

## Homework for Lecture 23 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file and/or .txt file) to

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by 8:00pm Monday, Dec. 8, 2025.

Subject: hw21

with an attachment hw23FirstLast.pdf and/or hw23FirstLast.txt

**1.** In the Maple package DMB.txt run

`ExpGenLotka(2,50);`

ten times.

In how many cases did all the sepecies survive? In how many cases was there agreement with the positive stable equilibrium points, and between the three initial conditions?

**2.** In the Maple package DMB.txt run

`ExpGenLotka(3,50);`

ten times.

In how many cases did all the sepecies survive? In how many cases was there agreement with the positive stable equilibrium points, and between the three initial conditions?

**3.** In the Maple package DMB.txt run

`ExpGenLotka(4,50);`

ten times.

In how many cases did all the sepecies survive? In how many cases was there agreement with the positive stable equilibrium points, and between the three initial conditions?

**4.** In the Maple package DMB.txt run

`ExpGenLotka(5,50);`

ten times.

In how many cases did all the sepecies survive? In how many cases was there agreement with the positive stable equilibrium points, and between the three initial conditions?

The Equilibrium points are

$\{[0, 0], [0, 0.8474576271], [0.5208333333, 0], [0.5220646178, -0.01970055162]\}$

The stable Equilibrium points are

$\{[0.5208333333, 0]\}$

at time  $t=, 50$ , with initial conditions,  $[0.5, 0.5]$ , the ultimate behavior is

$[0.5189098018, 0.02997972993]$

and with initial conditions,  $[0.1, 0.1]$

$[0.5190809992, 0.02735103168]$

and with initial conditions,  $\left[\frac{1}{2}, 1\right]$

$[0.5188099176, 0.03150983934]$

The Equilibrium points are

$\{[0, 0], [0, 1.162790698], [0.2104770814, 0.8886810103], [2.777777778, 0]\}$

The stable Equilibrium points are

$\{[0, 1.162790698], [2.777777778, 0]\}$

at time  $t=, 50$ , with initial conditions,  $[0.5, 0.5]$ , the ultimate behavior is

$[2.777777723, 7.455287790 \times 10^{-15}]$

and with initial conditions,  $[0.1, 0.1]$

$[2.777777723, 2.596930501 \times 10^{-15}]$

and with initial conditions,  $\left[\frac{1}{2}, 1\right]$

$[2.777777723, 2.263413087 \times 10^{-12}]$

The Equilibrium points are

$\{[0, 0], [0, 0.6493506494], [0.5244755245, 0.5880483153], [1.086956522, 0]\}$

The stable Equilibrium points are

$\{[0.5244755245, 0.5880483153]\}$

at time  $t=, 50$ , with initial conditions,  $[0.5, 0.5]$ , the ultimate behavior is

$[0.5301601430, 0.5836721247]$

and with initial conditions,  $[0.1, 0.1]$

$[0.6311761384, 0.4963779390]$

and with initial conditions,  $\left[\frac{1}{2}, 1\right]$

$[0.5092912292, 0.5992974787]$

$\{[0, 0], [0, 4.166666667], [0.6410256410, 0], [1.103565365, -0.7074136955]\}$

The stable Equilibrium points are

$\{[0, 4.166666667], [1.103565365, -0.7074136955]\}$

at time  $t=, 50$ , with initial conditions,  $[0.5, 0.5]$ , the ultimate behavior is

$[3.521629977 \times 10^{-46}, 4.166666592]$

and with initial conditions,  $[0.1, 0.1]$

$[1.294779416 \times 10^{-43}, 4.166666592]$

and with initial conditions,  $\left[\frac{1}{2}, 1\right]$

$[3.825889182 \times 10^{-48}, 4.166666592]$

The Equilibrium points are

$\{[0, 0], [0, 0.6944444444], [0.6756756757, 0], [0.8389261745, -0.4474272931]\}$

The stable Equilibrium points are

$\{[0.6756756757, 0]\}$

at time  $t=, 50$ , with initial conditions,  $[0.5, 0.5]$ , the ultimate behavior is

$[0.6756755828, 1.531484977 \times 10^{-7}]$

and with initial conditions,  $[0.1, 0.1]$

$[0.6756754648, 3.492703019 \times 10^{-7}]$

and with initial conditions,  $\left[\frac{1}{2}, 1\right]$

$[0.6756755522, 2.046475062 \times 10^{-7}]$

The Equilibrium points are

$\{[0, 0], [0, 0.7812500000], [2.532833021, 0.1876172608], [2.631578947, 0]\}$

The stable Equilibrium points are

$\{[2.532833021, 0.1876172608]\}$

at time  $t=, 50$ , with initial conditions,  $[0.5, 0.5]$ , the ultimate behavior is

$[2.503751554, 0.2348318993]$

and with initial conditions,  $[0.1, 0.1]$

$[2.526871595, 0.1981735395]$

and with initial conditions,  $\left[\frac{1}{2}, 1\right]$

$[2.499420252, 0.2455921211]$

agreement  
between  
I.C.  
→ similar  
values

All  
survive

match

ExpGenLotka(2, 50);

ExpGenLotka(2, 50);

> ExpGenLotka(2, 50);

{[0, 0.], [0, 0.9615384615], [1.250000000, 0.], [1.966292135, -1.685393258]}

The stable Equilibrium points are

{[1.250000000, 0.]}

at time  $t=, 50$ , with initial conditions, [0.5, 0.5], the ultimate behavior is

$[1.249999883, 2.412660284 \times 10^{-9}]$

and with initial conditions, [0.1, 0.1]

