Homework for Lecture 3 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Sept. 15, 2025.

Subject: hw3

with an attachment hw3FirstLast.pdf and/or hw3FirstLast.txt

1. (a) Prove that $a_1(n) = 2^{2^n}$ satisfies the non-linear recurrence equation

$$a(n) = a(n-1)^2 \quad .$$

Is the following constant multiple of the seqeuncec $a_1(n)$, given by $a_2(n) = 3 \cdot 2^{2^n}$, also a solution? Why?

2. Solve the following recurrence with the given initial conditions

$$a(n) = 3a(n-1) - 2a(n-2) + n \quad ; \quad a(0) = 2 \quad , \quad a(1) = 3 \quad .$$

3. Solve the following recurrence with the given initial conditions

$$a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3) + 3 \quad ; \quad a(0) = 3 \quad , \quad a(1) = 2 \quad , \quad a(2) = 6 \quad .$$

Daniyal Chaudhry Hw3

1. $\underbrace{a_1(n) = 2^{2^n}}$ $a(n) = a(n-1)^2$ and $a_2(n) = 3 \cdot a_1(n)$

→ Check if $[a_1(n-1)]^2 = a_1(n)$: $\underbrace{\left(2^{2^{n-1}}\right)^2}_{LHS} \Rightarrow 2^{2^{(n-1)} \cdot 2^1} \Rightarrow \underbrace{2^{2^n}}_{RHS}$

→ Check $[a_2(n-1)]^2 = a_2(n)$: $\left(3 \cdot 2^{2^{n-1}}\right)^2 \Rightarrow 9 \cdot 2^{2^{n-1} \cdot 2^1} \Rightarrow 9 \cdot 2^{2^n} \neq 3 \cdot 2^{2^n}$ This is not a solution.

→ The reason $a_2(n)$ is not a sol'n is the constant coefficient is squared, so $a(n-1)^2$ will be a constant multiple of $a(n)$, but not $a(n)$ itself as we desire.

2. $a(n) = 3a(n-1) - 2a(n-2) + n$ $a(0) = 2$ $a(1) = 3$

$a(n) - 3a(n-1) + 2a(n-2) = n$

① homogeneous form: $z^2 - 3z + 2 = 0 \rightarrow (z-2)(z-1) \rightarrow z = 1, 2$

homo. sol'n : $c_1(1)^n + c_2(2)^n \rightarrow c_1 + c_2(2)^n$

② particular sol'n: guess $d_1n^2 + d_0n$: $d_1n^2 + d_0n - 3(d_1(n-1)^2 + d_0(n-1)) + 2(d_1(n-2)^2 + d_0(n-2)) = n$

$d_1n + d_0$ did not work?

$d_1n^2 + d_0n - 3d_1n^2 + 6d_1n - 3d_1 - 3d_0n + 3d_0 + 2d_1n^2$
$-8d_1n + 8d_1 + d_0n - 4d_0 = n$

$\rightarrow d_0 + 6d_1 - 3d_0 - 8d_1 + d_0 = 1 = -d_0 - 2d_1$

$\rightarrow d_1 - 3d_1 + 2d_1 = 0$ ☑

$\rightarrow -3d_1 + 3d_0 + 8d_1 - 4d_0 = 0 = -d_0 + 5d_1$

$d_0 = 5d_1$: $d_1 = -1/7$, $d_0 = -5/7$

gen sol'n: $c_1 + c_2(2)^n - \frac{1}{7}n^2 - \frac{5}{7}n$ (??)

3. $a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3) + 3$ $a(0) = 3$ $a(1) = 2$ $a(2) = 6$

$a(n) - 2a(n-1) - 2a(n-2) + 2a(n-3) = 3$

① $a(n) - 2a(n-1) - 2a(n-2) + 2a(n-3) = 0$: $r^3 - 2r^2 - 2r + 2 = 0$? seems very difficult to factor

② y_p: guess c_1 : $c_1 - 2c_1 - 2c_1 + 2c_1 = 3$: $c_1 = -3$

③ so the final sol'n is: $y_h - 3$ where y_h is some $b_1(x)^n + b_2(y)^n + b_3(z)^n$ and x, y, z are roots of $r^3 - 2r^2 - 2r + 2 = 0$... ?