Homework for Lecture 21 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file and/or .txt file) to

ShaloshBEkhad@gmail.com

by 8:00pm Monday, Dec. 1,, 2025.

Subject: hw21

with an attachment hw21FirstLast.pdf and/or hw21FirstLast.txt

1. By hand solve the system

$$\frac{dx}{dt} = x - y \quad , \frac{dy}{dt} = y - x \quad , \quad x(0) = 1 \quad , \quad y(0) = 1 \quad .$$

Plot, by hand, the phase-plane diagram.

2. Now use Maple with the command

```
S:=dsolve({diff(x(t),t)=x(t)-y(t),diff(y(t),t)=y(t)-x(t),x(0)=1,y(0)=0},{x(t),y(t)});
plot([subs(S,x(t)),subs(S,y(t)),t=0..10]);
```

did you get the same thing?

3. Use Maple to solve and then plot the phase-plane diagram for the system

$$\frac{dx}{dt} = a_{11}x + a_{12}y \quad , \frac{dy}{dt} = a_{21}x + a_{22}y \quad , \quad x(0) = 1 \quad , \quad y(0) = 1 \quad ,$$

for three randomly chosen matrices

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad .$$

4. Carefully read, and understand, the Maple code for the following procedures (type Help(ProcedureName); for instructions)

`Lotka, Volterra, VolterraM`

in the Maple package

`https://sites.math.rutgers.edu/~zeilberg/Bio25/DMB.txt` ,

For **each of them**, experiment with **three** randmom choices of parameters, and random initial conditions, using `Dis` (with $h = 0.01$), of *each* of the quantities in question.

Send me these nice plots.

Confirm the numerics by using `SEquP`.

1. By hand solve the system

$$\frac{dx}{dt} = x - y \quad , \frac{dy}{dt} = y - x \quad , \quad x(0) = 1 \quad , \quad y(0) = 1 \quad .$$

Plot, by hand, the phase-plane diagram.

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A - \lambda I = (1-\lambda)(1-\lambda) - 1$$

$$\lambda^2 - 2\lambda$$

$$\lambda(\lambda - 2) = 0$$

$\lambda = 0 \qquad \lambda = 2$

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad A - 2I = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

$$\vec{V}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \vec{V}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$x(t) = y(t) = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{0t} + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{2t}$$

$$x(0) = C_1 - C_2 = 1 \qquad C_1 = 1 + C_2$$

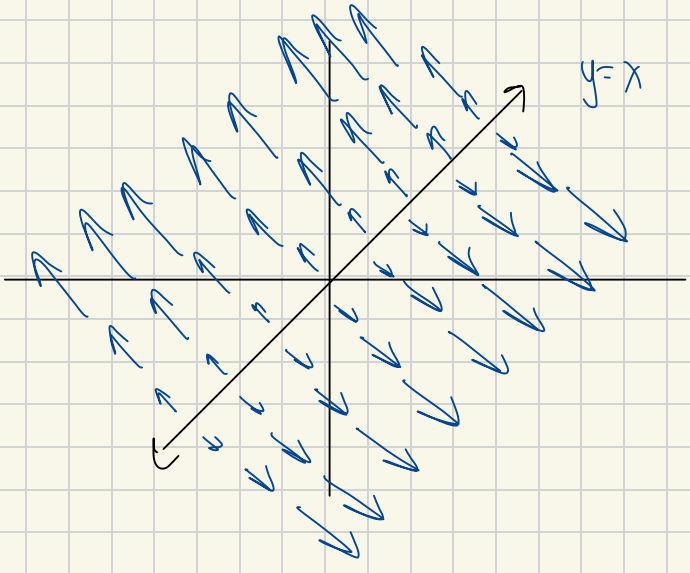
$$y(0) = C_1 + C_2 = 1 \qquad 1 - C_2 = C_1$$

$$1 - C_2 = 1 + C_2$$

$$0 = C_2$$

$$C_1 = 1$$

$$\boxed{x(t) = y(t) = 1}$$



2. Now use Maple with the command

```
S:=dsolve({diff(x(t),t)=x(t)-y(t),diff(y(t),t)=y(t)-x(t),x(0)=1,y(0)=0},{x(t),y(t)});
plot([subs(S,x(t)),subs(S,y(t)),t=0..10]);
```

did you get the same thing?

