HW21 ZACHARIAH ALI

1. $\frac{dx}{dt} = x - y$, $\frac{dy}{dt} = y - x$, $x(0) = 1$, $y(0) = 1$.

Let $\underline{z} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$, $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ and $\dot{\underline{z}} = \begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix}$

thus $\dot{\underline{z}} = A\underline{z}$.

Eigenvalues of A. $(1-\lambda)^2 - (1^2) = 0$ $\lambda_1 = 0$, $\lambda_2 = 2$.

Eigenvectors $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2$. Use $\underline{v_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = 2\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} \Rightarrow x_3 = -x_4$. Use $\underline{v_2} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

So general solution is $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = C_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{2t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\Rightarrow x(t) = C_1 + C_2 e^{2t}$, $y(t) = C_1 - C_2 e^{2t}$

$\Rightarrow 2C_1 = x(t) + y(t)$, $\Rightarrow 2C_1 = x(0) + y(0) = 2$

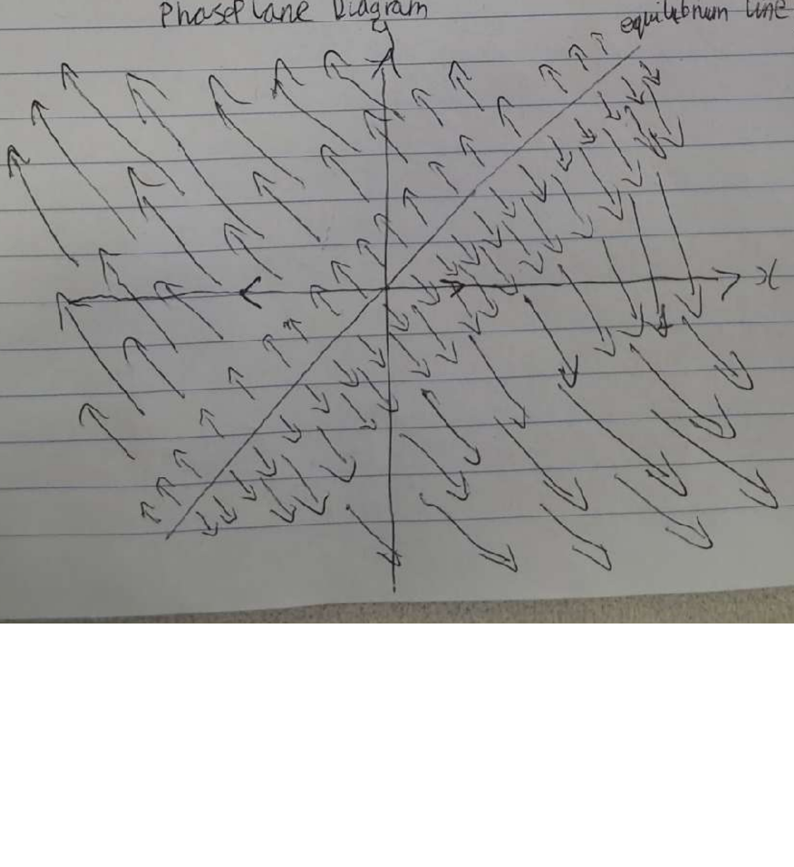
So $C_1 = 1$.

$2C_2 e^{2t} = x(t) - y(t)$. So for $t = 0$

$2C_2 = x(0) - y(0) = 0 \Rightarrow C_2 = 0$.