$[1.249999878, 1.553914733 \times 10^{-8}]$

and with initial conditions,  $[\frac{1}{2}, 1]$

$[1.249999883, 3.636927514 \times 10^{-9}]$

The Equilibrium points are

{[-1.574074074, 2.962962963], [0, 0.], [0, 1.388888889], [0.500000000, 0.]}

The stable Equilibrium points are

{[0, 1.388888889]}

at time  $t=, 50$ , with initial conditions, [0.5, 0.5], the ultimate behavior is

$[3.300988967 \times 10^{-17}, 1.388888810]$

and with initial conditions, [0.1, 0.1]

$[3.632411978 \times 10^{-16}, 1.388888810]$

and with initial conditions,  $[\frac{1}{2}, 1]$

$[6.915095200 \times 10^{-18}, 1.388888810]$

The Equilibrium points are

{[0, 0.], [0, 10.], [1.785714286, 0.], [41.17647059, -14.70588235]}

The stable Equilibrium points are

{[0, 10.], [41.17647059, -14.70588235]}

at time  $t=, 50$ , with initial conditions, [0.5, 0.5], the ultimate behavior is

$[6.440307627 \times 10^{-168}, 9.999999894]$

and with initial conditions, [0.1, 0.1]

$[2.352550413 \times 10^{-154}, 9.999999894]$

and with initial conditions,  $[\frac{1}{2}, 1]$

$[4.074199769 \times 10^{-174}, 9.999999894]$

The Equilibrium points are

{[-1.356852103, 1.797829037], [0, 0.], [0, 0.5102040816], [1.250000000, 0.]}

The stable Equilibrium points are

{[1.250000000, 0.]}

at time  $t=, 50$ , with initial conditions, [0.5, 0.5], the ultimate behavior is

$[1.249998775, 5.741105880 \times 10^{-7}]$

and with initial conditions, [0.1, 0.1]

$[1.249999517, 2.204198505 \times 10^{-7}]$

and with initial conditions,  $[\frac{1}{2}, 1]$

$[1.249996443, 1.667139436 \times 10^{-6}]$

• # cases where all species survived = 3

• # times there was agreement w/ positive stable equ pts = 4

• # times there was agreement between 3 initial conditions = 4

For a list of the Continuous Dynamical Models procedures type: *HelpC()*; for help with a specific procedure type: *Help(ProcedureName)*; for example *Help(Feig)*:

> ExpGenLotka(3, 50);

The Equilibrium points are

{[-1.569373440, 2.205445250, -0.08322434907], [-1.553166069, 2.090800478, 0.], [0, 0, 0.], [0, 0, 25.], [0, 0.7241063245, 0.01833180568], [0, 0.7462686567, 0.], [0.7498500300, 0, 0.6298740252], [2.173913043, 0, 0.]}

The stable Equilibrium points are

{[-1.569373440, 2.205445250, -0.08322434907], [0, 0, 25.], [2.173913043, 0, 0.]}  
at time t=, 50, with initial conditions, [0.5, 0.5, 0.5], the ultimate behavior is

[2.170935716, 1.339759022 × 10<sup>-26</sup>, 0.0009866029924]

and with initial conditions, [0.1, 0.1, 0.1]

[2.173837281, 1.202560617 × 10<sup>-28</sup>, 0.00002509691146]

and with initial conditions, [1/3, 2/3, 1]

[2.487416341 × 10<sup>-173</sup>, 2.526112754 × 10<sup>-358</sup>, 22.23601387]

The Equilibrium points are

{[-0.6818181818, 4.924242424, 0.], [0, -10.41666667, 37.50000000], [0, 0, 0.], [0, 0, 2.173913043], [0, 2.083333333, 0.], [0.1800720288, 0, 1.680672269], [0.4129734085, -0.1152744203, 1.433655654], [1.428571429, 0, 0.]}

The stable Equilibrium points are

{[-0.6818181818, 4.924242424, 0.], [0, -10.41666667, 37.50000000], [1.428571429, 0, 0.]}  
at time t=, 50, with initial conditions, [0.5, 0.5, 0.5], the ultimate behavior is

[1.419157437, 7.185853403 × 10<sup>-15</sup>, 2.676216565 × 10<sup>-11</sup>]

and with initial conditions, [0.1, 0.1, 0.1]

[1.297471798, 3.887689892 × 10<sup>-6</sup>, 2.412369628 × 10<sup>-9</sup>]

and with initial conditions, [1/3, 2/3, 1]

[1.409030965, 2.475792473 × 10<sup>-12</sup>, 3.053392176 × 10<sup>-10</sup>]

> ExpGenLotka(3, 50);

The Equilibrium points are

{[-0.01521606817, 0.9890444309, 0.], [0, 0, 0.], [0, 0, 1.250000000], [0, 0.9803921569, 0.], [0, 1.005981512, -0.4350190321], [0.07504344621, 0.9641546860, -0.4493829761], [0.3947368421, 0, 0.8059210526], [0.5319148936, 0, 0.]}

The stable Equilibrium points are

{[0, 0.9803921569, 0.]}

at time t=, 50, with initial conditions, [0.5, 0.5, 0.5], the ultimate behavior is

[0.01263777132, 0.9725043212, 0.00004051307062]

and with initial conditions, [0.1, 0.1, 0.1]

[0.01615395280, 0.9701603897, 0.0001396370501]

and with initial conditions, [1/3, 2/3, 1]

[0.01107189505, 0.9735219694, 0.00003587365085]

