```
read`DMB.txt`
```

For a list of the Main procedures type: Help(); for help with a specific procedure type: Help (ProcedureName); for example Help(Feig);

For a list of the Continuous Dynamical Models procedures type: HelpC(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig); **(1)**

#Problem 1

```
ExpGenLotka(2, 50);
```

The Equilibrium points are

$$\{[-3.908355795, 4.043126685], [0., 0.], [0., 0.9615384615], [4.545454545, 0.]\}$$

The stable Equilibrium points are

$$\{[-3.908355795, 4.043126685], [4.545454545, 0.]\}$$

at time t=, 50, *with initial conditions*, $[0.5, 0.5]$, *the ultimate behavior is*

$$[4.545454483, 7.077924258 \times 10^{-54}]$$

and with initial conditions, $[0.1, 0.1]$

$$[4.545454483, 2.467371674 \times 10^{-51}]$$

and with initial conditions, $\left[\frac{1}{2}, 1\right]$

$$[4.545454483, 2.326224095 \times 10^{-53}]$$ **(2)**

```
ExpGenLotka(2, 50);
```

The Equilibrium points are

$$\{[0., 0.], [0., 1.162790698], [0.2104770814, 0.8886810103], [2.777777778, 0.]\}$$

The stable Equilibrium points are

$$\{[0., 1.162790698], [2.777777778, 0.]\}$$

at time t=, 50, *with initial conditions*, $[0.5, 0.5]$, *the ultimate behavior is*

$$[2.777777723, 7.455287790 \times 10^{-15}]$$

and with initial conditions, $[0.1, 0.1]$

$$[2.777777723, 2.596930501 \times 10^{-15}]$$

and with initial conditions, $\left[\frac{1}{2}, 1\right]$

$$[2.777777723, 2.263413087 \times 10^{-12}] \quad \textbf{(3)}$$

ExpGenLotka$(2, 50)$;

The Equilibrium points are

$$\{[0., 0.], [0., 0.6493506494], [0.5244755245, 0.5880483153], [1.086956522, 0.]\}$$

The stable Equilibrium points are

$$\{[0.5244755245, 0.5880483153]\}$$

at time t=, 50, *with initial conditions*, $[0.5, 0.5]$, *the ultimate behavior is*

$$[0.5301601430, 0.5836721247]$$

and with initial conditions, $[0.1, 0.1]$

$$[0.6311761384, 0.4963779390]$$

and with initial conditions, $\left[\frac{1}{2}, 1\right]$

$$[0.5092912292, 0.5992974787] \quad \textbf{(4)}$$

ExpGenLotka$(2, 50)$;

The Equilibrium points are

$$\{[0., 0.], [0., 4.166666667], [0.6410256410, 0.], [1.103565365, -0.7074136955]\}$$

The stable Equilibrium points are

$$\{[0., 4.166666667], [1.103565365, -0.7074136955]\}$$

at time t=, 50, *with initial conditions*, $[0.5, 0.5]$, *the ultimate behavior is*

$$[3.521629977 \times 10^{-46}, 4.166666592]$$

and with initial conditions, $[0.1, 0.1]$

$$[1.294779416 \times 10^{-43}, 4.166666592]$$

and with initial conditions, $\left[\frac{1}{2}, 1\right]$

$$[3.825889182 \times 10^{-48}, 4.166666592] \quad \textbf{(5)}$$

ExpGenLotka$(2, 50)$;

The Equilibrium points are

$$\{[0., 0.], [0., 0.6944444444], [0.6756756757, 0.], [0.8389261745, -0.4474272931]\}$$

The stable Equilibrium points are

$$\{[0.6756756757, 0.]\}$$

at time t=, 50, *with initial conditions*, $[0.5, 0.5]$, *the ultimate behavior is*

$$[0.6756755828, 1.531484977 \times 10^{-7}]$$

and with initial conditions, $[0.1, 0.1]$

$$[0.6756754648, 3.492703019 \times 10^{-7}]$$

and with initial conditions, $\left[\frac{1}{2}, 1\right]$

$$[0.6756755522, 2.046475062 \times 10^{-7}] \quad \textbf{(6)}$$

ExpGenLotka$(2, 50)$;

The Equilibrium points are

$$\{[0.,\ 0.],\ [0.,\ 0.7812500000],\ [2.532833021,\ 0.1876172608],\ [2.631578947,\ 0.]\}$$

The stable Equilibrium points are

$$\{[2.532833021,\ 0.1876172608]\}$$

at time t=, 50, *with initial conditions*, $[0.5,\ 0.5]$, *the ultimate behavior is*

$$[2.505751554,\ 0.2348318993]$$

and with initial conditions, $[0.1,\ 0.1]$

$$[2.526871595,\ 0.1981735395]$$

and with initial conditions, $\left[\frac{1}{2},\ 1\right]$

$$[2.499420252,\ 0.2455921211] \qquad \textbf{(7)}$$

ExpGenLotka(2, 50);

The Equilibrium points are

$$\{[0.,\ 0.],\ [0.,\ 0.9615384615],\ [1.250000000,\ 0.],\ [1.966292135,\ -1.685393258]\}$$

The stable Equilibrium points are

$$\{[1.250000000,\ 0.]\}$$

at time t=, 50, *with initial conditions*, $[0.5,\ 0.5]$, *the ultimate behavior is*

$$[1.249999883,\ 2.412660284 \times 10^{-9}]$$

and with initial conditions, $[0.1,\ 0.1]$

$$[1.249999878,\ 1.553914733 \times 10^{-8}]$$

and with initial conditions, $\left[\frac{1}{2},\ 1\right]$

$$[1.249999883,\ 3.636927514 \times 10^{-9}] \qquad \textbf{(8)}$$

ExpGenLotka(2, 50);

The Equilibrium points are

$$\{[-1.574074074, 2.962962963], [0., 0.], [0., 1.388888889], [0.5000000000, 0.]\}$$

The stable Equilibrium points are

$$\{[0., 1.388888889]\}$$

at time t=, 50, *with initial conditions*, $[0.5, 0.5]$, *the ultimate behavior is*

$$[3.300988967 \times 10^{-17}, 1.388888810]$$

and with initial conditions, $[0.1, 0.1]$

$$[3.632411978 \times 10^{-16}, 1.388888810]$$

and with initial conditions, $\left[\frac{1}{2}, 1\right]$

$$[6.915095200 \times 10^{-18}, 1.388888810] \quad \textbf{(9)}$$

ExpGenLotka(2, 50);

The Equilibrium points are

$$\{[0., 0.], [0., 10.], [1.785714286, 0.], [41.17647059, -14.70588235]\}$$

The stable Equilibrium points are

$$\{[0., 10.], [41.17647059, -14.70588235]\}$$

at time t=, 50, *with initial conditions*, $[0.5, 0.5]$, *the ultimate behavior is*

$$[6.440307627 \times 10^{-168}, 9.999999894]$$

and with initial conditions, $[0.1, 0.1]$

$$[2.352550413 \times 10^{-154}, 9.999999894]$$

and with initial conditions, $\left[\frac{1}{2}, 1\right]$

$$\left[4.074199769 \times 10^{-174},\ 9.999999894\right] \tag{10}$$

ExpGenLotka(2, 50);

The Equilibrium points are

$$\{[-1.356852103,\ 1.797829037],\ [0.,\ 0.],\ [0.,\ 0.5102040816],\ [1.250000000,\ 0.]\}$$

The stable Equilibrium points are

$$\{[1.250000000,\ 0.]\}$$

at time t=, 50, *with initial conditions*, $[0.5,\ 0.5]$, *the ultimate behavior is*

$$\left[1.249998775,\ 5.741105880 \times 10^{-7}\right]$$

and with initial conditions, $[0.1,\ 0.1]$

$$\left[1.249999517,\ 2.204198505 \times 10^{-7}\right]$$

and with initial conditions, $\left[\frac{1}{2},\ 1\right]$

$$\left[1.249996443,\ 1.667139436 \times 10^{-6}\right] \tag{11}$$

 # for all the species to survuie, they both have positive population values at long time, the x>0 and y>0. In my 10 cases 2 out of 10 had specie survive (run 3 and 6).

#in the cases that the agreement was positve stable equilibrium--> all three initial conditions converge to the same stable equilibrium (within small numerical error). All 10 cases A single attracting (stable) equilibrium pt.