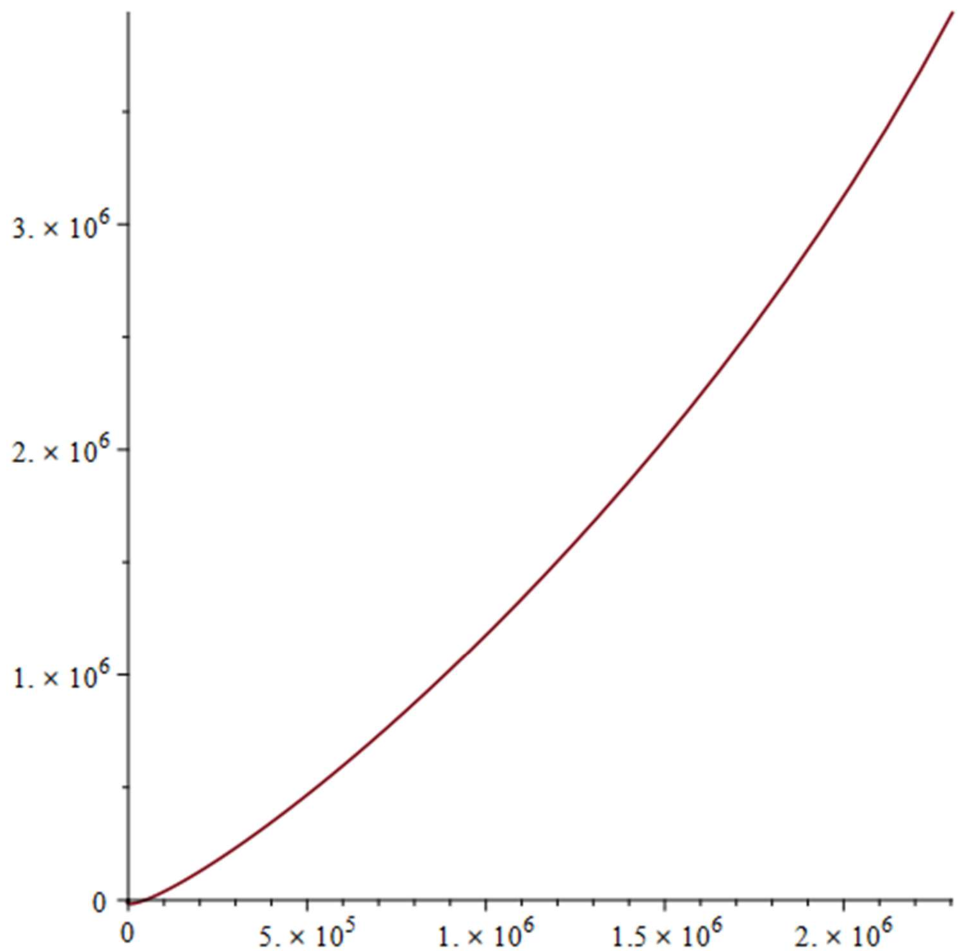
So $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ i.e. $x(t) = 1$, $y(t) = 1$ $\forall t \in \mathbb{R}$ (presumably).