The Equilibrium points are

{[0, 0, 0.], [0, 0, 1.], [0, 0.03559563360, 0.9373516849], [0, 5.555555556, 0.], [0.2227171492, 0, 0.8351893096], [0.4826538612, 0.2462269698, 0.2094766758], [0.6244109331, 0.2827521206, 0.], [0.9615384615, 0, 0.]}

The stable Equilibrium points are

{[0, 5.555555556, 0.], [0.2227171492, 0, 0.8351893096]}

at time t=, 50, with initial conditions, [0.5, 0.5, 0.5], the ultimate behavior is

[5.570115013 × 10<sup>-64</sup>, 5.555555208, 8.028999371 × 10<sup>-86</sup>]

and with initial conditions, [0.1, 0.1, 0.1]

[1.668081084 × 10<sup>-44</sup>, 5.555501899, 4.886069652 × 10<sup>-60</sup>]

and with initial conditions, [1/3, 2/3, 1]

[1.303277672 × 10<sup>-68</sup>, 5.555555440, 1.998589910 × 10<sup>-91</sup>]

> ExpGenLotka(3, 50);

The Equilibrium points are

{[-1.504424779, 0, 1.799410029], [-1.361111111, -2.313492063, 2.898148148], [0, -1.495016611, 1.937984496], [0, 0, 0.], [0, 0, 1.190476190], [0, 0.8928571429, 0.], [0.09633911368, 0.8670520231, 0.], [0.6410256410, 0, 0.]}

The stable Equilibrium points are

{[0, 0, 1.190476190]}

at time t=, 50, with initial conditions, [0.5, 0.5, 0.5], the ultimate behavior is

[4.490317784 × 10<sup>-19</sup>, 2.270247180 × 10<sup>-9</sup>, 1.190379535]

and with initial conditions, [0.1, 0.1, 0.1]

[2.686779155 × 10<sup>-14</sup>, 1.048175637 × 10<sup>-6</sup>, 1.188986057]

and with initial conditions, [1/3, 2/3, 1]

[1.336042711 × 10<sup>-21</sup>, 7.887464056 × 10<sup>-11</sup>, 1.190460799]

The Equilibrium points are

{[-10.07604563, 3.041825095, 0.], [-1.360544218, 0, 1.870748299], [-0.2173646828, 0.3926340926, 0.4951934852], [0, -0.8522727273, 1.014610390], [0, 0, 0.], [0, 0, 0.5102040816], [0, 1.515151515, 0.], [2.380952381, 0, 0.]}

The stable Equilibrium points are

{[-1.360544218, 0, 1.870748299], [0, 1.515151515, 0.]}

at time t=, 50, with initial conditions, [0.5, 0.5, 0.5], the ultimate behavior is

[3.213217917 × 10<sup>-22</sup>, 1.515151413, 1.326396896 × 10<sup>-10</sup>]

and with initial conditions, [0.1, 0.1, 0.1]

[1.745127132 × 10<sup>-20</sup>, 1.515151413, 3.808195337 × 10<sup>-10</sup>]

and with initial conditions, [1/3, 2/3, 1]

[6.499788911 × 10<sup>-22</sup>, 1.515151413, 4.597429474 × 10<sup>-10</sup>]

> ExpGenLotka(3, 50);

• `ExpGenLotka(3, 50);`

*The Equilibrium points are*  
 $\{[-0.1667284179, 0, 1.130048166], [0, -0.9391771020, 1.319320215], [0, 0, 0], [0, 0, 1], [0, 0.6578947368, 0], [0.08196721311, 0.6147540984, 0], [0.4092405552, -0.6485357774, 0.9012945313], [0.5000000000, 0, 0]\}$

*The stable Equilibrium points are*  
 $\{[0, 0, 1]\}$   
*at time  $t=, 50$ , with initial conditions,  $[0.5, 0.5, 0.5]$ , the ultimate behavior is*  
 $[0.00004608033477, 8.328159460 \times 10^{-7}, 0.9999559754]$

*and with initial conditions  $[0.1, 0.1, 0.1]$*

$[0.00009453898335, 7.186145671 \times 10^{-7}, 0.9999101640]$

*and with initial conditions  $\left[\frac{1}{3}, \frac{2}{3}, 1\right]$*

$[0.00002507449630, 6.352220998 \times 10^{-7}, 0.9999759542]$

• `ExpGenLotka(3, 50);`

*The Equilibrium points are*  
 $\{[-0.5943536404, 1.560178306, 0], [-0.1869060594, 0.7442780933, 0.2705596665], [0, 0, 0], [0, 0, 1.086956522], [0, 0.7246376812, 0], [0, 1.465201465, -0.8241758242], [0.7936507936, 0, 0.4313319531], [3.846153846, 0, 0]\}$

*The stable Equilibrium points are*  
 $\{[-0.5943536404, 1.560178306, 0], [0, 0, 1.086956522], [3.846153846, 0, 0]\}$   
*at time  $t=, 50$ , with initial conditions,  $[0.5, 0.5, 0.5]$ , the ultimate behavior is*  
 $[1.567945019 \times 10^{-13}, 0.03416802557, 1.039914366]$

*and with initial conditions  $[0.1, 0.1, 0.1]$*

$[7.917811303 \times 10^{-18}, 0.01251869442, 1.069728425]$

*and with initial conditions  $\left[\frac{1}{3}, \frac{2}{3}, 1\right]$*

$[8.906618458 \times 10^{-13}, 0.09594490233, 0.9546785552]$

`> ExpGenLotka(3, 50);`

*The Equilibrium points are*  
 $\{[0, 0, 0], [0, 0, 0.9090909091], [0, 0.1611374408, 0.6161137441], [0, 1.428571429, 0], [0.2152176564, 0.005487135715, 0.5273747104], [0.2187500000, 0, 0.5312500000], [0.6336939722, 0.4327666151, 0], [1.851851852, 0, 0]\}$

