Homework for Lecture 3 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Sept. 15, 2025.

Subject: hw3

with an attachment hw3FirstLast.pdf and/or hw3FirstLast.txt

1. (a) Prove that $a_1(n) = 2^{2^n}$ satisfies the non-linear recurrence equation

$$a(n) = a(n-1)^2 \quad .$$

Is the following constant multiple of the seqeuncec $a_1(n)$, given by $a_2(n) = 3 \cdot 2^{2^n}$, also a solution? Why?

2. Solve the following recurrence with the given initial conditions

$$a(n) = 3a(n-1) - 2a(n-2) + n \quad ; \quad a(0) = 2 \quad , \quad a(1) = 3 \quad .$$

3. Solve the following recurrence with the given initial conditions

$$a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3) + 3 \quad ; \quad a(0) = 3 \quad , \quad a(1) = 2 \quad , \quad a(2) = 6 \quad .$$

1. (a) Prove that $a_1(n) = 2^{2^n}$ satisfies the non-linear recurrence equation

$$a(n) = a(n-1)^2 \quad .$$

Is the following constant multiple of the seqeuncec $a_1(n)$, given by $a_2(n) = 3 \cdot 2^{2^n}$, also a solution? Why?

a.)

$a(n) = a(n-1)^2$

$$a_1(n) = 2^{2^n}$$
$$= \left(2^{2^{(n-1)}}\right)^2$$
$$= 2^2 \cdot 2^{2^n} \cdot 2^{-2} = 2^{2^n} \checkmark$$

b.)

$a(n) = a(n-1)^2$

$$a_2(n) = 3 \cdot 2^{2^n}$$
$$= \left(3 \cdot 2^{2^{(n-1)}}\right)^2$$
$$= 3^2 \cdot 2^{2^{(n-1)\cdot 2}}$$
$$= 9 \cdot 2^{2^n} \cdot 2^{-2} \cdot 2^2$$
$$= 9 \cdot 2^{2^n} \neq 3 \cdot 2^{2^n}$$

not a solution bc not equivalent

2. Solve the following recurrence with the given initial conditions

$$a(n) = 3a(n-1) - 2a(n-2) + n \quad ; \quad a(0) = 2 \quad , \quad a(1) = 3 \quad .$$

$a(n) - 3a(n-1) + 2a(n-2) = n$ non-homog!

$a(n) - 3a(n-1) + 2a(n-2) = 0$

use:
$a(n) = \alpha n^2 + \beta n$

$a(n) = 3a(n-1) - 2a(n-2)$

$\alpha n^2 + \beta n - 3(\alpha(n-1)^2 + \beta(n-1)) + 2(\alpha(n-2)^2 + \beta(n-2)) = n$

$\alpha n^2 + \beta n - 3(\alpha(n^2 - 2n + 1) + \beta n - \beta) + 2(\alpha(n^2 - 4n + 4) + \beta n - 2\beta) = n$

$z^2 = 3z - 2$

$\alpha n^2 + \beta n - 3\alpha n^2 + 6\alpha n - 3\alpha - 3\beta n + 3\beta + 2\alpha n^2 - 8\alpha n + 8\alpha + 2\beta n - 4\beta = n$

$z^2 - 3z + 2 = 0$

$-2\alpha n = n$ $\quad$ $5\alpha - \beta = 0$

$(z-2)(z-1) = 0$ $\quad$ $\alpha = -1/2$ $\quad$ $5(-1/2) = \beta$

$z = 1, 2$ $\quad$ $\beta = -5/2$

$a(n) = c_1(1)^n + c_2(2)^n$ $\quad$ $a_p(n) = -1/2 n^2 - 5/2 n$

$a(n) = c_1 + c_2(2)^n - 1/2 n^2 - \frac{5}{2} n$

$a(0) = 2 = c_1 + c_2$ $\quad$ $c_1 + c_2 = 2$ $\quad$ $c_1 = 2 - c_2$

$a(1) = 3 = c_1 + 2c_2 - 3$ $\quad$ $6 = c_1 + 2c_2$

$6 = 2 + c_2$

$c_2 = 4$

$c_1 = 2 - 4 = -2$

$$\boxed{a(n) = -2 + 4(2)^n - \frac{1}{2} n^2 - \frac{5}{2} n}$$

3. Solve the following recurrence with the given initial conditions

non-homog

$$a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3) + 3 \; ; \quad a(0) = 3 \quad , \quad a(1) = 2 \quad , \quad a(2) = 6 \quad .$$

<u>1st do homog</u>

$a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3)$

$z^3 = 2z^2 + 2z - 2$

$z^3 - 2z^2 - 2z + 2 = 0$

$z = -1.17, .689, 2.48$

$a_p(n) = \alpha n + \beta$

$\alpha(n) - 2\alpha(n-1) - 2\alpha(n-2) + 2\alpha(n-3) = 3$

$\alpha n + \beta - 2(\alpha(n-1) + \beta) - 2(\alpha(n-2) + \beta) + 2(\alpha(n-3) + \beta) = 3$

$\alpha n + \beta - 2(\alpha n - \alpha + \beta) - 2(\alpha n - 2\alpha + \beta) + 2((\alpha n - 3\alpha) + \beta) = 3$

$\alpha n + \beta - 2\alpha n + 2\alpha - 2\beta - 2\alpha n + 4\alpha - 2\beta + 2\alpha n - 6\alpha + 2\beta = 3$

$-\alpha n = 0$ $\quad -\beta = 3$

$\beta = 3$

$a(n) = c_1(-1.17)^n + c_2(.689)^n + c_3(2.48)^n$ $\qquad a_p(n) = -3$

$a(n) = c_1(-1.17)^n + c_2(.689)^n + c_3(2.48)^n - 3$

$a(0) = 3 = c_1 + c_2 + c_3 - 3$ $\qquad 6 = c_1 + c_2 + c_3$

$a(1) = 2 = -1.17c_1 + .689c_2 + 2.48c_3 - 3$ $\qquad 5 = -1.17c_1 + .689c_2 + 2.48c_3$

$a(2) = 6 = c_1(-1.17)^2 + c_2(.689)^n + c_3(2.48)^2$

$c_1 = .502$ $\qquad c_2 = 4.49$ $\qquad c_3 = 1.00$

$$\boxed{a(n) = .502(-1.17)^n + 4.49(6.89)^n + (2.48)^n - 3}$$