Phase Plane Diagram

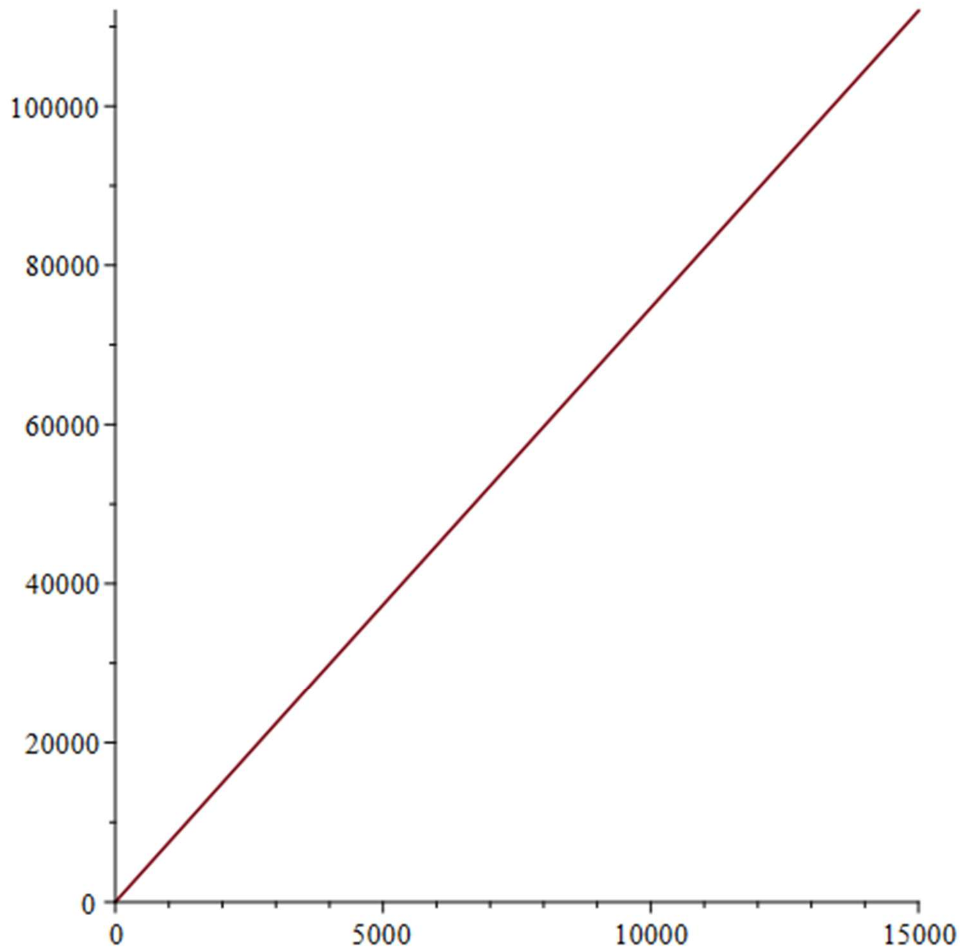


2, No. I presume this is as the ICs were different. I tried to make ICs the same to explore further but the solution was a single point so nothing was graphed.

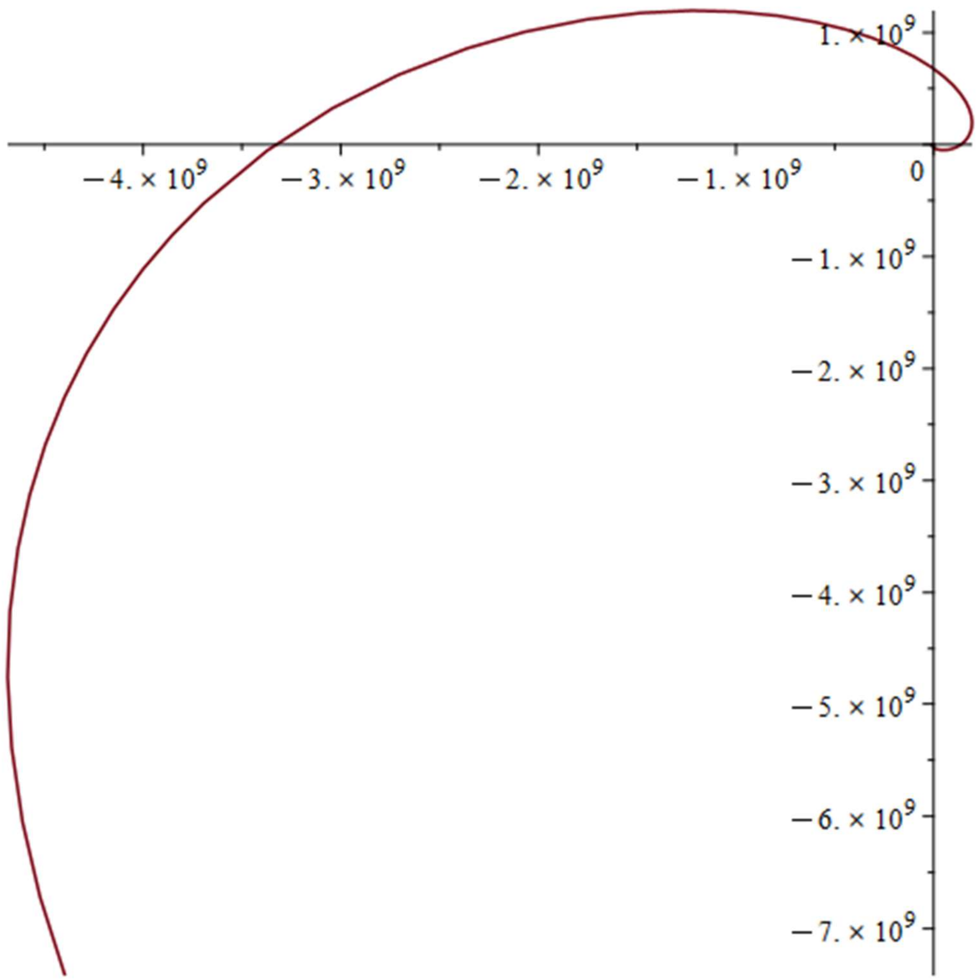
3.