*The stable Equilibrium points are*  
 $\{[0, 0, 0.9090909091], [0, 1.428571429, 0], [1.851851852, 0, 0]\}$   
*at time  $t=, 50$ , with initial conditions,  $[0.5, 0.5, 0.5]$ , the ultimate behavior is*  
 $[8.762401328 \times 10^{-20}, 1.428571365, 5.362429701 \times 10^{-10}]$

*and with initial conditions  $[0.1, 0.1, 0.1]$*

$[4.847190098 \times 10^{-19}, 1.428571365, 1.772378037 \times 10^{-10}]$

*and with initial conditions  $\left[\frac{1}{3}, \frac{2}{3}, 1\right]$*

$[7.311639811 \times 10^{-18}, 1.428571151, 5.068568484 \times 10^{-8}]$

`> ExpGenLotka(3, 50);`

*The Equilibrium points are*  
 $\{[-0.3253796095, 0, 2.422270427], [0, 0, 0], [0, 0, 2.083333333], [0, 0.3972602740, 0.5273972603], [0, 2.941176471, 0], [0.05065130858, 0.3921951431, 0.4944739097], [0.5434782609, 0, 0], [1.137270551, -0.7382282522, 0]\}$

*The stable Equilibrium points are*  
 $\{[0, 0, 2.083333333], [0, 2.941176471, 0]\}$   
*at time  $t=, 50$ , with initial conditions,  $[0.5, 0.5, 0.5]$ , the ultimate behavior is*  
 $[4.504041613 \times 10^{-6}, 1.149774671 \times 10^{-44}, 2.083326705]$