#Problem 2
ExpGenLotka(3, 50);

The Equilibrium points are

$$\{[-0.1959247649,\ 0.6922675026,\ 0.],\ [0.,\ 0.,\ 0.],\ [0.,\ 0.,\ 0.8474576271],\ [0.,\ 0.5181347150,\ 0.2590673575],\ [0.,\ 0.6172839506,\ 0.],\ [0.5952380952,\ 0.,\ 0.],\ [14.58333333,\ 0.,\ -12.50000000],\ [16.99738903,\ -0.4569190601,\ -14.19060052]\}$$

The stable Equilibrium points are

$\{[0., 0.5181347150, 0.2590673575]\}$

at time t=, 50, *with initial conditions*, $[0.5, 0.5, 0.5]$, *the ultimate behavior is*

$[0.2698751015, 0.4956717528, 0.03794795750]$

and with initial conditions, $[0.1, 0.1, 0.1]$

$[0.06323003779, 0.5103364470, 0.2121400251]$

and with initial conditions, $\left[\frac{1}{3}, \frac{2}{3}, 1\right]$

$[0.1827382861, 0.5070459903, 0.09981819900]$ **(12)**

ExpGenLotka(3, 50);

The Equilibrium points are

$\{[-15.24390244, 5.487804878, 0.], [-0.5756349428, 0.6244766955, 0.8024002233], [0., 0., 0.], [0., 0., 0.5102040816], [0., 0.4646549024, 0.4959799520], [0., 0.5102040816, 0.], [2.173913043, 0., 0.], [3.492647059, 0., -1.378676471]\}$

The stable Equilibrium points are

$\{[-0.5756349428, 0.6244766955, 0.8024002233], [2.173913043, 0., 0.]\}$

at time t=, 50, *with initial conditions*, $[0.5, 0.5, 0.5]$, *the ultimate behavior is*

$[1.099093757, 0.1739430220, 0.01928230253]$

and with initial conditions, $[0.1, 0.1, 0.1]$

$[0.1969532015, 0.4109528869, 0.3929650746]$

and with initial conditions, $\left[\frac{1}{3}, \frac{2}{3}, 1\right]$

$[0.6250677145, 0.2967432980, 0.1798827464]$ **(13)**

ExpGenLotka(3, 50);

The Equilibrium points are

$$\{[0.,\ 0.,\ 0.],\ [0.,\ 0.,\ 1.666666667],\ [0.,\ 1.020408163,\ 0.],\ [0.,\ 2.941176471,\ -1.568627451],\ [0.7095880928,\ 0.,\ 0.03461405331],\ [0.7462686567,\ 0.,\ 0.],\ [0.9276592204,\ -0.5511876534,\ 0.1393568784],\ [1.393188854,\ -1.083591331,\ 0.]\}$$

The stable Equilibrium points are

$$\{[0.,\ 0.,\ 1.666666667],\ [0.7462686567,\ 0.,\ 0.]\}$$

at time t=, 50, *with initial conditions*, $[0.5,\ 0.5,\ 0.5]$, *the ultimate behavior is*

$$[2.925768987 \times 10^{-11},\ 0.1651939973,\ 1.471201249]$$

and with initial conditions, $[0.1,\ 0.1,\ 0.1]$

$$[8.636650151 \times 10^{-12},\ 0.03751672748,\ 1.622414419]$$

and with initial conditions, $\left[\frac{1}{3},\ \frac{2}{3},\ 1\right]$

$$[2.559246633 \times 10^{-12},\ 0.1960611204,\ 1.434476835] \qquad \textbf{(14)}$$

ExpGenLotka(3, 50);

The Equilibrium points are

$$\{[0.,\ 0.,\ 0.],\ [0.,\ 0.,\ 25.],\ [0.,\ 0.5263157895,\ 0.],\ [0.,\ 0.5494505494,\ -0.2747252747],\ [0.2137110292,\ 0.3208658589,\ 0.4094631483],\ [0.4934210526,\ 0.1315789474,\ 0.],\ [0.5421313507,\ 0.,\ 0.06195786865],\ [0.5681818182,\ 0.,\ 0.]\}$$

The stable Equilibrium points are

$$\{[0.,\ 0.,\ 25.],\ [0.4934210526,\ 0.1315789474,\ 0.]\}$$

at time t=, 50, *with initial conditions*, $[0.5,\ 0.5,\ 0.5]$, *the ultimate behavior is*

$$[0.4337317574,\ 0.1805897188,\ 0.03387614748]$$

and with initial conditions, $[0.1,\ 0.1,\ 0.1]$

$$[0.2236494874, 0.3340586383, 0.2138072103]$$

and with initial conditions, $\left[\frac{1}{3}, \frac{2}{3}, 1\right]$

$$[0.1328797512, 0.3250709069, 1.117263082] \quad \textbf{(15)}$$

ExpGenLotka(3, 50);

The Equilibrium points are

$\{[0., 0., 0.], [0., 0., 0.5813953488], [0., 2.261306533, 0.5025125628], [0., 2.380952381, 0.],$
$[0.04074887375, 2.185724311, 0.4776673534], [0.2721088435, 1.836734694, 0.],$
$[0.6891433418, 0., 0.1166242578], [0.7246376812, 0., 0.]\}$

The stable Equilibrium points are

$\{[0.04074887375, 2.185724311, 0.4776673534]\}$

at time t=, 50, *with initial conditions,* $[0.5, 0.5, 0.5]$, *the ultimate behavior is*

$[0.1857114069, 1.789443024, 0.3858400064]$

and with initial conditions, $[0.1, 0.1, 0.1]$

$[0.3748214225, 1.160063296, 0.2686759611]$

and with initial conditions, $\left[\frac{1}{3}, \frac{2}{3}, 1\right]$

$$[0.1524271488, 1.891670098, 0.4069881623] \quad \textbf{(16)}$$

ExpGenLotka(3, 50);

The Equilibrium points are

$\{[-2.631578947, 0., 4.355716878], [-2.259353836, -0.2525813256, 4.288331298], [0., 0.,$
$0.], [0., 0., 1.724137931], [0., 1.255230126, 0.2092050209], [0., 1.724137931, 0.],$
$[0.4680504680, 0.1424501425, 0.], [0.6493506494, 0., 0.]\}$

The stable Equilibrium points are

$\{[0., 0., 1.724137931], [0., 1.724137931, 0.]\}$

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5], *the ultimate behavior is*

$$[3.292963584 \times 10^{-10}, 0.05953224303, 1.636018660]$$

and with initial conditions, [0.1, 0.1, 0.1]

$$[3.365360246 \times 10^{-8}, 0.01265615803, 1.705331586]$$

and with initial conditions, $\left[\frac{1}{3}, \frac{2}{3}, 1\right]$

$$[2.306058392 \times 10^{-11}, 0.07202282881, 1.617634058] \quad \textbf{(17)}$$

ExpGenLotka(3, 50);

The Equilibrium points are

$$\{[0., 0., 0.], [0., 0., 0.6410256410], [0., 2.777777778, 0.], [0., 3.787878788, -1.010101010], [1.290322581, 2.741935484, -1.397849462], [2.305475504, 0.2161383285, 0.], [2.941176471, 0., 0.]\}$$

The stable Equilibrium points are

$$\{[0., 2.777777778, 0.], [2.941176471, 0., 0.]\}$$

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5], *the ultimate behavior is*

$$[5.933663152 \times 10^{-29}, 2.777777698, 2.585342260 \times 10^{-8}]$$

and with initial conditions, [0.1, 0.1, 0.1]

$$[7.256470862 \times 10^{-28}, 2.777777698, 2.560010357 \times 10^{-8}]$$

and with initial conditions, $\left[\frac{1}{3}, \frac{2}{3}, 1\right]$

$$[9.318958505 \times 10^{-30}, 2.777777693, 3.135793767 \times 10^{-8}] \quad \textbf{(18)}$$

ExpGenLotka(3, 50);

The Equilibrium points are

$$\{[0., 0., 0.], [0., 0., 0.6410256410], [0., 1.036036036, 0.03003003003], [0., 1.041666667, 0.],$$

$$[0.5164676996, 0.2222504445, 0.2384782547], [0.5649717514, 0.1942090395, 0.], [0.8090614887, 0., 0.2157497303], [0.8196721311, 0., 0.]\}$$

The stable Equilibrium points are

$$\{[0., 1.036036036, 0.03003003003], [0.8090614887, 0., 0.2157497303]\}$$

at time t=, 50, *with initial conditions*, $[0.5, 0.5, 0.5]$, *the ultimate behavior is*

$$[1.693397574 \times 10^{-7}, 1.033194848, 0.04356814511]$$

and with initial conditions, $[0.1, 0.1, 0.1]$

$$[0.00001256986805, 1.030803732, 0.05370670344]$$

and with initial conditions, $\left[\frac{1}{3}, \frac{2}{3}, 1\right]$

$$[1.257116307 \times 10^{-8}, 1.033916394, 0.04023791466] \quad \textbf{(19)}$$

ExpGenLotka(3, 50);

The Equilibrium points are

$$\{[-0.2475974139, 0.5245500612, 0.6624148174], [0., 0., 0.], [0., 0., 0.7462686567], [0., 0.02352941176, 0.7294117647], [0., 2.941176471, 0.], [1.315789474, 0., 0.], [2.432712215, 0., -2.122153209], [4.658792651, -1.443569554, 0.]\}$$

The stable Equilibrium points are

$$\{[0., 2.941176471, 0.], [4.658792651, -1.443569554, 0.]\}$$

at time t=, 50, *with initial conditions*, $[0.5, 0.5, 0.5]$, *the ultimate behavior is*

$$[4.767765490 \times 10^{-82}, 2.941176380, 3.159026385 \times 10^{-29}]$$

and with initial conditions, $[0.1, 0.1, 0.1]$

$$[1.041489084 \times 10^{-75}, 2.941176380, 2.892725145 \times 10^{-27}]$$

and with initial conditions, $\left[\frac{1}{3}, \frac{2}{3}, 1\right]$

$$\left[8.154631856 \times 10^{-81},\ 2.941176380,\ 1.600355557 \times 10^{-28}\right] \quad \textbf{(20)}$$

#out of all 10 runs, run 5 was the only one where all the species survived >0. 7 runs out of the 10 showed agreement between the stable equilibriums and the long-term behavior produced by all three initial conditions.