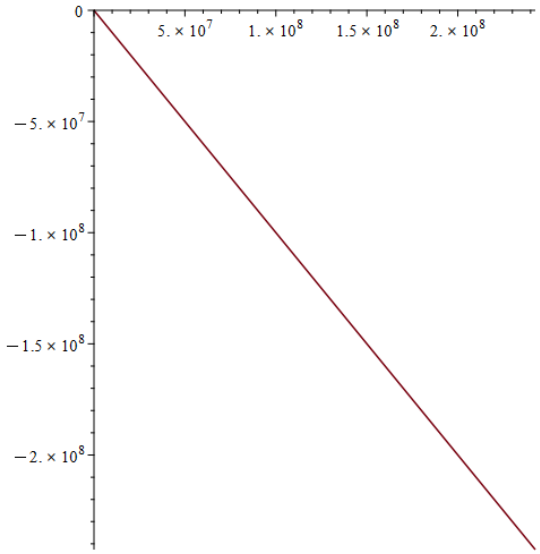
```
read("DMB.txt");
```

For a list of the Main procedures type: Help(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig);

For a list of the Continuous Dynamical Models procedures type: HelpC(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig); **(1)**

```
S := dsolve({diff(x(t), t) = x(t)-y(t), diff(y(t), t) = y(t)-x(t), x(0) = 1, y(0) = 0}, {x(t), y(t)});
plot([subs(S, x(t)), subs(S, y(t)), t = 0..10]);
```

$$S := \left\{x(t) = \frac{1}{2} + \frac{e^{2t}}{2}, y(t) = -\frac{e^{2t}}{2} + \frac{1}{2}\right\}$$



3. Use Maple to solve and then plot the phase-plane diagram for the system

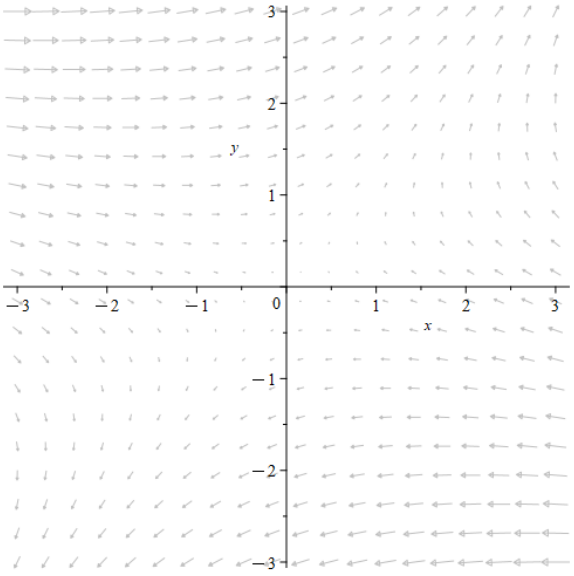
$$\frac{dx}{dt} = a_{11}x + a_{12}y \quad , \frac{dy}{dt} = a_{21}x + a_{22}y \quad , \quad x(0) = 1 \quad , \quad y(0) = 1 \quad ,$$