*and with initial conditions  $[0.1, 0.1, 0.1]$*

$[1.762291592 \times 10^{-42}, 2.941176420, 6.988269436 \times 10^{-73}]$

*and with initial conditions  $\left[\frac{1}{3}, \frac{2}{3}, 1\right]$*

$[1.559787299 \times 10^{-6}, 7.299556138 \times 10^{-49}, 2.083331033]$

`> |`

• # cases where all species survived = 0

• # times there was agreement w/ positive stable equ pts = 0

• # times there was agreement between 3 initial conditions = 2



```
> ExpGenLotka(4, 50);

                                The Equilibrium points are

{[-1.256281407, 0, 0, 1.356783920], [-0.8191049388, 0, 1.260842278, 0.4811638049], [0, -0.6723716381, 0, 1.079869601], [0, -0.5081292361, -0.2939831178, 1.181338101], [0, 0, -0.08375209380, 0.8793969849], [0, 0, 0, 0], [0, 0, 0, 0.8333333333], [0, 0, 0.6666666667, 0], [0, 0.009790483650, 0.6657528882, 0], [0, 0.6666666667, 0, 0], [0.1352296263, -0.4322366762, -0.4845686647, 1.201987203], [0.3863119348, 0.2524551961, 0.1692282084, 0], [0.9615384615, 0, 0, 0], [1.709401709, -0.4273504274, 0, 0], [6.356837607, -2.110042735, 0, -1.041666667], [13.28125000, 0, -15.62500000, 0]]

                                The stable Equilibrium points are
                                {[0, 0, 0, 0.8333333333], [6.356837607, -2.110042735, 0, -1.041666667]}
                                at time t=, 50, with initial conditions, [0.5, 0.5, 0.5, 0.5], the ultimate behavior is
                                [8.231414142 × 10-8, 3.584472476 × 10-13, 0.00305034505, 0.8312495747]

                                and with initial conditions, [0.1, 0.1, 0.1, 0.1]

                                [1.322833831 × 10-4, 1.257653608 × 10-12, 0.004463497440, 0.8306771989]
                                and with initial conditions, [1/4, 1/2, 3/4, 1]

                                [2.531265944 × 10-8, 1.089477809 × 10-13, 0.004023095570, 0.8309403926]

> ExpGenLotka(4, 50);

                                The Equilibrium points are

{[-6.396148556, 9.697386520, 0, -7.565337001], [-1.087278950, 1.707929264, 0.2304620650, -0.8077581289], [-0.5694423902, 0.7617216388, 0.3697677858, 0], [-0.1831501832, 0.7326007326, 0, 0], [-0.1300390117, 0, 0.4765547311, 0.6463703817], [0, -0.2255690270, 0.5157946933, 0.8113679011], [0, 0, 0, 0], [0, 0, 0, 1.470588235], [0, 0, 0.6250000000, 0], [0, 0, 0.7189542484, -0.3267973856], [0, 0.3359592215, 0.4402224282, 0], [0, 0.4182156134, 0, 0.3020446097], [0, 0.6097560976, 0, 0], [0.3000923361, 0, 0, 0.6232686981], [3.333333333, 0, 0, 0], [3.608247423, 0, -0.4123711340, 0]]

                                The stable Equilibrium points are
                                {[ -1.087278950, 1.707929264, 0.2304620650, -0.8077581289], [ -0.1300390117, 0, 0.4765547311, 0.6463703817], [3.333333333, 0, 0, 0]}
                                at time t=, 50, with initial conditions, [0.5, 0.5, 0.5, 0.5], the ultimate behavior is
                                [3.333333006, 1.543919566 × 10-32, 4.206374054 × 10-6, 2.715970445 × 10-55]

                                and with initial conditions, [0.1, 0.1, 0.1, 0.1]

                                [3.333331359, 4.787076447 × 10-29, 2.692139426 × 10-7, 9.132737852 × 10-30]
                                and with initial conditions, [1/4, 1/2, 3/4, 1]

                                [3.333331373, 3.739572829 × 10-29, 2.678013089 × 10-7, 1.391725983 × 10-49]

> ExpGenLotka(4, 50);

                                The Equilibrium points are

{[-5.178571429, 0, 0, 7.500000000], [-0.2311991463, -0.8587969120, 1.043112095, 0.7111588959], [0, -0.3274670139, 0.6204638159, 0.4663053678], [0, 0, 0, 0], [0, 0, 0, 0.9090909091], [0, 0, 0.6097560976, 0], [0, 1.063829787, 0, 0], [0, 1.192052980, 0, -0.06622516556], [0, 2.625437573, -1.983663944, 0], [0.6785214536, -0.3657573532, 0.7890106541, 0], [0.9314985123, 0, 2.353259400, -2.972004328], [1.444444444, 0, 0.2222222222, 0], [1.770428016, -2.665369650, 0, 0.8365758755], [1.785714286, 0, 0, 0], [2.246543779, -1.612903226, 0, 0]]

                                The stable Equilibrium points are
                                {[ -5.178571429, 0, 0, 7.500000000], [1.444444444, 0, 0.222222222, 0]}
                                at time t=, 50, with initial conditions, [0.5, 0.5, 0.5, 0.5], the ultimate behavior is
                                [1.368022132, 6.523209556 × 10-7, 0.2466909006, 0.00004974868319]

                                and with initial conditions, [0.1, 0.1, 0.1, 0.1]

                                [1.272724838, 0.000633214482, 0.2726459119, 0.001050995617]
                                and with initial conditions, [1/4, 1/2, 3/4, 1]

                                [1.260625586, 3.489063843 × 10-6, 0.2777152700, 0.001211678801]

> ExpGenLotka(4, 50);

                                The Equilibrium points are

{[-0.3360961235, 0, 0.4244713988, 0.4856889070], [-0.06429096253, 0, 0, 0.6612784717], [-0.01793721973, 0, 0.6188340807, 0], [0, -2.500000000, 0, 3.750000000], [0, -2.238781844, 1.795874822, 2.623496702], [0, 0, 0, 0], [0, 0, 0, 0.6578947368], [0, 0, 0.3981216823, 0.4797876684], [0, 0, 0.6172839506, 0], [0, 0.8196721311, 0, 0], [0, 1.003224651, -0.4478681476, 0], [0.2437592832, 0.2433999329, 0, 0.3440180154], [0.2496157168, 0.3340386273, 0.1699994779, 0.1555516306], [0.3506319568, 0.3332424845, 0.2331694153, 0], [0.6413612565, 0.1047120419, 0, 0], [0.6578947368, 0, 0, 0]]

                                The stable Equilibrium points are
                                {[0, -2.238781844, 1.795874822, 2.623496702], [0.2496157168, 0.3340386273, 0.1699994779, 0.1555516306]}
                                at time t=, 50, with initial conditions, [0.5, 0.5, 0.5, 0.5], the ultimate behavior is
                                [0.2562740946, 0.4338734324, 0.1032287819, 0.08692505903]

                                and with initial conditions, [0.1, 0.1, 0.1, 0.1]

                                [0.3279452003, 0.2912329466, 0.1654058591, 0.1338659866]
                                and with initial conditions, [1/4, 1/2, 3/4, 1]

                                [0.3509488616, 0.3719697803, 0.09260953566, 0.06862130035]
```

} survived

- # cases where all species survived = 1
- # times there was agreement w/ positive stable equ pts = 0
- # times there was agreement between 3 initial conditions = 1

➤ *ExpGenLotka*(5, 50);

*The Equilibrium points are*

$\{[-30.43478261, 0, 0, 23.55072464, 0, 0], [-6.578390805, -1.378390805, 2.221379310, 5.539310345, 0, 0], [-4.528366193, -1.095700443, 0.096079322, 3.990244461, -0.4990053142], [-1.960751734, 0, 0.5019816335, 1.831767613, 0.2324498406], [-1.491748765, 0, 0.5986418871, 1.496410480, 0, 0], [-1.232169954, 0, 0, 1.145675266, 1.614567527], [-1.176803236, 0.2389121742, 0, 1.150098222, 0.8170431605], [-0.1527016246, 0.6484102262, 0.04065267876], [-0.1242750621, 0, 0.6628003314, 0, 0, 0], [-0.09287925697, 0, 0, 0, 1.733746130], [-0.06727719283, 0.2141448963, 0.3031356007, 0, 0.2277798581], [-0.002102448091, 0.3628825404, 0, 0, 0.5230890849], [0, -13.91982183, 3.034521158, 6.726057906, 0, 0], [-1.486822580, 1.911067302, 1.033452099, -1.480354458], [0, 0, 0, 0, 1.562500000], [0, 0, 0, 0.3819036428, 0.8225616921], [0, 0, 0, 0.555555556, 0, 0], [0, 0, 0.2049180328, 0.4918032787, 0, 0], [0, 0, 0.2304451568, 0.4895440069, -0.02691710573], [0, 0, 0.5574406205, 0.01211827436], [0, 0, 0.56818182, 0, 0, 0], [0.09741269694, 0.3977002963, 0.5118941722], [0, 0.1392165937, 0.3660599807, 0, 1.441479717], [0, 0.1872659176, 0.4681647940, 0, 0], [0.3617363344, 0, 0, 0.5225080386], [0, 1.851851852, 0, 0, 0, 0], [0, 2.317880795, 0, -0.6291390728, 0, 0], [0.0254959066, 0.1483002510, 0.4695615412, 0, 0, 0], [1.022222222, 0.4888888889, 0, 0, 0, 0], [1.439304073, 0.5258959560, 0, -0.8007117438, 0, 0], [16.66666667, 0, 0, 0, 0, 0]]$

