Homework for Lecture 3 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Sept. 15, 2025.

Subject: hw3

with an attachment hw3FirstLast.pdf and/or hw3FirstLast.txt

1. (a) Prove that $a_1(n) = 2^{2^n}$ satisfies the non-linear recurrence equation

$$a(n) = a(n-1)^2 \quad .$$

Is the following constant multiple of the seqeuncec $a_1(n)$, given by $a_2(n) = 3 \cdot 2^{2^n}$, also a solution? Why?

2. Solve the following recurrence with the given initial conditions

$$a(n) = 3a(n-1) - 2a(n-2) + n \quad ; \quad a(0) = 2 \quad , \quad a(1) = 3 \quad .$$

3. Solve the following recurrence with the given initial conditions

$$a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3) + 3 \quad ; \quad a(0) = 3 \quad , \quad a(1) = 2 \quad , \quad a(2) = 6 \quad .$$

1. (a) Prove that $a_1(n) = 2^{2^n}$ satisfies the non-linear recurrence equation

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Is the following constant multiple of the seqeuncec $a_1(n)$, given by $a_2(n) = 3 \cdot 2^{2^n}$, also a solution? Why?

$a_1(n-1) = \left(2^{2(n-1)}\right)^2$

$= 2^2 \cdot 2^{2^n} \cdot 2^{-2}$

$= 2^{2n} = 2^{2n}$

$a_2(n-1) = \left[3\left(2^{2(n-1)}\right)\right]^2$

$= 3^2 \cdot 2^{2(n-1)2}$

$= 9 \cdot 2^{2^n} \neq 3 \cdot 2^{2n}$

2. Solve the following recurrence with the given initial conditions

$$a(n) = 3a(n-1) - 2a(n-2) + n \quad ; \quad a(0) = 2 \quad , \quad a(1) = 3 \quad .$$

$a(n) - 3a(n-1) + 2a(n-2) = n$

$z^2 - 3z + 2 = 0$

$(z-2)(z-1) = 0$

$z = 2, 1$ $\qquad a(n) = C_1 2^n + C_2$

$a(n) = C_1 n + C_0$

$(C_1 n + C_0) - 3(C_1(n-1) + C_0) + 2(C_1(n-2) + C_0) = n$

$C_1 n + C_0 - 3C_1 n + 3C_1 - 3C_0 + 2C_1 n - 4C_1 + 2C_0 = n$ → this didnt work, all of the n cancelled

$a(n) = C_2 n^2 + C_1 n$

$n^2 - 4n + 4$

$(C_2 n^2 + C_1 n) - 3(C_2(n-2)^2 + C_1(n-1)) + 2(C_2(n-2)^2 + C_1(n-1)) = n$

$C_2 n^2 + C_1 n - 3C_2 n^2 + 12C_2 n - 12C_2 - 3C_1 n + 3C_1 + 2C_2 n^2 - 8C_2 n + 8C_2 + 2C_1 n - 2C_1 = n$

$0 + 0 + 4C_2 n - 4C_2 + C_1 = n$

$4C_2 n = n$ $\qquad -4C_2 + C_1 = 0$

$C_2 = 1/4$ $\qquad -1 + C_1 = 0$

$\qquad C_1 = 1$

$a(n) = 1/4 n^2 + n$

$a(n) = C_1 2^n + C_2 + 1/4 n^2 + n$

$n=0: \; 2 = C_1 + C_2$

$C_1 = 2 - C_2$

$C_1 = 2 - 9/4$

$C_1 = -1/4$

$n=1: \; 3 = C_1(2) + C_2 + 1/4 + 1$

$3 = 2(2 - C_2) + C_2 + 5/4$

$7/4 = 4 - C_2$

$-9/4 = -C_2$

$9/4 = C_2$

$a(n) = -1/4(2)^n + 9/4 + 1/4 n^2 + n$

3. Solve the following recurrence with the given initial conditions

$$a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3) + 3 \quad ; \quad a(0) = 3 \quad , \quad a(1) = 2 \quad , \quad a(2) = 6 \quad .$$

$a(n) - 2a(n-1) - 2a(n-2) + 2a(n-3) = 3$

$z^3 - 2z^2 - 2z + 2 = 0$

$x = -1.17, 0.69, 2.48$ $\qquad$ $a(n) = C_1(-1.17)^n + C_2(0.69)^n + C_3(2.48)^n$

$a(n) = A$

$A - 2A - 2A + 2A = 3$

$-A = 3$

$A = -3$

$a(n) = -3$

$a(n) = C_1(-1.17)^n + C_2(0.69)^n + C_3(2.48)^n - 3$

$n=0 = 3 = C_1 + C_2 + C_3 - 3$ $\quad$ $n=1 = 2 = C_1(-1.17) + C_2(0.69) + C_3(2.48) - 3$ $\quad$ $n=2 = 6 = C_1(-1.17)^2 + C_2(0.69)^2 + C_3(2.48)^2 - 3$

$C_1 = 0.501 \qquad C_2 = 4.50 \qquad C_3 = 1.00$

$a(n) = 0.501(-1.17)^n + 4.5(0.69)^n + 1.00(2.48)^n - 3$