for three randomly chosen matrices

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} .$$

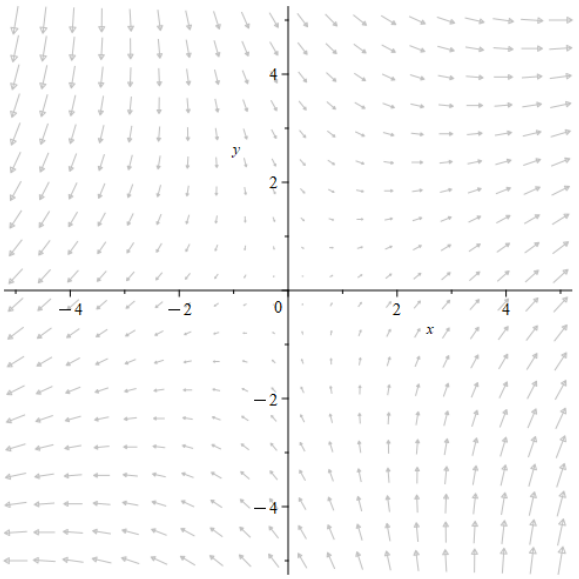
```
"A =", [ -2  3 ]
       [  1  1 ]
```

$$\left\{x(t) = \frac{\left(\frac{1}{2} + \frac{5\sqrt{21}}{42}\right) e^{\frac{(-1+\sqrt{21})t}{2}} \sqrt{21}}{2} - \frac{\left(\frac{1}{2} - \frac{5\sqrt{21}}{42}\right) e^{-\frac{(1+\sqrt{21})t}{2}} \sqrt{21}}{2} - \frac{3\left(\frac{1}{2} + \frac{5\sqrt{21}}{42}\right) e^{\frac{(-1+\sqrt{21})t}{2}}}{2} - \frac{3\left(\frac{1}{2} - \frac{5\sqrt{21}}{42}\right) e^{-\frac{(1+\sqrt{21})t}{2}}}{2}, y(t) = \left(\frac{1}{2} + \frac{5\sqrt{21}}{42}\right) e^{\frac{(-1+\sqrt{21})t}{2}} + \left(\frac{1}{2} - \frac{5\sqrt{21}}{42}\right) e^{-\frac{(1+\sqrt{21})t}{2}}\right\}$$




```
"A =", [ 3   2 ]
       [ 3  -3 ]
```

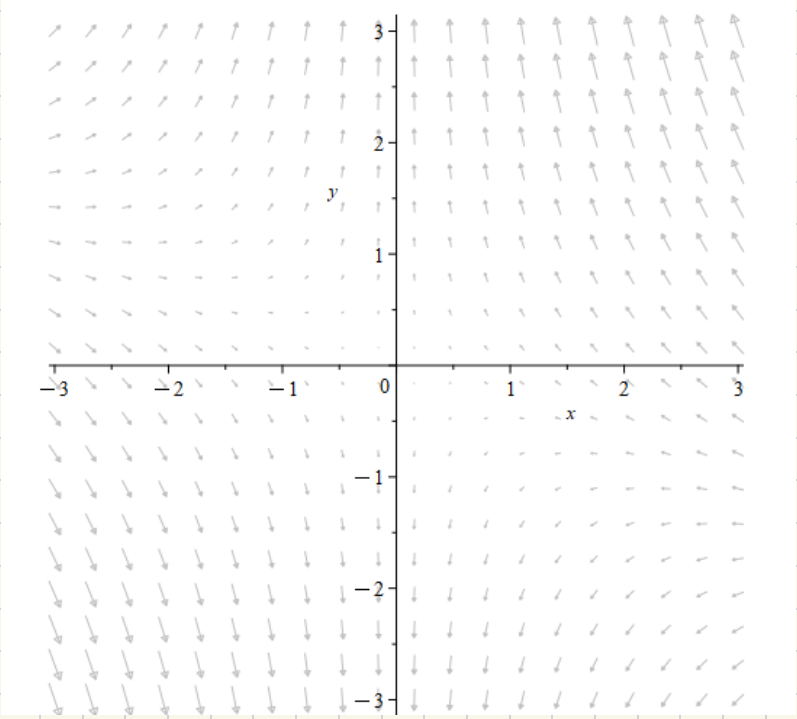
$$\left\{x(t) = \frac{\sqrt{15}\, e^{\sqrt{15}\, t}}{6} - \frac{\sqrt{15}\, e^{-\sqrt{15}\, t}}{6} + \frac{e^{\sqrt{15}\, t}}{2} + \frac{e^{-\sqrt{15}\, t}}{2}, y(t) = \frac{e^{\sqrt{15}\, t}}{2} + \frac{e^{-\sqrt{15}\, t}}{2}\right\}$$



not a valid dsolve/numeric solution

$$\text{"A ="}, \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\left\{x(t) = e^{-t}, y(t) = -\frac{e^{-t}}{3} + \frac{4\,e^{2t}}{3}\right\}$$