*The stable Equilibrium points are*

$\{[-1.232169954, 0, 0, 1.145675266, 1.614567527], [0, 1.851851852, 0, 0, 0, 0], [16.66666667, 0, 0, 0, 0, 0]\}$   
*at time  $t \rightarrow 50$ , with initial conditions, [0.5, 0.5, 0.5, 0.5, 0.5], the ultimate behavior is*

$[16.66666598, 4.257236015 \times 10^{-132}, 1.133632193 \times 10^{-49}, 7.282079148 \times 10^{-135}, 9.905505656 \times 10^{-26}]$

*and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]*

$[4.776934081 \times 10^{-28}, 1.841715511, 0.002408007943, 7.670225693 \times 10^{-6}, 3.837254274 \times 10^{-8}]$

*and with initial conditions,  $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$*

$[16.66666598, 1.420538390 \times 10^{-134}, 4.690985485 \times 10^{-46}, 8.675375780 \times 10^{-141}, 3.585536792 \times 10^{-21}]$

*The Equilibrium points are*

$\{[0, 0, -0.9459459459, 0.1621621622], [0, 0, 0, -26.66666667, 14.44444444], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0.6172839506], [0, 0, 0, 0.3125000000, 0, 0], [0, 0.3476033663, 1.408708379, 0, 0, 0], [0, 0, 0.7483525075, 2.943147548, -1.703339663], [0, 0, 25, 0, 0, 0], [0.4193244136, 0.2967780314, 0.1295972216, 0.1573350591], [0, 0.4641665644, 0.3323179690, 0.1497011556, 0, 0], [0, 0.5150554675, 0.5348652932, 0, 0, 0], [0.5443338075, 0.5704215178, 0, -0.0891477335], [0, 0.7439031419, 0.03945612612, 0.4590653775], [0, 0.8226037196, 0, 0, 0.4644999285], [0.1060968320, 0, 0.07471607890, 0, 0, 1.219512195, 0, 0, 0, 0], [0.1448376828, 0.5091895515, 0.4817936949, 0, -0.006425652594], [0.1458112407, 0.5070254507, 0.4788573701, 0, 0, 0], [0.1958225453, 1.09660574, 0, 0, 0], [0.2434902174, 0.6908438130, 0, 0, 0.4592893905], [0.3759398496, 0, 0, 0.3759398496, 0.07471607890], [0.4451256516, 0.3670363656, -0.1276877519, 0.3633190532, 0, 0], [0.4551413843, 0.1169512231, 0.3992729656, 0, 0], [0.4947195092, -0.2374077793, 0.3972174168, 0.3942072851], [0.5030893471, 0.1383839277, -0.3587846946, 0.2942956212, 0.7578816699], [0.5170316302, 0, 0, 0.4105839416, 0, 0], [0.6311682350, 0, 0, 0, 0.5393619463], [0.7938345092, 0, -0.7634683348, 0.2285985811, 1.211343102], [0.8771729825, 0, 0, 0, 0], [0.9983640253, 0, 0, -0.8819581358, 0.1430429129], [12.90322581, 0, -13.70967742, 0, 0, 0], [60.12216405, 0, -62.21644089, -5.322862129, 0, 0]]$

*The stable Equilibrium points are*

$\{[0, 0, 0, 3.125000000, 0, 0], [0, 0, 25, 0, 0, 0], [0.2434902174, 0.6908438130, 0, 0, 0.4592893905]\}$   
*at time  $t \rightarrow 50$ , with initial conditions, [0.5, 0.5, 0.5, 0.5, 0.5], the ultimate behavior is*

$[1.869743777 \times 10^{-142}, 7.739103963 \times 10^{-132}, 24.999999922, 2.051901662 \times 10^{-106}, 1.546738906 \times 10^{-132}]$

*and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]*

$[1.089560128 \times 10^{-362}, 1.051970879 \times 10^{-138}, 24.999999922, 1.445108214 \times 10^{-402}, 1.195020186 \times 10^{-144}]$

*and with initial conditions,  $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$*

$[1.479480643 \times 10^{-373}, 1.417477576 \times 10^{-142}, 24.999999922, 1.584067203 \times 10^{-416}, 1.967788778 \times 10^{-264}]$

➤ *ExpGenLotka*(5, 50);