#Problem 3

ExpGenLotka(4, 50);

The Equilibrium points are

$$\{[-1.017441860,\ 0.,\ 0.,\ 1.889534884],\ [-0.3032855939,\ 0.,\ 0.1853411963,\ 1.053074979],\ [0.,\ 0.,\ 0.,\ 0.],\ [0.,\ 0.,\ 0.,\ 0.9090909091],\ [0.,\ 0.,\ 0.4214328718,\ 0.5719446117],\ [0.,\ 0.,\ 0.7936507936,\ 0.],\ [0.,\ 0.1120000000,\ 0.,\ 0.8480000000],\ [0.,\ 0.1648933441,\ 0.08489558310,\ 0.7512326186],\ [0.,\ 0.6024096386,\ 0.,\ 0.],\ [0.,\ 0.6027727547,\ -0.01004621258,\ 0.],\ [0.3193150376,\ 0.1248428637,\ 0.4152059216,\ 0.2011264827],\ [0.5755395683,\ 0.,\ 0.6474820144,\ 0.],\ [0.6329113924,\ 0.,\ 0.,\ 0.],\ [0.7428667905,\ -0.2363667061,\ 0.9201418200,\ 0.],\ [1.191969887,\ -0.6900878294,\ 0.,\ 0.],\ [2.013147083,\ -1.437962202,\ 0.,\ -0.2465078061]\}$$

The stable Equilibrium points are

$$\{[0.,\ 0.1648933441,\ 0.08489558310,\ 0.7512326186],\ [0.5755395683,\ 0.,\ 0.6474820144,\ 0.]\}$$

at time t=, 50, *with initial conditions*, $[0.5,\ 0.5,\ 0.5,\ 0.5]$, *the ultimate behavior is*

$$[0.1391428322,\ 0.1843162240,\ 0.2501900208,\ 0.4636947628]$$

and with initial conditions, $[0.1,\ 0.1,\ 0.1,\ 0.1]$

$$[0.02848119021,\ 0.1491128922,\ 0.1540011892,\ 0.6762832307]$$

and with initial conditions, $\left[\frac{1}{4},\ \frac{1}{2},\ \frac{3}{4},\ 1\right]$

$$[0.06014356407,\ 0.2221879165,\ 0.1804153724,\ 0.5771963325] \quad \textbf{(21)}$$

ExpGenLotka(4, 50);

The Equilibrium points are

{[−2., 0., 0., 3.], [0., 0., 0., 0.], [0., 0., 0., 0.9259259259], [0., 0., 0.5775991964, 0.4018081366], [0., 0., 2.941176471, 0.], [0., 0.2793296089, 1.396648045, 0.], [0., 1.282051282, 0., 0.], [0., 2.173913043, 0., −2.173913043], [0., 3.752493637, −1.895164064, −2.705166128], [0.4409649350, 0.2417053788, 0.5671233523, 0.], [0.7919169853, 0.2457673403, 0., −0.2457673403], [0.8096628617, 0.1194584550, 0., 0.], [0.9111617312, 0., 0.7972665148, 0.], [0.9423986643, 0., −0.2365272238, 0.1632501623], [0.9936557956, −0.02554468826, −0.2639745542, 0.1714253629], [1.063829787, 0., 0., 0.]}

The stable Equilibrium points are

{[0., 1.282051282, 0., 0.], [0.9111617312, 0., 0.7972665148, 0.]}

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

$[6.895423150 \times 10^{-14}, 1.275490777, 0.0008008182308, 0.009720712388]$

and with initial conditions, [0.1, 0.1, 0.1, 0.1]

$[7.918295549 \times 10^{-12}, 1.277277715, 0.0009228455988, 0.006287363804]$

and with initial conditions, $\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right]$

$[1.238365343 \times 10^{-14}, 1.272938269, 0.0005809761274, 0.01468247664]$ **(22)**

ExpGenLotka(4, 50);

The Equilibrium points are

{[0., 0., −2.256699577, 3.279266573], [0., 0., 0., 0.], [0., 0., 0., 2.777777778], [0., 0., 0.5154639175, 0.], [0., 1.617720259, 0.2986560478, 0.], [0., 2., 0., 0.], [0., 9.506172840, 0., −1.975308642], [0., 10.16201688, 1.355187947, −2.604383541], [0.4641497835, 0.5624076278, 0., 0.1242528480], [0.4703177258, 0., 0.4912207358, 0.], [0.5001602784, 0.05891936547, 0.3934344679, 0.1045134570], [0.5127122201, 0., 0.4384745336, 0.05980986758], [0.9018538446, −0.5747407552, 0.5460037175, 0.], [0.9615384615, 0., 0., 0.], [4.166666667, 0., 0., −18.51851852]}

The stable Equilibrium points are

$$\{[0.,\ 10.16201688,\ 1.355187947,\ -2.604383541],\ [0.5001602784,\ 0.05891936547,\ 0.3934344679,\ 0.1045134570],\ [0.9018538446,\ -0.5747407552,\ 0.5460037175,\ 0.]\}$$

at time t=, 50, *with initial conditions*, $[0.5,\ 0.5,\ 0.5,\ 0.5]$, *the ultimate behavior is*

$$[0.5219723895,\ 0.0003537386549,\ 0.4282837549,\ 0.07020861979]$$

and with initial conditions, $[0.1,\ 0.1,\ 0.1,\ 0.1]$

$$[0.5318102105,\ 0.005334274315,\ 0.4142061018,\ 0.08354725963]$$

and with initial conditions, $\left[\frac{1}{4},\ \frac{1}{2},\ \frac{3}{4},\ 1\right]$

$$[0.4980857702,\ 2.118212625 \times 10^{-7},\ 0.4562311095,\ 0.04039498431] \qquad \textbf{(23)}$$

ExpGenLotka$(4,\ 50)$;

The Equilibrium points are

$$\{[-0.2518468771,\ 0.,\ 0.,\ 0.7891202149],\ [0.,\ 0.,\ 0.,\ 0.],\ [0.,\ 0.,\ 0.,\ 0.6329113924],\ [0.,\ 0.,\ 0.5094614265,\ 0.2911208151],\ [0.,\ 0.,\ 0.6849315068,\ 0.],\ [0.,\ 0.3207698476,\ 0.,\ 0.3608660786],\ [0.,\ 0.4875227235,\ 0.6114691786,\ 0.],\ [0.,\ 0.5882352941,\ 0.,\ 0.],\ [0.,\ 1.695955875,\ 1.535922056,\ -1.835859653],\ [0.1339515714,\ 0.1895574624,\ 0.,\ 0.3890635826],\ [0.2613879996,\ 0.,\ 0.4390731005,\ 0.1762166290],\ [0.4305684313,\ 0.1627641183,\ 0.4303756970,\ 0.],\ [0.4308281474,\ 0.,\ 0.4547630445,\ 0.],\ [0.5157232704,\ 0.1635220126,\ 0.,\ 0.],\ [0.5208333333,\ 0.,\ 0.,\ 0.],\ [0.5792837954,\ 0.1523885099,\ 0.4456996220,\ -0.1546457734]\}$$

The stable Equilibrium points are

$$\{[0.4305684313,\ 0.1627641183,\ 0.4303756970,\ 0.]\}$$

at time t=, 50, *with initial conditions*, $[0.5,\ 0.5,\ 0.5,\ 0.5]$, *the ultimate behavior is*

$$[0.2080869661,\ 0.2002366269,\ 0.4213055434,\ 0.2034318700]$$

and with initial conditions, $[0.1,\ 0.1,\ 0.1,\ 0.1]$

$$[0.3615168669,\ 0.1696270695,\ 0.4242936308,\ 0.06933581622]$$

and with initial conditions, $\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right]$

$$[0.01399640615, 0.2687487974, 0.4478652616, 0.3009322840] \quad \textbf{(24)}$$

ExpGenLotka(4, 50);

