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hw 21 Dynamical Models in Biology

1) $\frac{dx}{dt} = x - y$, $\frac{dy}{dt} = y - x$, $x(0) = 1$, $y(0) = 1$

$$\leadsto \frac{d}{dt} \vec{x}(t) = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \vec{x}(t)$$

$$(1-\lambda)^2 - 1 = 0 \rightarrow \lambda^2 - 2\lambda = 0, \lambda_1 = 0, \lambda_2 = 2$$

$$\lambda_1 = 0: \left(\begin{array}{cc|c} 1 & -1 & 0 \\ -1 & 1 & 0 \end{array} \right) \rightarrow \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right) \begin{array}{l} a=b \\ b=b \end{array} \rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 = 2: \left(\begin{array}{cc|c} -1 & -1 & 0 \\ -1 & -1 & 0 \end{array} \right) \rightarrow \left(\begin{array}{cc|c} -1 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right) \begin{array}{l} a=-b \\ b=b \end{array} \rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

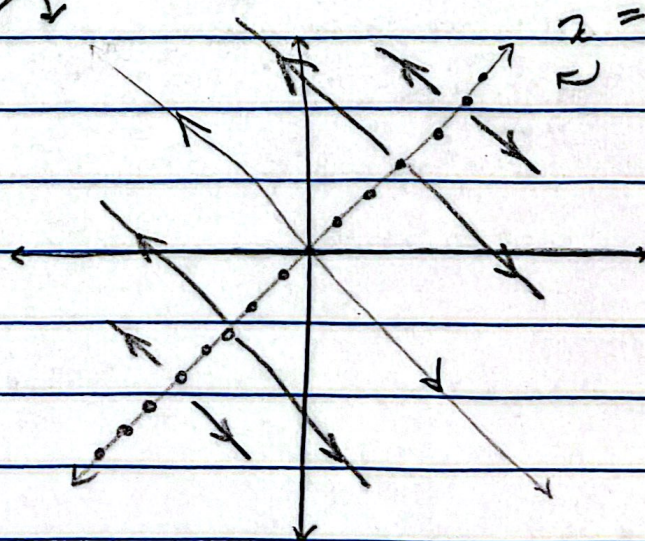
$$\vec{x}(t) = C_1 e^{0t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{aligned} 1 &= C_1(1) + C_2(-1) \\ 1 &= C_1(1) + C_2(1) \end{aligned} \rightarrow C_1 = 1, C_2 = 0$$

$$\vec{x}(t)_{\text{part}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \leadsto x(t) = 1, y(t) = 1$$