*The Equilibrium points are*

$\{[-3.474903475, 0, 3.523166023, 0, 0, 0], [-1.530612245, 0, 0, 0, 8.928571429], [-0.3949855553, 1.014010145, 0.7438781367, -0.5488725351, 0.4247001406], [-0.2812751138, 0, 2.782614519, 0, -2.996919368], [-0.2656339002, 1.611502156, 0, -0.8746389047, 0.5265352226], [-0.1670592058, 1.621018541, 0, 0, -1.486988628], [0, 0, -0.7017767422, 1.119566009, 0.7136095536], [0, 0, 0, 0, 0, 0], [0, 0, 0, 2.500000000], [0, 0, 0.6439894320, 0.5036327609], [0, 0, 0, 50, 0, 0], [0, 0, 0.5000000000, 0, 0, 0], [0, 1.30769231, -1.923076923, 0, 0, 0], [1.898734177, 0, -2.059662025], [0, 0.09359400998, 1.559900166, 0, -1.580698835], [0, 0.6048387097, 0.4032238065, 0, 0, 0], [0, 0.7343499197, 1.504815409, -1.735611557, 0, 0], [0.3766754334, 0.5595512855, -0.5716279319, 0.2770919435], [0, 0.9259259259, 0, 0, 0, 0], [0.1300156915, 0, -0.8322125084, 0.3992938803], [0, 4.166666667, 0, 0, -12.50000000], [0, 6.666666667, 0, -10, 0, 0], [0.1711614439, 0, 0.4435473976, 0.4061250031], [0.2208723443, 0.4755121397, 0.2317591181, 0, 0, 0], [0.3250157266, 0.5347032921, 0, 0, 0, 0], [0.3462460605, 0, -0.4471032217, 0.5415017438, 0.4401588720], [0.8705592684, 0, 0, 0.3781217024, 0, 0], [1.008998914, -0.9658563741, 0, 1.179717944, 0, 0], [1.502734568, 0, -1.209471317, 0.6282691292, 0, 0], [1.754176611, 1.145584726, 0, 0, -8.991646778], [2.031327566, -0.6855531725, -2.032773454, 1.367510904, 0, 0], [3.571428571, 0, 0, 0, 0, 0]]$

*The stable Equilibrium points are*

$\{[0, 0, 0, 50, 0, 0], [3.571428571, 0, 0, 0, 0, 0]\}$   
*at time  $t \rightarrow 50$ , with initial conditions, [0.5, 0.5, 0.5, 0.5, 0.5], the ultimate behavior is*

$[3.32426145 \times 10^{-100}, 0.00002478786387, 1.374991439 \times 10^{-62}, 49.99895894, 4.096865645 \times 10^{-1100}]$

*and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]*

$[3.391112284, 0.02859025538, 2.622055672 \times 10^{-73}, 5.335547191 \times 10^{-19}, 1.546047463 \times 10^{-73}]$

*and with initial conditions,  $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$*

$[5.3955145 \times 10^{-419}, 0.00001466570880, 1.372037458 \times 10^{-67}, 49.99937240, 1.774709446 \times 10^{-1218}]$

*The Equilibrium points are*

$\{[-0.9064688916, 1.524515863, 1.297896440, 0, 0, 0], [-0.02146576326, 0.3720529944, -1.431071120, 0.6803411426, 1.921138292], [0, 0, -5.113636364, 7.765151515, 0, 0], [0, 0, -3.181060680, 0.7548279580, 5.001960592], [0, 0, 0, 0, 0, 0], [0, 0, 0, 3.125000000], [0, 0, 0, 0.5140186916, 0.7476635514], [0, 0, 0, 1.190476190, 0, 0], [0, 0.3846153846, 0, 0, 0], [0.3833313299, -1.159975272, 0.6390882760, 1.551678000], [0, 0.4094631483, 0.5080376099, 0, 0], [0.4302944459, 0.5558417032, -0.09120954588], [0.5253940455, 0.6125997198, 0, 0], [0.5340857036, -0.4403864573, 0.8665445106, 0, 0], [0.1074938575, 4.299754300, 0, -5.743243243], [0.1488095238, 0, 0.8928571429], [0.1666666667, 0, 0, 0.3740488212, 0.8444101971], [0.1805712089, 0.3270074564, 0.3987925373, 0.0639414698, 0, 0], [0.2322202661, 0.4335687337, 0, 0.3200331764, 0.06561239405], [0.2500007728, 0.4515469909, 0, 0.3426594827, 0, 0], [0.3718020804, 0.3450941805, 0.6051447849, 0, 0], [0.4889006342, 0, 0.1004228330, 0, 0], [0.4938271605, 0.1234567901, 0, 0], [0.5133701907, 0.1246756177, 0, -1.032381812], [0.5434782609, 0, 0, 0, 0, 0], [0.6049708211, 0, 2.783366868, -0.3118326819, -2.549588720], [0.8148831488, -0.4458794588, 0, 0, 0, 0], [1.069078947, 0, 0, 0, -1.151315789], [1.132039574, 0, -0.6835725158, 0, 0, -0.3777995216], [1.137710588, -0.6934246796, -0.01068728990, 0, -0.3676705317]]$

*The stable Equilibrium points are*

$\{[0, 0, 3.846153846, 0, 0, 0], [0, 16.66666667, 0, 0, 0, 0], [0.4889006342, 0, 0, 1.004228330, 0, 0]\}$   
*at time  $t \rightarrow 50$ , with initial conditions, [0.5, 0.5, 0.5, 0.5, 0.5], the ultimate behavior is*

$[0.0008688864704, 4.244066250 \times 10^{-29}, 3.818424622, 1.45369356 \times 10^{-15}, 0.007563235670]$

*and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]*

$[3.120938678 \times 10^{-6}, 2.029128674 \times 10^{-45}, 3.845769491, 2.830824185 \times 10^{-29}, 0.0001793847265]$

*and with initial conditions,  $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$*

$[7.095359351 \times 10^{-71}, 1.317073093 \times 10^{-49}, 3.844828964, 4.505901542 \times 10^{-32}, 0.0007034398503]$