The Equilibrium points are

$\{[-15.32153647, 1.273198101, 0., 10.35822184], [-1.445086705, 0., -0.2890173410, 1.734104046], [-0.2049180328, 0., 0., 0.8196721311], [0., 0., -0.1844640295, 0.7481041197], [0., 0., 0., 0.], [0., 0., 0., 0.6849315068], [0., 0., 0.5102040816, 0.], [0., 0.2685907157, -0.4590552347, 0.7575177316], [0., 0.3293297172, 0., 0.5811700891], [0., 0.6024096386, 0., 0.], [0., 0.6067647818, -0.09036922282, 0.], [0.4122011542, 0.4534212696, 0., 0.], [0.4638249377, 0.4438931280, -0.1894694387, 0.], [0.9615384615, 0., 0., 0.], [0.9686038744, 0., -0.03340013360, 0.], [96.35275876, -4.147399042, 11.02679858, -65.13989324]\}$

The stable Equilibrium points are

$\{[0., 0.3293297172, 0., 0.5811700891]\}$

at time t=, 50, *with initial conditions,* $[0.5, 0.5, 0.5, 0.5]$, *the ultimate behavior is*

$[0.0001697563426, 0.3293399276, 1.425633096 \times 10^{-15}, 0.5810281957]$

and with initial conditions, $[0.1, 0.1, 0.1, 0.1]$

$[0.0001580683171, 0.3293386694, 7.790117080 \times 10^{-15}, 0.5810382222]$

and with initial conditions, $\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right]$

$$[0.00006475412616, 0.3293335881, 9.382420443 \times 10^{-16}, 0.5811159761] \quad \textbf{(25)}$$

ExpGenLotka(4, 50);

The Equilibrium points are

$\{[-1.952227071, 0., 0.4913882133, 3.181530466], [-0.1694915254, 0., 0.6779661017, 0.], [0., -0.8865248227, 0., 1.950354610], [0., -0.07941000723, -0.1281567443,$

1.533163506], [0., −0.01517911354, 0.5616272010, 0.], [0., 0., −0.2012650949, 1.552616446], [0., 0., 0., 0.], [0., 0., 0., 1.351351351], [0., 0., 0.5494505494, 0.], [0., 1.388888889, 0., 0.], [0.1221560544, −0.2252252252, 0.6375019087, 0.], [0.4989002498, −0.9112869875, 0.4606952346, 0.9131037843], [0.5367998965, 2.208640538, 0., −0.7793299702], [0.7174392936, 1.269315673, 0., 0.], [1.724137931, 0., 0., 0.], [5.369127517, 0., 0., −5.033557047]}

The stable Equilibrium points are

{[0.7174392936, 1.269315673, 0., 0.]}

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

[0.7647208766, 1.090963497, 0.04796306234, 0.009424493133]

and with initial conditions, [0.1, 0.1, 0.1, 0.1]

[0.7156890634, 1.263171445, 0.002195598380, 0.00007547352577]

and with initial conditions, $\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right]$

$[0.1965076232, 3.692611187 \times 10^{-8}, 0.1858518047, 0.9664629493]$ **(26)**

ExpGenLotka(4, 50);

The Equilibrium points are

{[−0.2504173623, −5.843071786, 0., 10.85141903], [0., −3.989955357, 0.3069196429, 5.943080357], [0., 0., 0., 0.], [0., 0., 0., 6.250000000], [0., 0., 0.5298651252, 0.5539499037], [0., 0., 2.941176471, 0.], [0., 0.3259452412, 1.043024772, 0.], [0., 16.66666667, 0., 0.], [0.2694919457, −0.2961210933, 0.5064820379, 0.7916034600], [0.4803607607, 0.2418142605, 0.2614208222, 0.], [0.6363027461, 0.3348961822, 0., 0.], [0.9259259259, 0., 0.4901960784, 0.], [0.9919944309, 0., 0.5046989210, −0.04350852767], [1.851851852, 0., 0., 0.], [13.15789474, 0., 0., −5.263157895]}

The stable Equilibrium points are

{[0., 0., 2.941176471, 0.], [0., 16.66666667, 0., 0.], [13.15789474, 0., 0., −5.263157895]}

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

$$\left[5.061480682 \times 10^{-99},\ 3.671648688,\ 3.444866285 \times 10^{-83},\ 3.725893621\right]$$

and with initial conditions, $\left[0.1,\ 0.1,\ 0.1,\ 0.1\right]$

$$\left[6.261538917 \times 10^{-76},\ 0.4507321910,\ 3.043581110 \times 10^{-69},\ 5.915016708\right]$$

and with initial conditions, $\left[\frac{1}{4},\ \frac{1}{2},\ \frac{3}{4},\ 1\right]$

$$\left[2.861369682 \times 10^{-99},\ 3.384355337,\ 2.603263817 \times 10^{-83},\ 3.904875701\right] \quad \textbf{(27)}$$

ExpGenLotka$(4,\ 50)$;

The Equilibrium points are

$\{[-0.9822051640,\ 0.,\ 1.443068155,\ 0.09566178181],\ [-0.8771929825,\ 0.,\ 1.403508772,\ 0.],$
$[-0.2111295431,\ 0.2507163324,\ 0.6529935153,\ 0.],\ [0.,\ -4.071428571,\ 6.261904762,$
$-0.9047619048],\ [0.,\ -0.01960784314,\ 0.,\ 0.5686274510],\ [0.,\ 0.,\ 0.,\ 0.],\ [0.,\ 0.,\ 0.,$
$0.5617977528],\ [0.,\ 0.,\ 0.7714037375,\ 0.2064319861],\ [0.,\ 0.,\ 0.8333333333,\ 0.],\ [0.,$
$0.8333333333,\ 0.,\ 0.],\ [0.1647220316,\ 0.,\ 0.,\ 0.5284831846],\ [0.1681706317,$
$-0.03015972591,\ 0.,\ 0.5382907880],\ [0.2766798419,\ 0.7411067194,\ 0.,\ 0.],\ [0.5263157895,$
$0.,\ 0.,\ 0.],\ [1.813300327,\ 1.332854009,\ -2.266003501,\ 0.7746995883]\}$

The stable Equilibrium points are

$\{[0.,\ -4.071428571,\ 6.261904762,\ -0.9047619048],\ [0.,\ 0.,\ 0.7714037375,\ 0.2064319861]\}$

at time t=, 50, *with initial conditions,* $\left[0.5,\ 0.5,\ 0.5,\ 0.5\right]$, *the ultimate behavior is*

$$\left[1.020081177 \times 10^{-10},\ 0.007817802238,\ 0.7464822148,\ 0.2463101216\right]$$

and with initial conditions, $\left[0.1,\ 0.1,\ 0.1,\ 0.1\right]$

$$\left[1.585316855 \times 10^{-8},\ 0.02471539729,\ 0.7337929044,\ 0.2117003831\right]$$

and with initial conditions, $\left[\frac{1}{4},\ \frac{1}{2},\ \frac{3}{4},\ 1\right]$

$$\left[3.388214393 \times 10^{-13},\ 0.002277140306,\ 0.7547147990,\ 0.2465334766\right] \quad \textbf{(28)}$$

ExpGenLotka$(4,\ 50)$;

The Equilibrium points are

{[0., 0., 0., 0.], [0., 0., 0., 1.], [0., 0., 0.4687500000, 0.2500000000], [0., 0., 0.5681818182, 0.], [0., 0.4395171871, 0.1967987405, 0.1049593283], [0., 0.7066462949, 0.03819709702, 0.], [0., 0.7352941176, 0., 0.], [0., 0.7833787466, 0., −0.03405994550], [0.04623263313, 0.7260894305, −0.009483617052, 0.], [0.04638218924, 0.7189239332, 0., 0.], [0.05042998195, 0.7100010617, 0., 0.005308419153], [0.07974684834, 0.6299491455, 0.02140171010, 0.04331298472], [0.4733674118, 0., 0.1270378996, 0.2571005112], [0.8030431107, 0., 0., 0.08453085376], [0.9090909091, 0., 0., 0.], [1.112378779, 0., −0.2281802624, 0.]}

The stable Equilibrium points are

{[0.04638218924, 0.7189239332, 0., 0.], [0.07974684834, 0.6299491455, 0.02140171010, 0.04331298472]}

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

[0.2065994183, 0.3740822769, 0.09613364850, 0.1386192925]

and with initial conditions, [0.1, 0.1, 0.1, 0.1]

[0.1725044619, 0.5418046580, 0.03180462935, 0.06673197586]

and with initial conditions, $\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right]$

[0.3126704451, 0.1028649461, 0.2057943274, 0.2062988648] **(29)**

ExpGenLotka(4, 50);