phase plane Diagram

$\lambda = 2$



$\lambda = 0$, eq. line

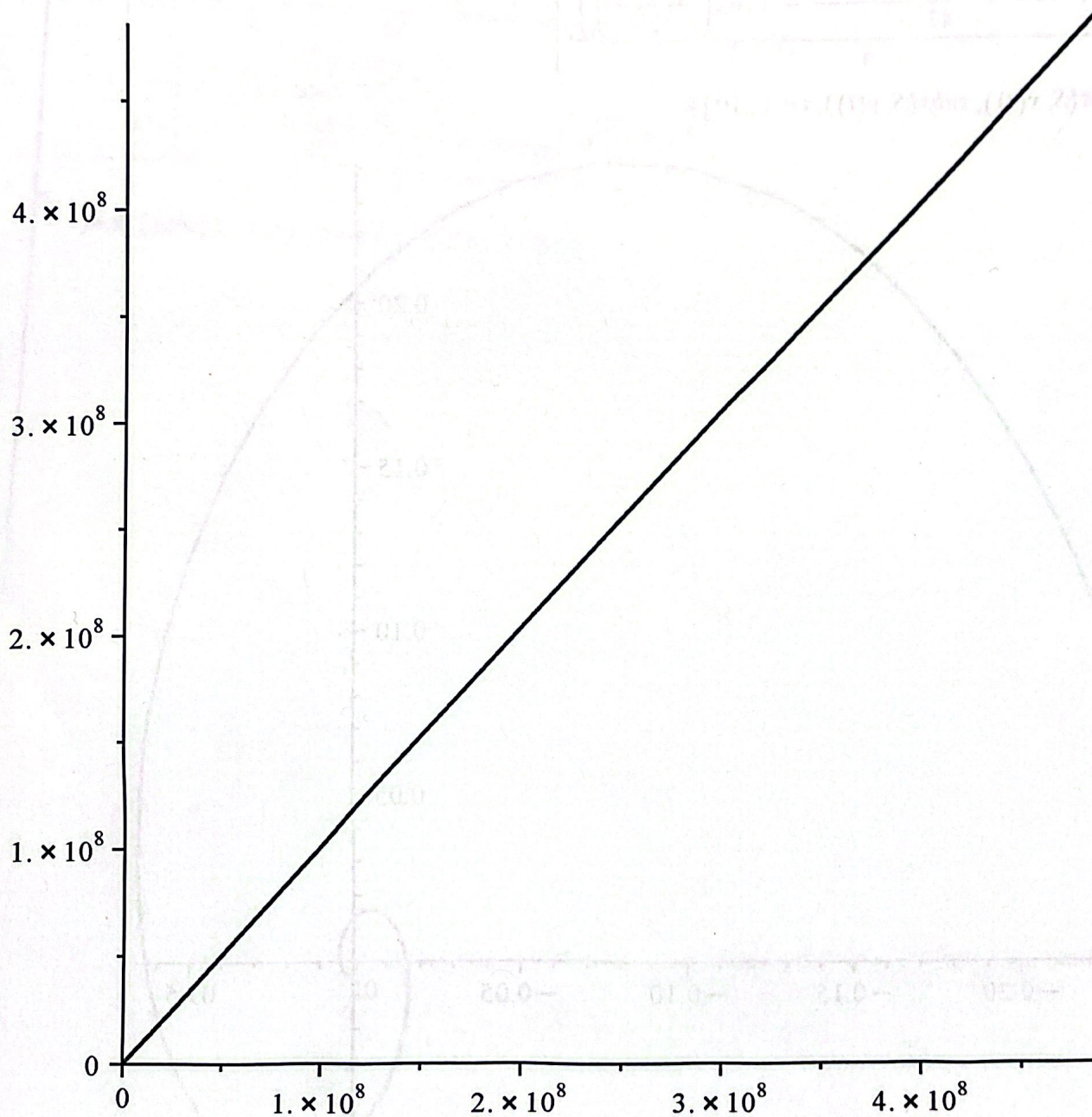
2. I Got Some Sol. plot to me was blank!

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> S := dsolve({diff(x(t), t) = 1·x(t) + 1·y(t), diff(y(t), t) = 1·x(t) + 1·y(t), x(0) = 1, y(0) = 1}, {x(t), y(t)});
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$$S := \{x(t) = e^{2t}, y(t) = e^{2t}\}$$

(1)