5, −2, 3, 1



$-5, 1, -4, 3$

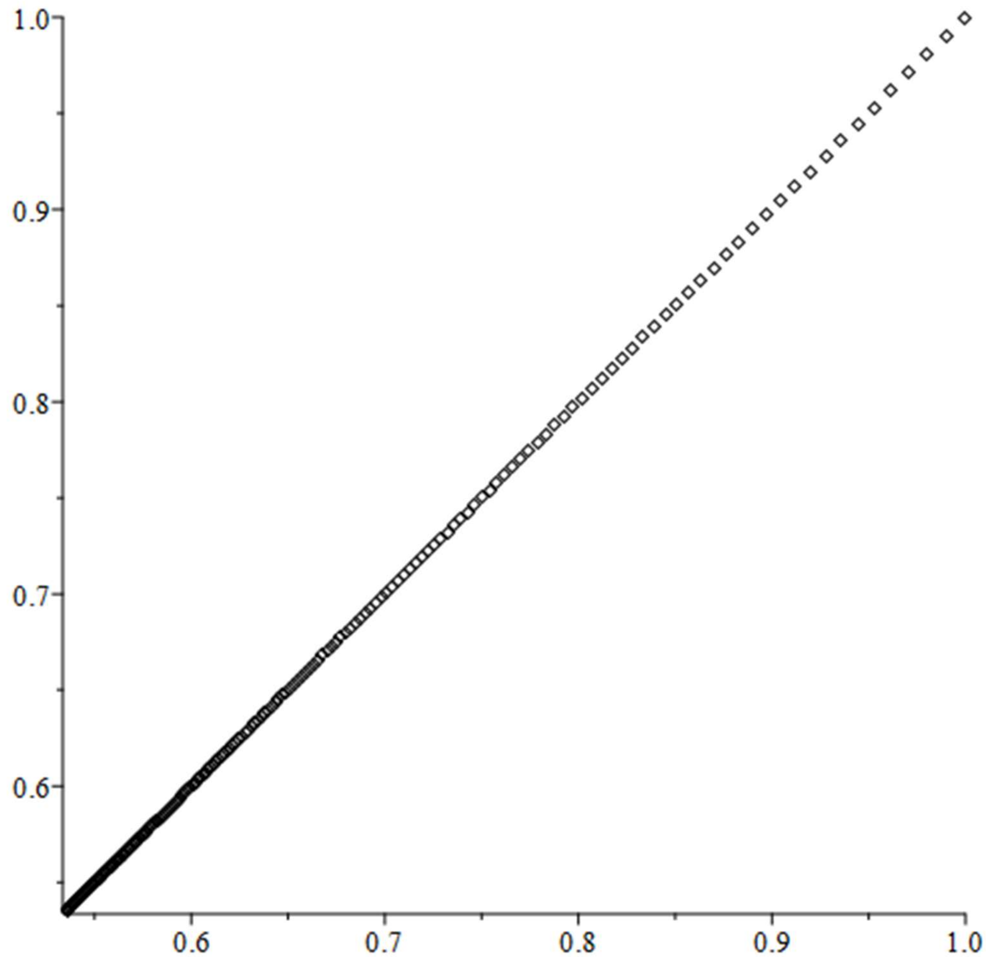


$4, -4, 5, 5$

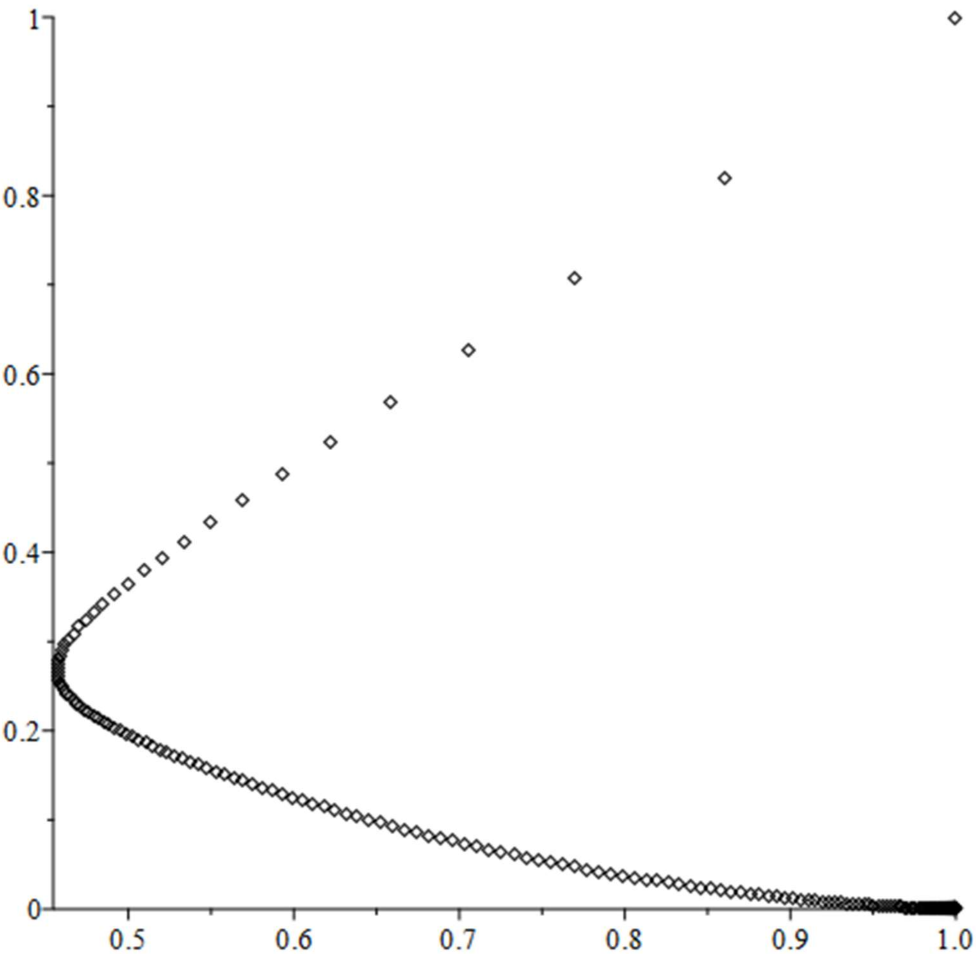
4.