The Equilibrium points are

{[−0.6728045326, 0., 0., 1.735127479], [−0.6024096386, 1.204819277, 0., 0.], [−0.2209164106, 0., 0.3711844260, 0.7064839527], [−0.1863468824, 0.5972853931, 0., 0.2605262888], [−0.1670172124, −0.2977578429, 0.6146557391, 0.7668809727], [0., 0., 0., 0.], [0., 0., 0., 1.190476190], [0., 0., 0.3777971520, 0.5158384191], [0., 0., 12.50000000, 0.], [0., 0.1716781119, 0.2383492233, 0.4541969465], [0., 0.5154639175, 0., 0.2577319588], [0., 0.9259259259, 0., 0.], [0., 3.488372093, −19.76744186, 0.], [0.2382863449,

0.9004715561, −0.6546603793, 0.], [0.6024096386, 0., 0., 0.], [0.6182380216, 0., −0.01931993818, 0.]}

The stable Equilibrium points are

{[0., 0., 0., 1.190476190], [0., 0., 12.50000000, 0.]}

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

$[0.01382287109, 0.002574988417, 3.428426964 \times 10^{-8}, 1.050977405]$

and with initial conditions, [0.1, 0.1, 0.1, 0.1]

$[1.818562208 \times 10^{-91}, 0.00006419021493, 12.49910645, 3.311489685 \times 10^{-35}]$

and with initial conditions, $\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right]$

$[0.0002604118025, 3.058536107 \times 10^{-7}, 1.005152857 \times 10^{-17}, 1.183945068]$ **(30)**

#out of all 10 runs, 4 runs where all the species survived (1,4,7,8). Runs where the three ICs agreed (converged to the same attractor / matched stable equilibrium): 6 runs (Runs 1, 2, 4, 5, 7, 8)

#Problem 4

ExpGenLotka(5, 50);

The Equilibrium points are

{[−4.560563552, 0., 8.489113198, −7.585691632, 10.18388631], [−2.676659529, 8.779443255, 0., 0., 0.], [−1.379997901, 0.01049428062, 2.565851611, 0., 0.], [−1.376146789, 0., 2.568807339, 0., 0.], [−1.254198081, 0.5091424876, 1.989240799, 0., 0.1618178301], [−0.2522339221, 0.2254897848, 0., 0., 0.9403710618], [−0.08070278628, 0.2678564160, 0., 0.2768727588, 0.4796034576], [−0.05353319058, 0., 0., 0.5888650964, 0.], [0., 0., 0., −0.09621552277, 1.667735728], [0., 0., 0., 0., 0.], [0., 0., 0., 0., 1.562500000], [0., 0., 0., 0.5747126437, 0.], [0., 0., 0.1937984496, 0., 1.550387597], [0., 0., 0.4500075632, −0.5747995765, 2.163061564], [0., 0., 1., 0., 0.], [0., 0., 1.121212121, −0.1212121212, 0.], [0., 0.2996316820, 0., 0.2906725951, 0.3550452932], [0., 0.3291880029, 0., 0., 0.5852231163], [0., 0.5711115462, 0.6918608446, 0., −0.1762287057], [0., 0.6241540996, 0.8019262451, −0.1431504245, −0.1840070967],

[0., 1.014492754, 0.2898550725, 0., 0.], [0., 1.131639723, 0.4387990762, −0.2309468822, 0.], [0., 1.612903226, 0., 0., 0.], [0., 4.166666667, 0., −1.388888889, 0.], [0.2660811160, −0.2014983208, 0., 0.5993283389, 0.], [0.6994963626, 0., −0.4935646335, 0.6961387801, 0.], [0.9543679844, 0.2289521435, −1.103208016, 1.050674788, −0.3763707219], [1.037245119, 1.278722518, −2.044360949, 0.9667957500, 0.], [1.287243275, 0., −1.193231125, 0., 1.395718831], [1.564972745, 0., 0., −0.6242307016, 1.951819940], [3.571428571, 0., 0., 0., 0.8928571429], [50., 0., 0., 0., 0.]}

The stable Equilibrium points are

{[−2.676659529, 8.779443255, 0., 0., 0.], [0., 0., 0.1937984496, 0., 1.550387597], [50., 0., 0., 0., 0.]}

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

$[39.56545257, 2.116257296 \times 10^{-128}, 6.124553382 \times 10^{-193}, 1.927253435 \times 10^{-155}, 0.1907673879]$

and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]

$[49.12280311, 2.256069455 \times 10^{-233}, 3.504367166 \times 10^{-359}, 5.026804095 \times 10^{-288}, 0.01601852927]$

and with initial conditions, $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$

$[6.502161790 \times 10^{-8}, 0.0001207597111, 0.4295030407, 0.00005574285454, 1.114406981]$ **(31)**

ExpGenLotka(5, 50);

The Equilibrium points are

{[−17.11298606, 0., 6.685619956, 0., 12.79805576], [−0.9241245136, 0., 2.638618677, −0.1094357977, 0.], [−0.6390977444, 0., 2.105263158, 0., 0.], [0., −1.851851852, 7.407407407, 0., 0.], [0., −1.187939698, 1.962311558, 0.9603015075, 0.], [0., 0., 0., 0., 0.], [0., 0., 0., 0., 0.7352941176], [0., 0., 0., 1.612903226, 0.], [0., 0., 0., 2.227722772, −1.361386139], [0., 0., 1.005244755, 0.2185314685, 0.], [0., 0., 1.618171021, 0.1929928741, −1.826009501], [0., 0., 1.785714286, 0., 0.], [0., 0., 2.717391304, 0., −3.260869565], [0., 0.1927710843, 0., 1.277108434, 0.], [0., 0.5186323473,

0.06221917854, 0., 0.5217641851], [0., 0.7614843427, 0., 0.06138777762, 0.4983445993], [0., 0.8318244927, 0.1218913148, −0.2919911012, 0.6351339282], [0., 0.9052924791, 0., 0., 0.5222841226], [0., 50., 0., 0., 0.], [0.1787842670, 0., 0., 0., 0.5065554231], [0.4643226552, 0.3110945689, −0.05783831896, 0.1887214379, −0.02452829754], [0.4806559675, 0.4445708573, −0.2024759672, 0.1115162067, 0.], [0.5124291321, 0.2943741823, 0., 0., 0.], [0.5211815687, 0.2934059202, 0., 0.01609983858, −0.01570118923], [0.5376973386, 0.6308494399, −0.3982130403, 0., 0.], [0.5499487033, 0.2994297712, 0., 0., −0.03877078711], [0.5745903384, 0.3234730794, 0., −0.1553522026, 0.], [0.7142857143, 0., 0., 0., 0.], [0.9040511292, 0., −0.5555243599, 0.5378278951, −0.1106029049], [1.109503136, 0., 0., −0.7034893070, −0.02210966393], [1.180466285, 0.5056319502, −0.1449557534, 0., −0.6808103333], [1.276595745, 0., 0., −1.063829787, 0.]}

The stable Equilibrium points are

{[0., −1.851851852, 7.407407407, 0., 0.], [0., 0., 0., 1.612903226, 0.], [0., 50., 0., 0., 0.]}

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

$[0.00005349499125, 2.030825313 \times 10^{-6}, 2.616564058 \times 10^{-31}, 1.608900824, 0.007816761232]$

and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]

$[2.017189386 \times 10^{-421}, 49.99999927, 1.157446026 \times 10^{-1221}, 6.597007656 \times 10^{-884}, 1.723018169 \times 10^{-16}]$

and with initial conditions, $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$

$[1.130912372 \times 10^{-6}, 1.394071929 \times 10^{-8}, 1.541233760 \times 10^{-33}, 1.608241299, 0.009410955943]$ **(32)**

ExpGenLotka(5, 50);

The Equilibrium points are

{[−1.601423488, 0., 1.245551601, 0., 2.135231317], [−1.368654698, 1.031732834, 0., 0., 1.246104793], [−0.5376523477, 0.4223082137, 0.3188074097, 0., 0.7019648011], [−0.1755829878, 0.2070574790, 0.3965782459, 0.2989630717, 0.2033546553], [−0.1651969213, 0., 0.6188363616, 0.4748324664, 0.1830207996], [−0.06448367004, 0.,

0.4861076665, 0.6487340651, 0.], [−0.06389980281, 0.1504948452, 0.3138465708, 0.5349503387, 0.], [−0.003959434276, 0.4614900623, 0., 2.560326181, −1.887222354], [0., −0.3898235535, 0., 0.9437833402, 0.], [0., −0.01217137293, 0.9128529698, 0., 0.], [0., 0., 0., 0., 0.], [0., 0., 0., 0., 3.333333333], [0., 0., 0., 0.6329113924, 0.], [0., 0., 0., 64.28571429, −52.38095238], [0., 0., 0.4869000696, 0.6144215163, 0.], [0., 0., 0.5259288582, 0.5481455151, 0.05331997641], [0., 0., 0.8928571429, 0., 0.], [0., 0., 1.136363636, 0., −2.727272727], [0., 0.1496183905, 0.3156350582, 0.5016093193, 0.], [0., 0.1667011138, 0.3422870725, 0.4102646874, 0.06312560740], [0., 0.3980459562, 0., 0., 0.7327664194], [0., 0.4595326304, 0., 2.555164271, −1.883422220], [0., 0.9466786512, −0.7785090899, 0., 1.300414625], [0., 2.941176471, 0., 0., 0.], [0.1048732665, 0.004039562857, 0.8506387170, 0., 0.], [0.1056586053, 0., 0.8570086875, 0., 0.], [0.2731174405, 0., 0., 0.4877097152, 0.], [0.5434782609, 0., 0., 0., 0.], [0.6112558832, −0.4274470178, 0., 0.6488153801, 0.], [0.7589743590, 0., 0., 12.89230769, −10.42051282], [1.291989664, 0., 0., 0., −1.059431525], [3.135451505, −2.591973244, 0., 0., 0.]}