*The Equilibrium points are*

$\{[-1.808375635, 1.572697607, 0, 0, 2.234408992], [0, -0.1718213058, 0.6248047485, 0, 0, 0], [-1.909722222, 0.5, 0.34722222], [0, 0, -0.9875987523, 0.4723627746, 2.991097163], [0, 0, 0, 0, 0, 0], [0, 0, 0, 3.125000000], [0, 0, 0, 0.2491694352, 2.533222591], [0, 0, 0, 0.5050505050, 0, 0], [0, 0, 0.9531590414, 0.2450980392, 0, 0], [0, 1.111111111, 0, 0, 0, 0], [0.3936852935, 0, 0.1922747167, 0.3800517792], [0.4238164151, 0, 0.6615670870], [0.4868614112, 1.361377884, 0, -1.066259837], [0.4965422913, 1.230261962, -0.1005229338, -0.7326720624], [0.7497827659, 0.6340113118, -0.1904375380, 0, 0], [0.1039168665, 0.2797761791, 0, 0, 0], [0.1724137931, 0, 0, 0], [0.1957696897, 0, -0.5117755959, 0.4576793110, 1.613838023], [0.2270876096, 0.2870361320, 0.2981135764, 0, 0], [0.2376103191, 0, 0.4978501924, 0, 0], [0.2608474576, 0.2004347826, 0, 0.3679587325, -0.1040456890], [0.2687140115, 0.6333973129, 0, 0], [0.3201024328, 0, 0, 0.1664532650], [0.3236449332, 0.3613511390, 0.2466614297, 0, 0], [0.3462189505, 0.2233988112, 0.4219235917, 0, -0.1750531433], [0.5521472393, 0.4294478528, 0, 0, 0], [0.5545884079, 0.3797079981, -0.3914963508, 0.3303715593, 0, 0], [0.6883225138, 0, 0.6390664073, -1.533254187], [0.7073769308, 0, -0.5557961600, 0.6351956114, 0, 0], [1.06382987, 0, 0, 0], [1.344262295, 0.1315573770, 0, -4.323770492], [1.570361371, -0.8507160423, -0.9687016769, 1.480479985, -1.644245046]]$

*The stable Equilibrium points are*

$\{[0, 0, 0, 0.2491694352, 2.533222591], [0, 1.724137931, 0, 0, 0, 0], [0.7073769308, 0, -0.5557961600, 0.6351956114, 0, 0]\}$   
*at time  $t \rightarrow 50$ , with initial conditions, [0.5, 0.5, 0.5, 0.5, 0.5], the ultimate behavior is*

$[2.752663073 \times 10^{-6}, 2.731491985 \times 10^{-19}, 0.3562274546, 0.3082231456, 0.9737629331]$

*and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]*

$[7.930247786 \times 10^{-8}, 4.767223375 \times 10^{-15}, 0.8003860009, 0.2614313127, 0.2447297751]$

*and with initial conditions,  $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$*

$[8.313105571 \times 10^{-6}, 3.918623749 \times 10^{-25}, 0.03375344708, 0.3291229950, 1.656874864]$

*The Equilibrium points are*

$\{[-2.287142280, -0.8131269903, 0.6502889405, 0.200207601], [-1.883947833, -1.029613650, 0, 2.494361640], [-0.02618478260, 0.2018821317, 0.3919887378, 0.2946626973, 0, 0], [-73.75000000, 0, 0, 30, 0, 0], [-0.7773648833, 0.3975627083, 0.6598721242], [0, -0.3665689150, 0.7697947214, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1.219512195], [0, 0, 0, 0.5376344086, 0, 0], [0, 0, 0, 0.5889281508, -0.05889281508], [0, 0, 0.1623903865, 0.1120493667], [0, 0, 0.5166008823, 0.3598792663, 0, 0], [0, 0, 0.5319148936, 0, 0], [0, 0, 0.5597033572, 0.4714168158, -0.1450897691], [0, 0.006366509523, 0.5573031819, 0.4734879957, -0.1506067150], [0, 0.2693879718, 0.3456924465, 0.2680609374, 0, 0], [0, 1.250000000], [0, 0.1576834862, -0.3440366972, 0, 0], [0, 5.399675626, -3.353799815, 0, 1.157321594], [0.1567027256, 0.4151065579, 0.2408655802, 0, 0], [0.2074361870, 0.2335204159, 0.4380419361, 0.5030146931, -0.3267553731], [0.3038117944, 0, 0.4763583402, 0.3181906732, 0, 0], [0.3672716381, 0.06773591842, 0.4326248860, 0, 0], [0.3960128842, 0.4955120091, 0.3867070345, -0.1055846608], [0.4060734463, 0.3972457627, 0, 0, 0, 0], [0.4463106474, 0.07222044578, 0.4960490953, -0.09228777835], [0.4889975550, 0, 0.4482477588, 0, 0], [0.5086724483, 0, 0.4628085390, -0.02084723149], [0.9305210918, 0.4032258065, 0, 0], [1.282051282, 0, 0, 0, 0, 0], [71.04545455, 0.1454545455, 0, -32.59090909], [137.5000000, 0, 0, -62.50000000]]$

*The stable Equilibrium points are*

$\{[0, 0, 0.1623903865, 0.1120493667], [0,$

*The Equilibrium points are*

(8)

and with initial conditions,  $[0.1, 0.1, 0.1, 0.1, 0.1]$

and with initial conditions,  $\left[ \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1 \right]$

(9)

*The Equilibrium points are*

(9)

and with initial conditions,  $[0.1, 0.1, 0.1, 0.1, 0.1]$

and with initial conditions,  $\left[ \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1 \right]$

(9)

The Equilibrium points are

(9)

→ no agreement

all survived  
= agreement  
between  
I.C.

and with initial conditions,  $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$

10)

*The Equilibrium points are*

(9)

and with initial conditions,  $[0.1, 0.1, 0.1, 0.1, 0.1]$

0 survive

11)

- # cases where all species survived = 1
- # times there was agreement w/ positive stable eqn pts = 0
- # times there was agreement between 3 initial conditions = 0