

Katryn Keating

Homework #3

1) $a_n = 2^{2^n}$, $a_n = a_{n-1}^2$
(a) Let $n \in \mathbb{N}$. Then $a_0 = 2^{2^0} = 2$
For $n=1$, $a_1 = 2^{2^1} = 4$
Then $a_1 = 4 = 2^2 = a_0^2$
For the function $a_n = 2^{2^n}$,
 $a_{n+1} = 2^{2^{n+1}} = (2^{2^n \cdot 2}) = (2^{2^n})^2 = a_n^2$

(b) No, $a_2(n) = 3 \cdot 2^{2^n}$ is not a solution
 $a_{n+1} = 3 \cdot 2^{2^{n+1}} = 3 \cdot 2^{2^n \cdot 2} = 3(2^{2^n})^2 \neq (3 \cdot 2^{2^n})^2$

2.) $a_n = 3a_{n-1} - 2a_{n-2} + n$, $a_0 = 2$, $a_1 = 3$

$$a_n - 3a_{n-1} + 2a_{n-2} = n \Rightarrow r^2 - 3r + 2 = 0$$

$$r=1, 2 \quad a_{\text{homog}} = C_1(1)^n + C_2(2)^n$$

$$a_{\text{part}} = C_1 n + C_0$$

$$C_1 n + C_0 - 3(C_1(n-1) + C_0) + 2(C_1(n-2) + C_0) = n$$

$$C_1 n + C_0 - 3C_1 n + 3C_1 - 3C_0 + 2C_1 n - 4C_1 + 2C_0 = n$$

$$3C_1 - 4C_1 = n \Rightarrow C_1 = -n$$

$$a_{\text{part}} = -n^2 \quad a_{\text{gen}} = C_1(1)^n + C_2(2)^n - n^2$$

$$a(0) = C_1 + C_2 = 2 \quad \left. \begin{array}{l} C_1 = 0 \\ C_2 = 2 \end{array} \right\}$$

$$a(1) = C_1 + 2C_2 = 4$$

$$\text{IVP sol: } a(n) = 2(2)^n - n^2$$

3.) $a(n) - 2a(n-1) - 2a(n-2) + 2a(n-3) = 3$

$$r^3 - 2r^2 - 2r + 2 = 0 \Rightarrow r^2(r-2) - 2(r-1) = 0$$

not factorable $\therefore$