The stable Equilibrium points are

{[−1.601423488, 0., 1.245551601, 0., 2.135231317], [0., 0., 0.5259288582, 0.5481455151, 0.05331997641], [0., 2.941176471, 0., 0., 0.], [0.7589743590, 0., 0., 12.89230769, −10.42051282]}

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

[0.2256688052, 0.00003681737726, 0.4806217678, 0.4917102728, 0.0001369111526]

and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]

$[0.0004252112247, 2.940286610, 2.202528079 \times 10^{-6}, 8.272257728 \times 10^{-33}, 5.397127699 \times 10^{-54}]$

and with initial conditions, $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$

$[0.1149762755, 1.388534287 \times 10^{-7}, 0.5243201148, 0.5488943587, 0.001458614528]$ **(33)**

ExpGenLotka(5, 50);

The Equilibrium points are

{[−1.800394267, 2.694254960, 0.3087176920, −1.469772827, 0.], [−1.555371632, 2.334439656, 0.2775790400, −1.405756875, 0.1781801553], [−0.6585612969, −5.217831814, 5.319148936, 0., 0.], [−0.5501222494, 0., 0.9321515892, 0., 0.], [−0.3228782288, 1.063768450, 0., −0.2190959410, 0.], [−0.2385677490, 0.8921879618, 0., −0.2736667652, 0.1564941097], [−0.07002801120, 0.7177871148, 0., 0., 0.], [−0.03343148798, 0., 0.3303897253, 1.224889888, −0.5209820775], [−0.01440883742, 0., 0.2588254129, 0.6027696987, 0.], [−0.004277159966, 0.6027231252, 0., 0., 0.06986027944], [0., −10.30534351, 9.541984733, 0., 0.], [0., −5.959252971, −1.511035654, 7.028862479, 0.], [0., 0., 0., 0., 0.], [0., 0., 0., 0., 1.724137931], [0., 0., 0., 0.7042253521, 0.], [0., 0., 0., 1.551814834, −0.6838506049], [0., 0., 0.2649006623, 0.5960264901, 0.], [0., 0., 0.3356257773, 1.140977810, −0.4629837731], [0., 0., 0.8928571429, 0., 0.], [0., 0., 2.069256757, 0., −0.8445945946], [0., 0.5990701001, 0., 0., 0.07153075823], [0., 0.6944444444, 0., 0., 0.], [0., 0.7132923129, 0., −0.3497977073, 0.2992245448], [0., 0.9278350515, −1.752577320, 0., 1.340206186], [0., 1.019021739, 0., −0.2717391304, 0.], [0., 1.452373214, 0.4510696558, −2.872312749, 1.614679621], [0.3542234332, 0., 0., 0.5994550409, 0.], [0.3675935950, 0., 0., 0.6996396775, −0.08402139314], [0.7692307692, 0., 0., 0., 0.], [0.8090051295, 0., −0.8893515294, 0., 1.238046989], [0.9926262053, 0., 0., 0., −0.2268859898], [9.368788338, −5.998514951, −7.484878453, 0., 9.148823624]}

The stable Equilibrium points are

{[0., 0., 0., 0., 1.724137931], [0., 0., 0.3356257773, 1.140977810, −0.4629837731]}

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

$[0.00004499618795, 4.409040950 \times 10^{-13}, 0.0001607533322, 9.345299841 \times 10^{-29}, 1.723100328]$

and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]

$[0.0001317205206, 8.912560401 \times 10^{-12}, 0.0001531854808, 1.364172818 \times 10^{-25}, 1.722563603]$

and with initial conditions, $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$

$$[3.346428990 \times 10^{-6},\ 4.096856637 \times 10^{-16},\ 0.00001338441193,\ 6.809945778 \times 10^{-38},\ 1.724051730] \tag{34}$$

ExpGenLotka(5, 50);

The Equilibrium points are

$$\{[-2.780176245, 0., 0., 0.2253853773, 2.749701602],\ [-2.115003305, 2.214144085, 0., 0., 0.],\ [-0.5700196282, 0.6064297308, 0., 0.3091954845, 0.6187503113],\ [-0.4670005297, 0., 0.4463092708, 0.2468978083, 0.4235594904],\ [-0.009175353560, 0.2189402371, 0.3806874997, 0.2739928802, -0.003763592027],\ [-0.002650684788, 0.2032044059, 0.3695965569, 0.2907465923, 0.],\ [0., -2.078384798, 0., 3.503562945, 0.],\ [0., 0., 0., -4.434447301, 2.185089974],\ [0., 0., 0., 0., 0.],\ [0., 0., 0., 0., 1.612903226],\ [0., 0., 0., 1.315789474, 0.],\ [0., 0., 0.3441385435, 0.5550621670, 0.],\ [0., 0., 0.3892733564, 0., 0.4325259516],\ [0., 0., 0.6011819387, 0.8170082972, -0.3154559814],\ [0., 0., 1.470588235, 0., 0.],\ [0., 0.1832356077, 0.3648054371, 0.3164978678, 0.],\ [0., 0.2368598406, 0.3659325850, 0.2416666749, 0.006039355894],\ [0., 0.2796420582, 0., 0., 1.062639821],\ [0., 0.3840245776, 0.4992319508, 0., 0.],\ [0., 0.4380017520, 0.2302009208, 0., 0.05300021200],\ [0., 0.5050505050, 0., 0., 0.],\ [0., 0.6599238261, 0., -0.3120439656, 0.3546071765],\ [0.03549604899, 0.3353879022, 0.5815380737, 0., 0.],\ [0.1527143419, 0., 0.1978664141, 0.5408214868, 0.],\ [0.2718485703, 0., 0., 0.7148610552, 0.],\ [0.4142960248, -0.5570031963, 0., 0.9862963623, 0.],\ [0.5708196971, -0.4249435523, 0., 0., 2.209706472],\ [0.7405637840, 0., 0.6211180124, 0., 0.],\ [1.432538047, -0.9649514007, 0.7612019226, 0., 0.6027761326],\ [3.846153846, 0., 0., 0., 0.],\ [7.119963370, 0., 0.4960317460, 0., -2.876984127]\}$$

The stable Equilibrium points are

$$\{[-0.5700196282, 0.6064297308, 0., 0.3091954845, 0.6187503113],\ [0.03549604899, 0.3353879022, 0.5815380737, 0., 0.]\}$$

at time t=, 50, *with initial conditions*, $[0.5, 0.5, 0.5, 0.5, 0.5]$, *the ultimate behavior is*

$$[2.945046885, 8.668018366 \times 10^{-62},\ 3.429716229 \times 10^{-28},\ 8.550001767 \times 10^{-45},\ 0.4436136032]$$

and with initial conditions, $[0.1, 0.1, 0.1, 0.1, 0.1]$

$$[3.441352669, 8.500503781 \times 10^{-76},\ 4.517660471 \times 10^{-26},\ 2.850883447 \times 10^{-47},\ 0.2011421242]$$

and with initial conditions, $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$

$[2.421762116, 1.383434653 \times 10^{-37}, 3.508715012 \times 10^{-28}, 3.317121086 \times 10^{-37}, 0.6924760444]$ **(35)**

ExpGenLotka(5, 50);