Lotka

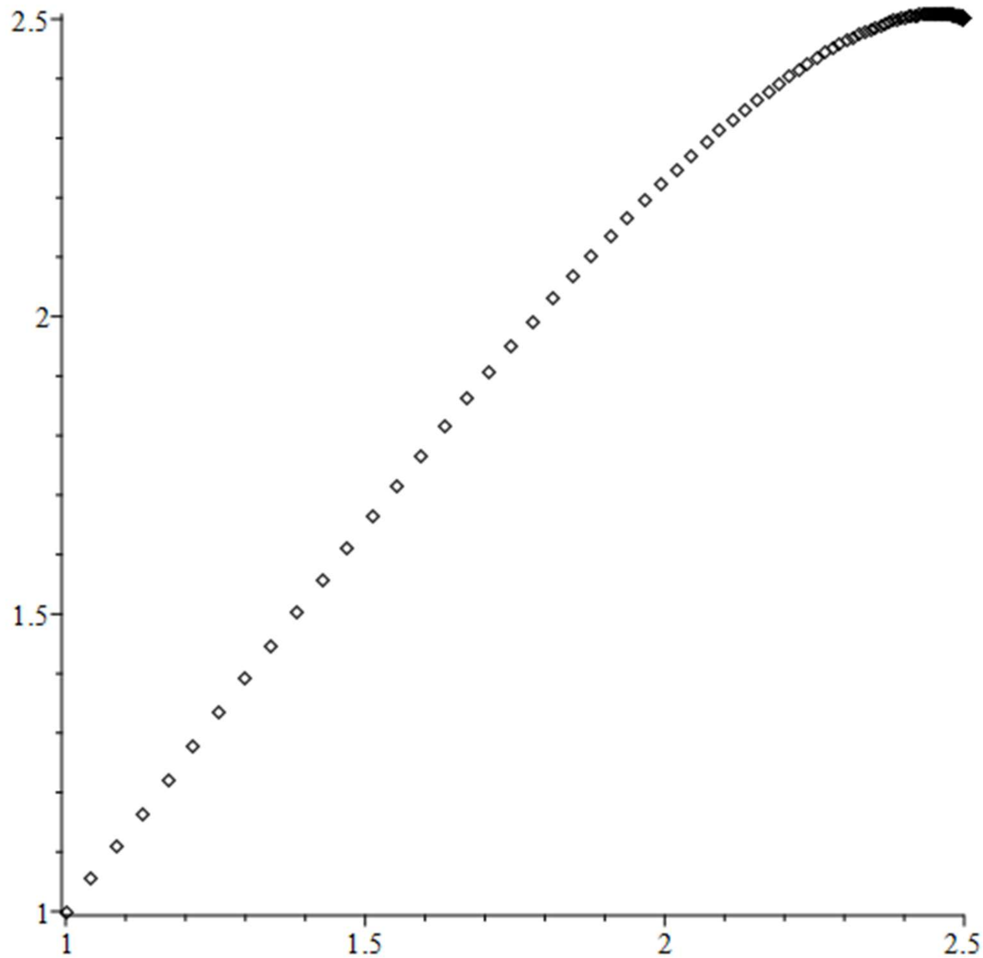
(r1,k1,r2,k2,b12,b21)=(1,1,1,1,1,1)



(r1,k1,r2,k2,b12,b