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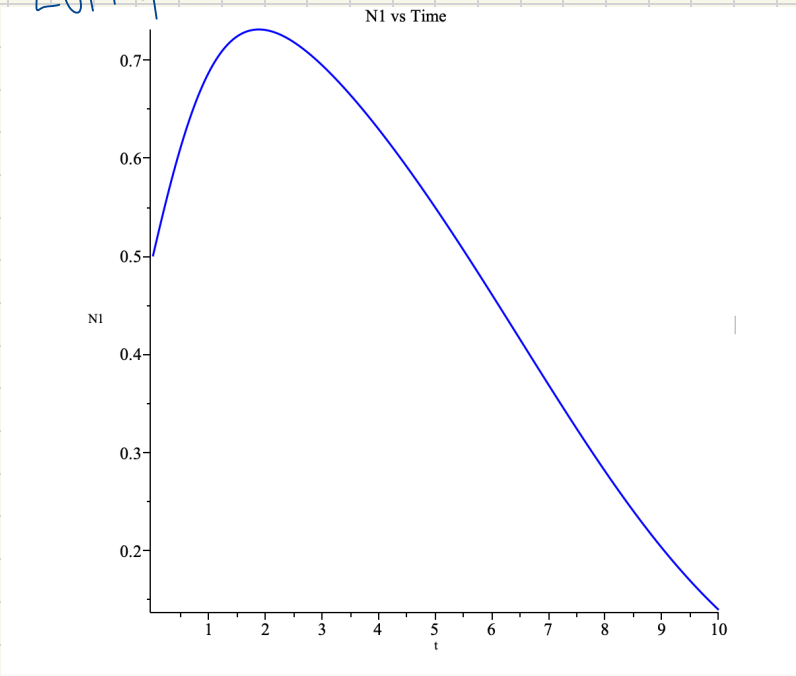
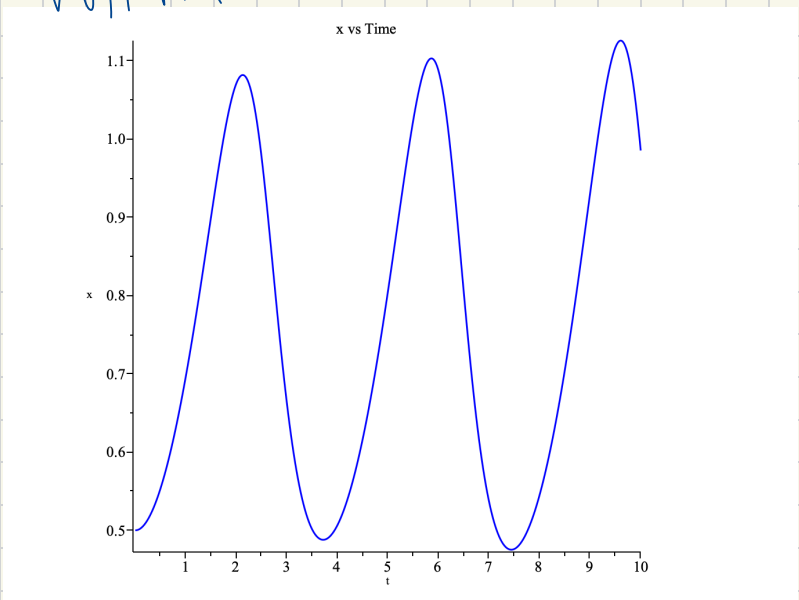
Lotka



Volterra



volterra M