The Equilibrium points are

{[−7.051282051, 0., 13.46153846, 0., 0.], [−1.863354037, 0., 0., 0., 3.467908903], [−1.847903340, 0., 2.036247335, 1.727078891, 0.], [−0.8596019835, 0., 23.46920943, −1.692443804, −15.45044455], [−0.2376640187, 1.226223371, −0.5273152076, −0.1983783403, 0.9587744330], [−0.1074556275, 0.02632467971, 0., 0.4692445623, 0.4136586887], [−0.1042266229, 0., 0., 0.4849649207, 0.4095172907], [0., −4.689608637, −1.309041835, 6.086369771, 0.], [0., −1.749435666, 0., 2.144469526, 0.], [0., −0.09442945564, 0., 0.5388033645, 0.3122520235], [0., 0., 0., 0., 0.], [0., 0., 0., 0., 1.666666667], [0., 0., 0., 0.4820936639, 0.3168044077], [0., 0., 0., 1.315789474, 0.], [0., 0., 0.2282157676, 0.8713692946, 0.], [0., 0., 1.562500000, 0., 0.], [0., 0., 3.097345133, 0., −1.327433628], [0., 0., 10.87712124, −0.5367000613, −7.345123697], [0., 0.8361496329, −0.5506351241, 0., 0.8332362196], [0., 0.8928571429, 0., 0., 0.], [0., 0.9385899333, −0.5170266391, −0.03315068798, 0.7262507863], [0., 1.204819277, 0., 0., −0.3012048193], [0., 1.376651982, −0.3303964758, 0., 0.], [0.2384696017, 1.101939203, −0.3550838574, 0., 0.], [0.2815721242, 1.100281414, −0.3399892494, 0., −0.07398975526], [0.3030303030, 0.4978354978, 0., 0., 0.], [0.5208333333, 0., 0., 0., 0.], [0.6067961165, 0., 0., −0.1213592233, 0.], [0.8250000000, 1.450000000, 0., −1.325000000, 0.], [1.072180451, 2.142105263, −0.1186466165, −2.007218045, 0.], [1.945412311, 0., 12.51451800, 0., −12.31126597], [5.614035088, 3.659147870, 0., 0., −9.736842105]}

The stable Equilibrium points are

{[0., 0., 0., 0., 1.666666667], [0., 0., 0., 1.315789474, 0.]}

at time t=, 50, *with initial conditions,* [0.5, 0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

[0.007120439403, 0.04114270096, 0.5729766885, 0.03043193071, 0.7644434424]

and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]

$$[0.0003628725187, 0.01529359716, 0.1125033006, 0.0006473616487, 1.515132095]$$

and with initial conditions, $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$

$$[0.003224023438, 0.02720010679, 0.7012947713, 0.05627226628, 0.5371542245] \quad \textbf{(36)}$$

ExpGenLotka(5, 50);

The Equilibrium points are

{[−0.2591939021, −0.8314897016, 2.322347684, −2.472785989, 2.065007640], [−0.1296724262, −0.8002487476, 0.8984504017, 0., 1.998546564], [−0.05963029219, 0.6857483602, 0., 0., 0.], [0., −0.8582168390, 1.852277388, −2.065454104, 1.862137346], [0., −0.8176250223, 0.7700541119, 0., 1.893916870], [0., 0., 0., 0., 0.], [0., 0., 0., 0., 25.], [0., 0., 0., 0.8333333333, 0.], [0., 0., 0., 1.263586957, −0.2717391304], [0., 0., 0.6183745583, 0., 1.192579505], [0., 0., 1.314080063, −1.337077473, 1.149467025], [0., 0., 2.034883721, 0.2906976744, 0.], [0., 0., 2.173913043, 0., 0.], [0., 0.2617801047, 1.832460733, 0., 0.], [0., 0.2675537939, 1.727774632, 0.2031426954, 0.], [0., 0.5984042553, 0., 0.4543439716, 0.], [0., 0.6329113924, 0., 0., 0.], [0., 0.8343393589, 0., 0.9361994472, −0.3987038155], [0., 3.682719547, 0., 0., −4.461756374], [0.2041232134, 0., 0., 0.8434703210, −0.03433495827], [0.2529312358, 0., 0.8390117405, −0.9174520886, 0.9298403306], [0.2948375328, 0., 0.3191092534, 0., 0.9207924334], [0.3549319685, 0.2095465608, 0., 0.4508587057, 0.1091746606], [0.4065040650, 0., 0., 0.7452574526, 0.], [0.5186586069, 0.6119079648, 0., 0., −0.6416079945], [0.5544354839, 0., 0., 0., 2.822580645], [0.7774227902, −0.1118210863, 0., 0.7357117501, 0.], [0.8418657565, 0., 0.3071672355, 0., 0.], [0.8682866429, 0., −0.06876423375, 0.6635416897, 0.], [0.9183269253, −0.2835838285, 0.5075148549, 0., 0.], [0.9485467001, −0.2790959252, 0.05714272808, 0.7893375735, 0.], [2.083333333, 0., 0., 0., 0.]}

The stable Equilibrium points are

{[0., 0., 0., 0., 25.], [0., 0.2675537939, 1.727774632, 0.2031426954, 0.], [0.8682866429, 0., −0.06876423375, 0.6635416897, 0.]}

at time t=, 50, *with initial conditions,* [0.5, 0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

$$[1.406742173 \times 10^{-6}, 0.2676269757, 1.727355232, 0.2036230264, 6.082501768 \times 10^{-29}]$$

and with initial conditions, $[0.1, 0.1, 0.1, 0.1, 0.1]$

$$[2.449147322 \times 10^{-7}, 0.2675894466, 1.727645163, 0.2032499380, 1.205699650 \times 10^{-28}]$$

and with initial conditions, $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$

$$[3.065975369 \times 10^{-7}, 0.2675588451, 1.727692923, 0.2032660031, 3.310931061 \times 10^{-29}] \quad \textbf{(37)}$$

ExpGenLotka$(5, 50)$;

The Equilibrium points are

$\{[-348.4931872, 35.05170654, 31.57982335, 0., 292.1419378], [-4.118913437, 1.326689931, 0., -1.963851256, 3.964937994], [-3.628246120, 0., 0., -2.417530999, 4.261873197], [-2.816002916, 0.4442723047, 0., 0., 3.169142441], [-2.586206897, 0., 0., 0., 3.201970443], [0., 0., 0., -0.1272074229, 0.6510026938], [0., 0., 0., 0., 0.], [0., 0., 0., 0., 0.6493506494], [0., 0., 0., 0.5681818182, 0.], [0., 0., 0.3102625298, 0.3460620525, 0.], [0., 0., 0.4969634925, 0.05731314350, 0.1451887590], [0., 0., 0.5377950403, 0., 0.1045712578], [0., 0., 0.5434782609, 0., 0.], [0., 0.2068014706, 0., 0., 0.5284926471], [0., 0.2094344837, 0., -0.006693267005, 0.5270407987], [0., 0.2177324436, 0.3806672678, 0.1394218404, 0.1346827442], [0., 0.2865836400, 0.4617180866, 0., 0.01415227852], [0., 0.2969457014, 0.4595588235, 0., 0.], [0., 0.3591461917, 0.1407180985, 0.4470344186, 0.], [0., 0.4117572533, 0., 0.5447865197, 0.], [0., 0.5494505494, 0., 0., 0.], [0.9846827133, 0., 0.3829321663, 0., 0.], [0.9916005600, 0., 0., 0.1399906673, 0.], [0.9932865501, 0., 0.5558579632, -0.2586811430, 0.], [1., 0., 0., 0., 0.], [1.146616541, -0.2443609023, 0., 0., 0.], [1.159631366, -0.2743542985, 0., 0.08302022267, 0.], [1.401122476, -0.7027805451, 0.5136462721, 0., 0.], [1.577982082, -0.9517275830, 1.149715178, -0.8822356524, 0.], [2.009197652, 0., -0.4015505750, 0.9920007064, -0.9398711353], [2.744843337, -0.5446870142, -0.4396009873, 1.128820350, -1.310871705], [40.50295027, 0., -4.151447036, 0., -35.12222534]\}$

The stable Equilibrium points are

$\{[-348.4931872, 35.05170654, 31.57982335, 0., 292.1419378], [0.9846827133, 0., 0.3829321663, 0., 0.]\}$

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

$$[0.8296310499, 0.2165322001, 0.3306228515, 0.01037923900, 3.003042341 \times 10^{-12}]$$

and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]

$$[0.8963913035, 0.07788904212, 0.3714368239, 0.004981665160, 1.213719712 \times 10^{-7}]$$

and with initial conditions, $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$

$$[0.8209597149, 0.2082340319, 0.3283820080, 0.01829025600, 1.157922995 \times 10^{-9}] \quad \textbf{(38)}$$

ExpGenLotka(5, 50);