21)=(7,1,9,1,2,2)



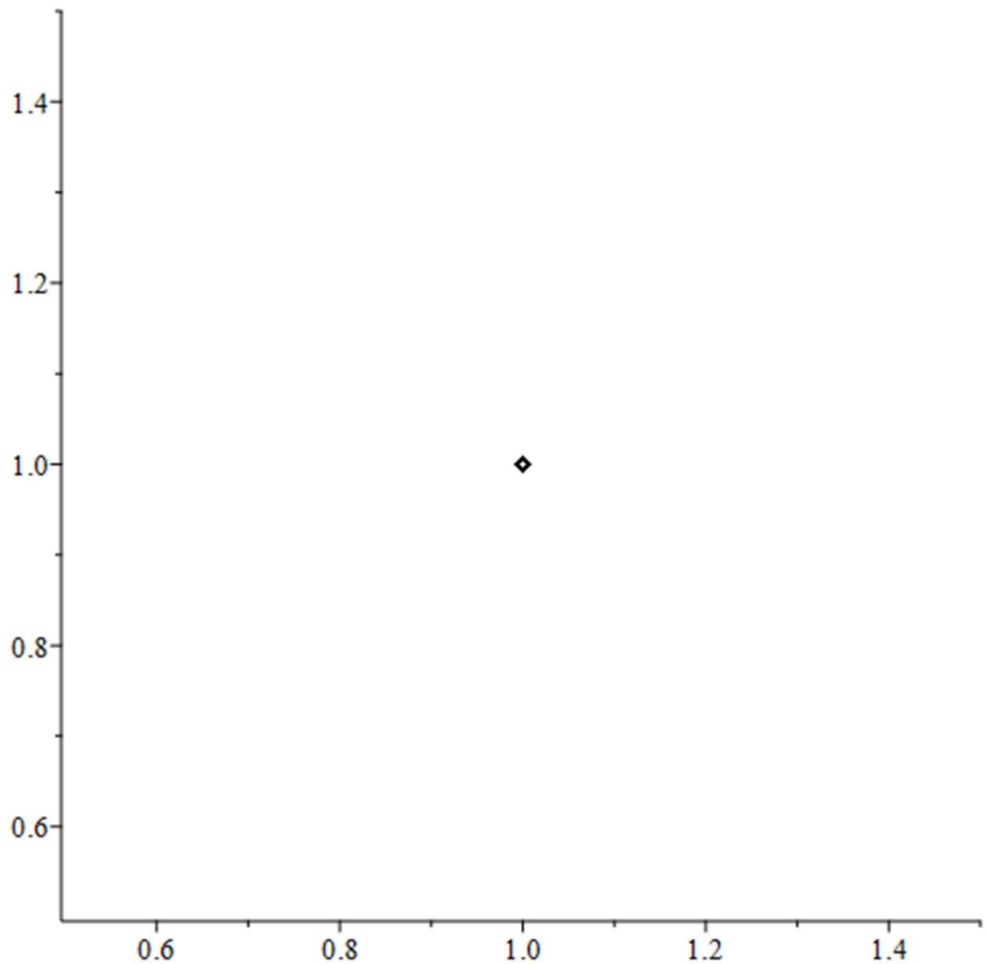
(r1,k1,r2,k2,b12,b21 =(7,3,9,3,0.2,0.2)