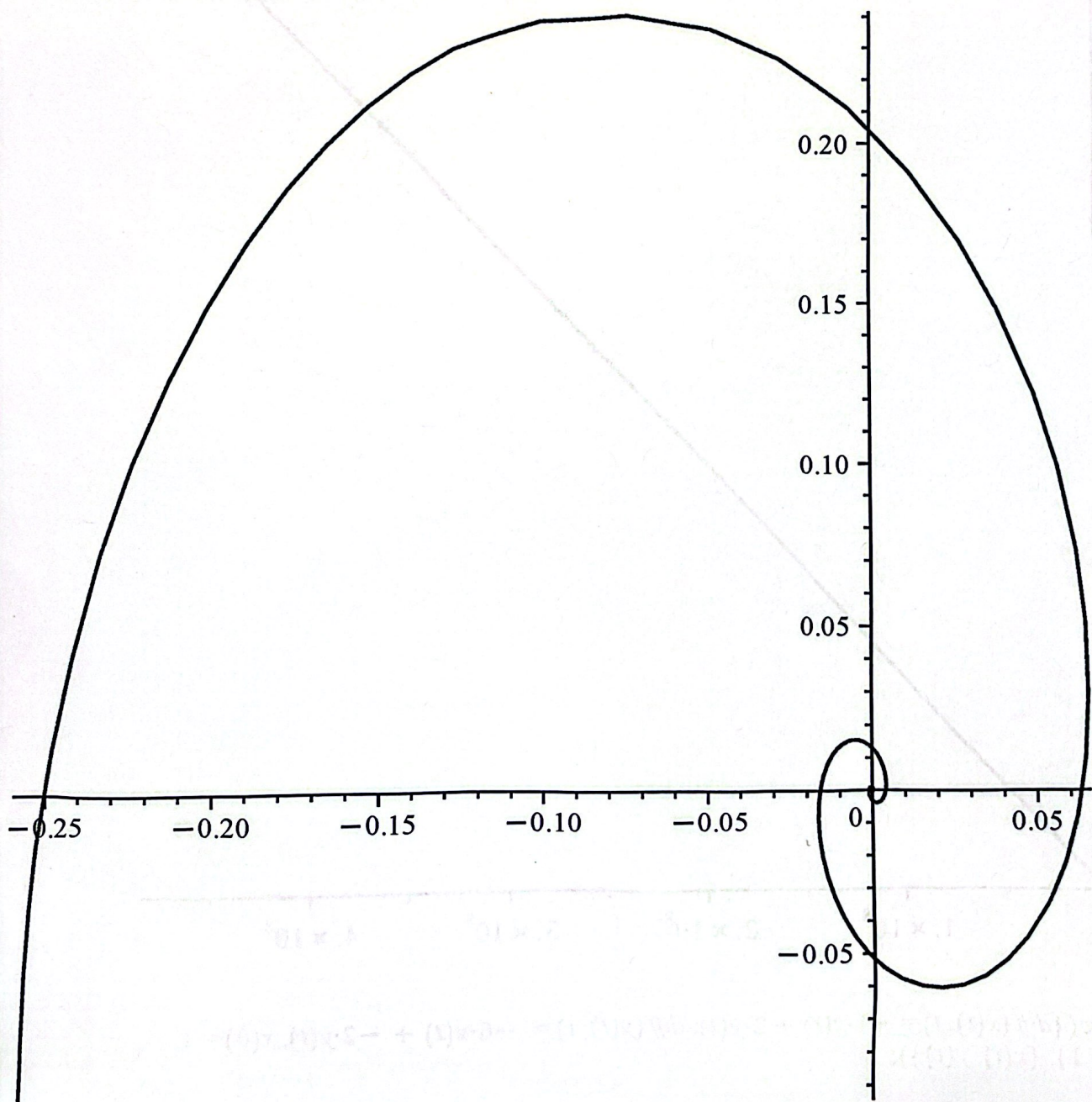
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> plot([subs(S, x(t)), subs(S, y(t)), t = 1..10]);
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> S := dsolve({diff(x(t), t) = -1·x(t) + 2·y(t), diff(y(t), t) = -6·x(t) + -2·y(t), x(0) = 1, y(0) = 1}, {x(t), y(t)});
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$$S := \left\{ \begin{aligned} x(t) &= e^{-\frac{3t}{2}} \left(\frac{5 \sin\left(\frac{\sqrt{47} t}{2}\right) \sqrt{47}}{47} + \cos\left(\frac{\sqrt{47} t}{2}\right) \right), y(t) = \\ &= \frac{e^{-\frac{3t}{2}} \left(\frac{52 \sin\left(\frac{\sqrt{47} t}{2}\right) \sqrt{47}}{47} - 4 \cos\left(\frac{\sqrt{47} t}{2}\right) \right)}{4} \end{aligned} \right. \quad (2)$$

> plot([subs(S, x(t)), subs(S, y(t)), t = 1 .. 10]);