The Equilibrium points are

{[−15.20210896, 14.23550088, 0., 0., 11.33567663], [−2.054794521, 0., 2.397260274, 0., 0.], [0., −3.636068939, 1.057143372, 3.171692750, −0.6252213638], [0., −0.8201438849, 0.4388489209, 1.051258993, 0.], [0., −0.1176470588, 0., 1.377450980, −0.4705882353], [0., 0., −0.8620689655, 0.7543103448, 0.], [0., 0., 0., 0., 0.], [0., 0., 0., 0., 1.851851852], [0., 0., 0., 0.6944444444, 0.], [0., 0., 0., 1.295045045, −0.4504504505], [0., 0., 1.694915254, 0., 0.6591337100], [0., 0., 1.923076923, 0., 0.], [0., 0., 2.262837250, −0.1396141572, 0.5076878445], [0., 0.3649635036, 0., 0., 1.459854015], [0., 0.5192459646, −0.05643977876, 0., 1.333860105], [0., 0.5494505494, 0., 0.4349816850, 0.], [0., 0.5930807249, 0.1894563427, 0., 0.], [0., 0.9433962264, 0., 0., 0.], [0.2735990974, 0., 0.2040240692, 0.4484768710, 0.], [0.3671071953, 0.5139500734, 0., 0., 0.], [0.4397937519, 0.6672732787, −0.1289050652, 0., 0.], [0.4486402900, 0.2339233126, 0., 0.2038826345, 0.], [0.4488887246, 0.6416186495, −0.1306907587, 0.02022510431, 0.], [0.5501349388, 0., 0., 0.2283578991, 0.], [0.6024096386, 0., 0., 0., 0.], [0.7919074224, 0., 0., 0.5289994641, −0.3791074394], [0.8241531431, 0., −0.02097790040, 0.5111069910, −0.3850849539], [0.9911680289, 0.7745173242, −0.4806538772, 0., −0.2570360002], [1.206825817, 1.219010426, −0.6771892327, −0.2968809947, −0.4198057449], [1.397913982, −1.180061685, 0., 0.7693583983, −0.5265047946], [2.729693742, 0., 0., 0., −2.596537949], [18.03334084, 0., 9.641811219, 0., −34.32079297]}

The stable Equilibrium points are

{[0., 0., 1.694915254, 0., 0.6591337100], [0.7919074224, 0., 0., 0.5289994641, −0.3791074394]}

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

[0.001162957630, 0.3681513611, 0.4572746787, $2.118364287 \times 10^{-13}$, 0.9335342416]

and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]

[0.01971931658, 0.08934855767, 0.9083377192, 0.06426196476, 0.4545452267]

and with initial conditions, $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$

[0.00001882553883, 0.2576473393, 0.6992594486, $3.312989851 \times 10^{-22}$, 1.084109781] **(39)**

ExpGenLotka(5, 50);

The Equilibrium points are

{[−1.191971576, 1.048775132, 1.097951584, 0., 0.6122447263], [−0.02714440825, 0.6029936405, 0.5254381883, 0., 0.], [0., 0., 0., 0., 0.], [0., 0., 0., 0., 3.333333333], [0., 0., 0., 0.7621121394, 0.8437670114], [0., 0., 0., 2.777777778, 0.], [0., 0., 0.6578947368, 0.5847953216, 0.], [0., 0., 1.020408163, 0.9276437848, −0.6493506494], [0., 0., 2.631578947, 0., 0.8771929825], [0., 0., 12.50000000, 0., 0.], [0., 0.2519101876, 0.4385742938, 0.4649234489, −0.2237718204], [0., 0.2667902761, 0.2933147919, 0.3327152864, 0.], [0., 0.3262130292, 0., 0.2851343514, 0.2923835299], [0., 0.3800743975, 0., 0.6873685913, 0.], [0., 0.4411583866, 0.2052078210, 0., 0.2889818003], [0., 0.4418177645, 0., 0., 0.4762451228], [0., 0.5854800937, 0.4976580796, 0., 0.], [0., 3.571428571, 0., 0., 0.], [0.2668334192, 0., 0.7132577694, 0.5586923208, −0.2959247546], [0.3386076708, 0., 0.5185054889, 0.3533992692, 0.], [0.4339440694, 0.6268081003, 0., 0., 0.], [0.4743833017, 0., 0., 1.802656546, 0.], [0.5204744115, 0., 0., 0.1396394763, 0.6564868884], [0.6889763780, 0., 0., 0., 0.3937007874], [0.7177885223, 0., 0.08154077613, 0., 0.1946642472], [0.7678726319, 0.2747861393, 0., −0.5580249006, 0.1030077452], [0.7790942627, 0.2925003214, 0., −0.4324455302, 0.], [0.8758852031, 0., 0.01863585539, 0., 0.], [0.8928571429, 0., 0., 0., 0.], [0.8957339300, 0.3551549634, −0.1961717839, −0.3628994305, 0.], [2.470588235, −0.8235294118, 0., 0., −1.882352941], [2.587912940, 0.7962994044, −3.797726306, −4.113351861,

4.123662605]}

The stable Equilibrium points are

{[0., 0., 0., 0., 3.333333333], [0., 0., 12.50000000, 0., 0.], [0., 3.571428571, 0., 0., 0.], [0.4743833017, 0., 0., 1.802656546, 0.]}

at time t=, 50, *with initial conditions*, [0.5, 0.5, 0.5, 0.5, 0.5], *the ultimate behavior is*

$[0.4919406763, 3.884204502 \times 10^{-33}, 9.024024116 \times 10^{-30}, 1.740877395, 4.369486974 \times 10^{-19}]$

and with initial conditions, [0.1, 0.1, 0.1, 0.1, 0.1]

$[0.7571320315, 2.117485674 \times 10^{-22}, 5.716201573 \times 10^{-8}, 0.6177484217, 0.004453690967]$

and with initial conditions, $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$

$[0.4862869246, 1.693226401 \times 10^{-35}, 1.262436546 \times 10^{-32}, 1.760968481, 1.355602166 \times 10^{-19}]$ **(40)**

ExpGenLotka(5, 50);

The Equilibrium points are

{[−6.448521358, 0., 0., −6.400602410, 7.756024096], [−0.6969274990, 0., 0.5682968744, −0.4377645727, 0.6240694968], [−0.2351037436, 0., 0.2553742949, 0., 0.5225297648], [−0.2066115702, 0., 0., 0., 0.8264462810], [0., 0., 0., −0.2515479876, 0.9481424149], [0., 0., 0., 0., 0.], [0., 0., 0., 0., 0.7575757576], [0., 0., 0., 50., 0.], [0., 0., 0.2198406156, 0., 0.4877713658], [0., 0., 0.2450095301, −0.04531899069, 0.4912148120], [0., 0., 0.4746835443, 3.481012658, 0.], [0., 0., 5., 0., 0.], [0., 0.3429383966, 0.4673581197, 1.455397098, 0.], [0., 0.5390835580, 0.8490566038, 0., 0.], [0., 2.848465808, 1.480085175, 0., −1.878905295], [0., 3.470100393, 0., −0.8948057617, 0.4364906155], [0., 3.571428571, 0., 0., 0.], [0., 3.673003074, 1.610069417, 0.4228066991, −2.596106305], [0., 5.755395683, 0., 0., −0.8992805755], [0., 6.603773585, 0., −2.830188679, 0.], [0.4044206973, −0.3580579553, 0.0003009916169, 0., 0.7254767498], [0.4044600306, −0.3580613254, 0., 0., 0.7258340077], [0.4329296710, −0.2555189893, 0., 0.2086425330, 0.5287618392], [0.4346785154, −0.2781292097, −0.05999286342, 0.1640043636, 0.6421322940], [0.6524317912, 0.07625826131, 0., 0., 0.], [0.7575757576, 0., 0., 0., 0.], [0.9082448642, 0.1775127533, 0., −1.373569557, 0.], [0.9193054137, 0., 0.,

$-0.5617977528, 0.]$, $[0.9971252009, 0.5256452129, -0.6428684610, 0., 0.]$, $[3.692666911, 2.064103246, -1.611475442, -11.68491278, 0.]$, $[4.816017316, 0., -2.705627706, 0., 0.]$, $[15.58898222, 0., -8.146122290, -9.074037898, 0.]\}$

The stable Equilibrium points are

$\{[0., 0., 0., 50., 0.]$, $[0., 0., 5., 0., 0.]$, $[0., 3.470100393, 0., -0.8948057617, 0.4364906155]$, $[0.4044600306, -0.3580613254, 0., 0., 0.7258340077]\}$

at time t=, 50, *with initial conditions*, $[0.5, 0.5, 0.5, 0.5, 0.5]$, *the ultimate behavior is*

$[1.154990462 \times 10^{-48}, 5.009458237 \times 10^{-22}, 2.131321226 \times 10^{-60}, 46.41311121, 3.512476048 \times 10^{-150}]$

and with initial conditions, $[0.1, 0.1, 0.1, 0.1, 0.1]$

$[2.116035376 \times 10^{-57}, 7.568533936 \times 10^{-18}, 4.999999931, 3.554851233 \times 10^{-27}, 2.032868832 \times 10^{-53}]$

and with initial conditions, $\left[\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$

$[7.083726402 \times 10^{-62}, 2.500533207 \times 10^{-30}, 2.937338312 \times 10^{-76}, 48.57277571, 3.800865693 \times 10^{-204}]$ **(41)**

#out of 10 runs, 6 runs where all of the species survived (3,5,6,7,9,10). For if they agree, 3 out of 10 runs did argee (1,2,7)

#This experiment simulates 2,3,4, and 5 competing species using the generalized Lotka-Volterra equations with random parameters, running 10 different scenarios.This shows that in ecology, competitive systems are often unpredictable. Even with identical competition rules, small differences in starting populations can determine which species dominate or go extinct.