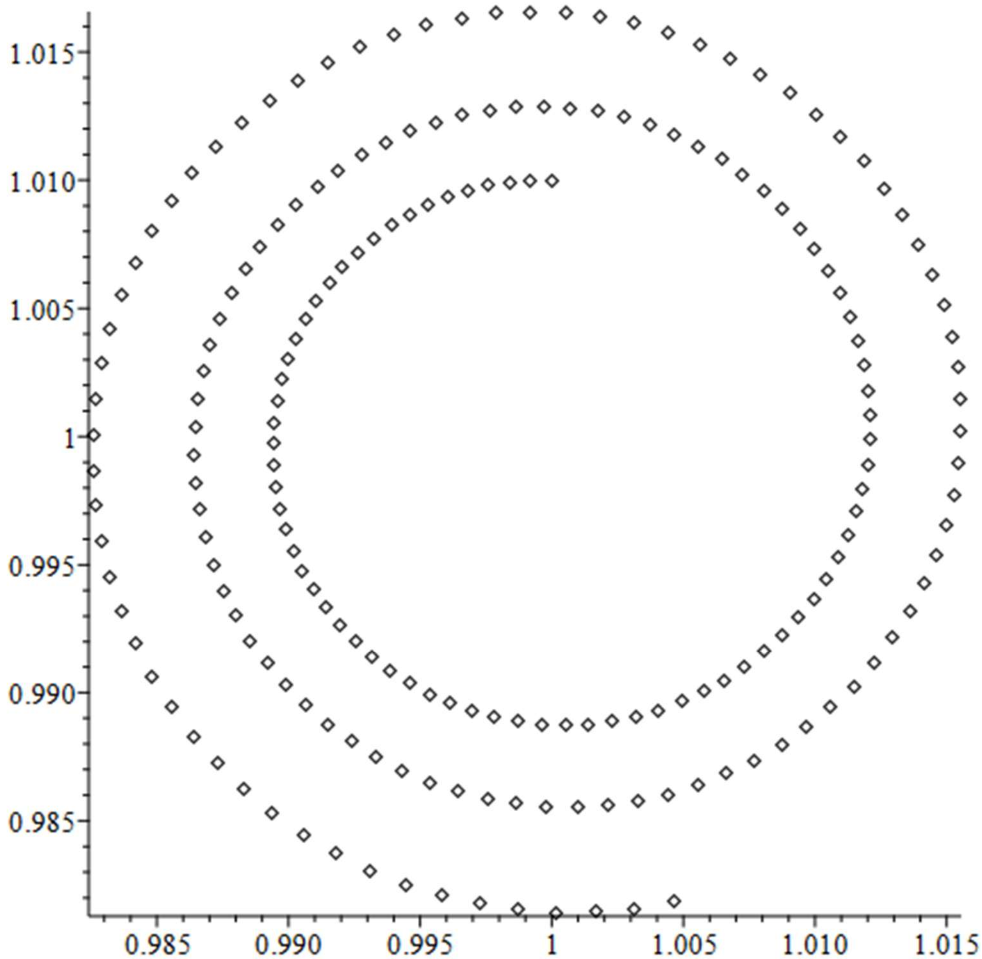
Volterra

(a,b,c,d)=(8,8,8,8)

Interestingly, I initially accidentally began