n-1)^2 = (2^{2^{n-1}})^2$$
$$= 2^{2^{n-1}\cdot 2}$$
$$= 2^{2^n}$$

Is $a_2(n) = 3 \cdot 2^{2^n}$ a solution?

$$a(n-1)^2 = (3 \cdot 2^{2^{n-1}})^2$$
$$= 9 \cdot 2^{2^n} \neq 3 \cdot 2^{2^n} = a(n)$$

No, the squared term means a constant multiple of a solution is not also a solution.

2. $a(n) = 3a(n-1) - 2a(n-2) + n$

$$a(n) - 3a(n-1) + 2a(n-2) = n$$

solve homogeneous eqn:

$$a(n) - 3a(n-1) + 2a(n-2) = 0$$
$$r^2 - 3r + 2 = 0$$
$$(r-2)(r-1) = 0$$
$$r = 1, 2$$
$$a(n) = c_1(1)^n + c_2(2)^n$$
$$a(n) = c_1 + c_2 \cdot 2^n$$

Particular solution $\alpha n^2 + b$

$$\alpha n^2 + b - 3(\alpha(n-1)^2 + b) + 2(\alpha(n-2)^2 + b)$$
$$\alpha n^2 + b - 3(\alpha(n^2 - 2n + 1) + b) + 2(\alpha(n^2 - 4n + 4) + b)$$
$$\alpha n^2 + b - 3\alpha n^2 + 6\alpha n + -3\alpha + -3b + 2\alpha n^2 - 8\alpha n + 8\alpha + 2b$$
$$b \quad + 6\alpha n - 3\alpha - 3b \quad - 8\alpha n + 8\alpha + 2b$$
$$-2\alpha n + 5\alpha = n$$

$-2\alpha = 1$ $\qquad$ $b - 3b + 2b$?

$\alpha = -\frac{1}{2}$

3. $a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3) + 3$

$a(n) - 2a(n-1) - 2a(n-2) + 2a(n-3) = 3$

$r^3 - 2r^2 - 2r + 2 = 3$

homogeneous:

$r^3 - 2r^2 - 2r + 2 = 0$

$r^2(r-1) - 2(r-1) = 0$

$(r^2 - 2)(r-1) = 0$

$(r+\sqrt{2})(r-\sqrt{2})(r-1) = 0$

$r = \sqrt{2}, -\sqrt{2}, 1$

$a(n) = c_1(\sqrt{2})^n + c_2(-\sqrt{2})^n + c_3(1)^n$

particular solution:

$c = 2c + 2c - 2c + 3$

$c = -3$

$a(n) = c_1(\sqrt{2})^n + c_2(-\sqrt{2})^n + c_3(1)^n - 3$

$a(0) = c_1 + c_2 + c_3 - 3 = 3$

$c_1 + c_2 + c_3 = 6$

$a(1) = c_1\sqrt{2} - c_2\sqrt{2} + c_3 - 3 = 2$

$c_1\sqrt{2} - c_2\sqrt{2} + c_3 = 5$

$a(2) = 2c_1 + 2c_2 + c_3 - 3 = 6$

$2c_1 + 2c_2 + c_3 = 9$

$-2c_1 - 2c_2 - 2c_3 = -12$

$-c_3 = -3$

$c_3 = 3$

$\sqrt{2}c_1 - \sqrt{2}c_2 = 2$

$2c_1 + 2c_2 = 6$

$2c_1 - 2c_2 = 2\sqrt{2}$

$4c_1 = 6 + 2\sqrt{2}$

$c_1 = \dfrac{3+\sqrt{2}}{2}$

$2c_2 = 3 + \sqrt{2} - 2\sqrt{2}$

$c_2 = \dfrac{3-\sqrt{2}}{2}$

$$\boxed{a(n) = \frac{3+\sqrt{2}}{2}(\sqrt{2})^n + \frac{3-\sqrt{2}}{2}(-\sqrt{2})^n}$$