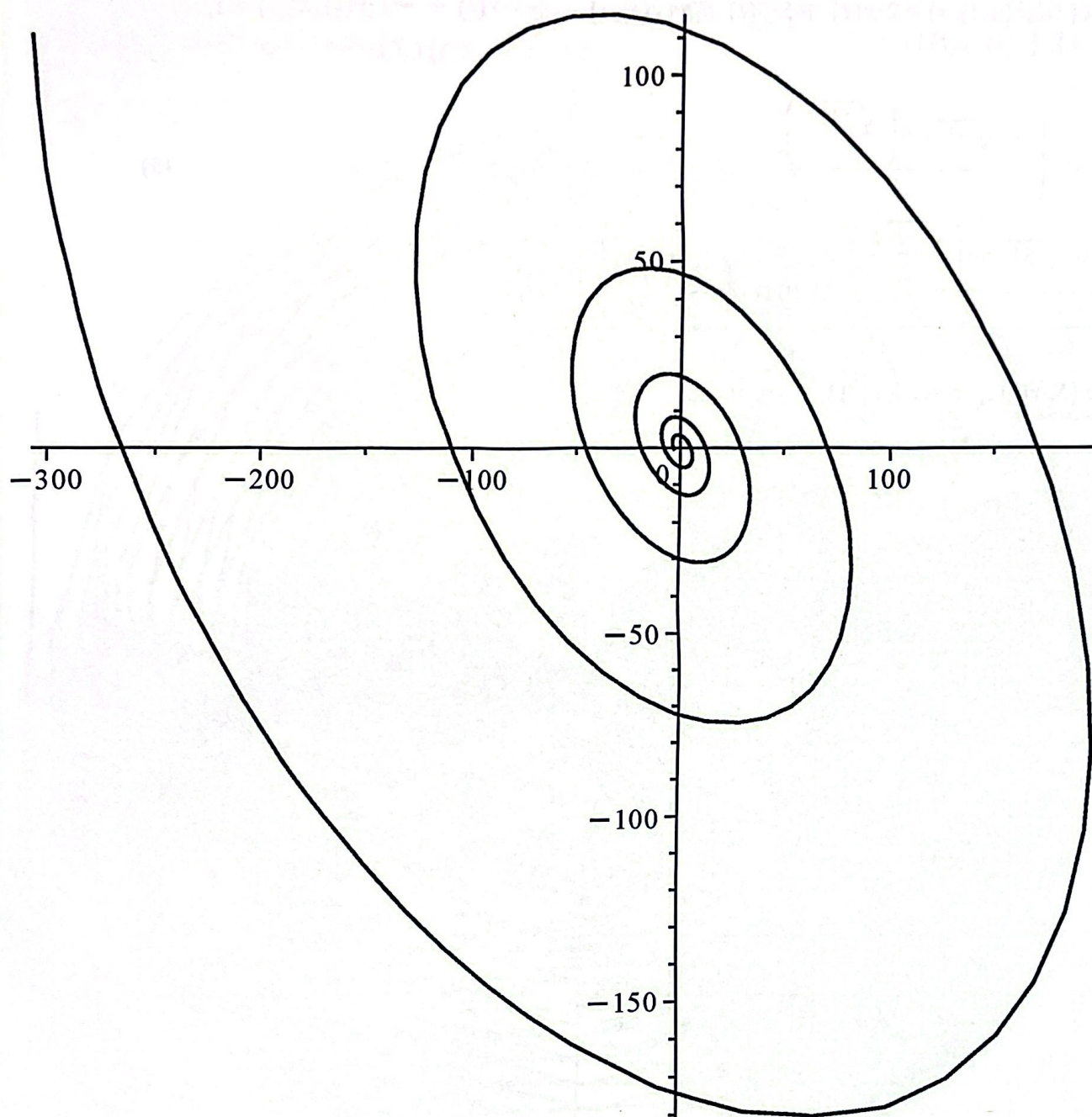


> $S := \text{dsolve}(\{\text{diff}(x(t), t) = 2 \cdot x(t) + 5 \cdot y(t), \text{diff}(y(t), t) = -3 \cdot x(t) + -1 \cdot y(t), x(0) = 1, y(0) = 1\}, \{x(t), y(t)\});$

$$S := \left\{ x(t) = e^{\frac{t}{2}} \left(\frac{13 \sqrt{51} \sin\left(\frac{\sqrt{51} t}{2}\right)}{51} + \cos\left(\frac{\sqrt{51} t}{2}\right) \right), y(t) = \right. \quad (3)$$

$$\left. e^{\frac{t}{2}} \left(\frac{30 \sqrt{51} \sin\left(\frac{\sqrt{51} t}{2}\right)}{17} - 10 \cos\left(\frac{\sqrt{51} t}{2}\right) \right) \right\}$$

> $\text{plot}([\text{subs}(S, x(t)), \text{subs}(S, y(t)), t = 1..10]);$



$\Rightarrow S := \text{dsolve}(\{\text{diff}(x(t), t) = 4 \cdot x(t) + 1000 \cdot y(t), \text{diff}(y(t), t) = -120 \cdot x(t) + -7 \cdot y(t), x(0) = 1, y(0) = 1\}, \{x(t), y(t)\});$

$$S := \left\{ x(t) = e^{-\frac{3t}{2}} \left(\frac{2011 \sqrt{479879} \sin\left(\frac{\sqrt{479879} t}{2}\right)}{479879} + \cos\left(\frac{\sqrt{479879} t}{2}\right) \right), y(t) = \right. \quad (4)$$

$$\left. \frac{e^{-\frac{3t}{2}} \left(\frac{502000 \sqrt{479879} \sin\left(\frac{\sqrt{479879} t}{2}\right)}{479879} - 2000 \cos\left(\frac{\sqrt{479879} t}{2}\right) \right)}{2000} \right\}$$

```
> plot([subs(S, x(t)), subs(S, y(t)), t = 1..10]);
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