at equilibrium (1,1) and got this:

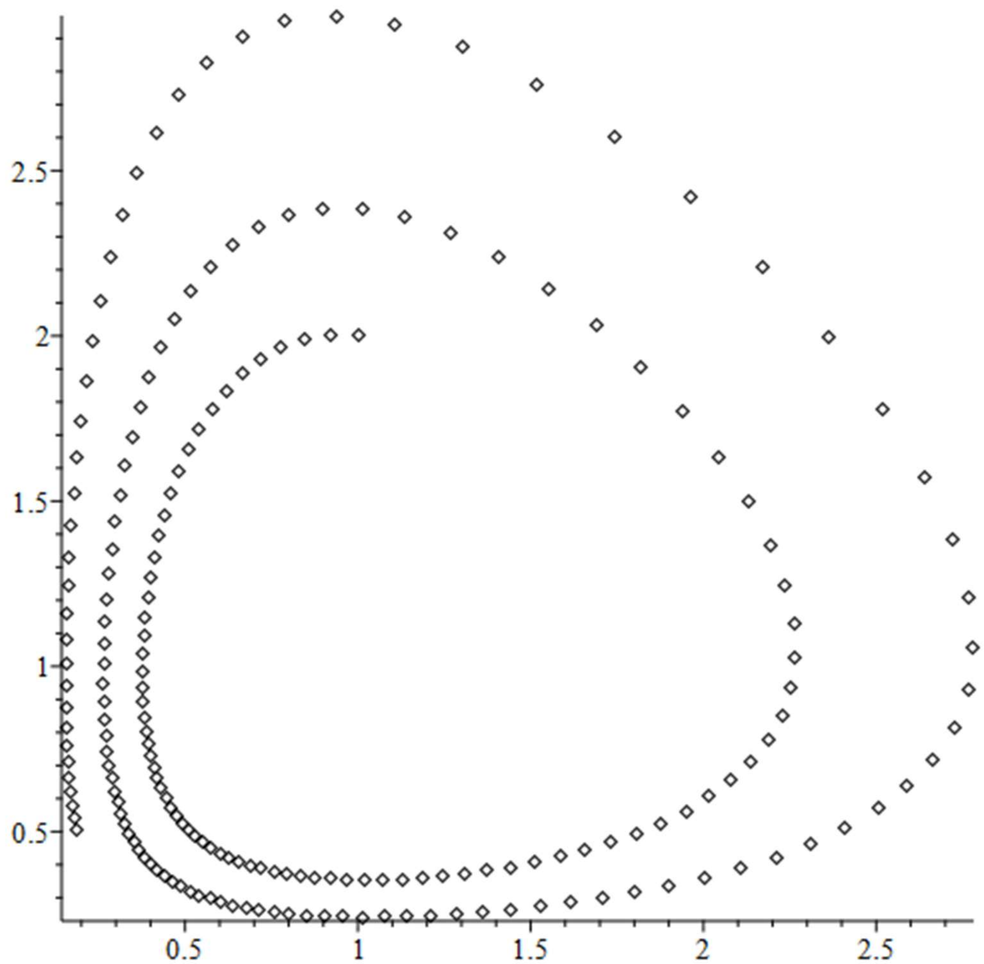


But then I perturbed to [1,1.01] and got

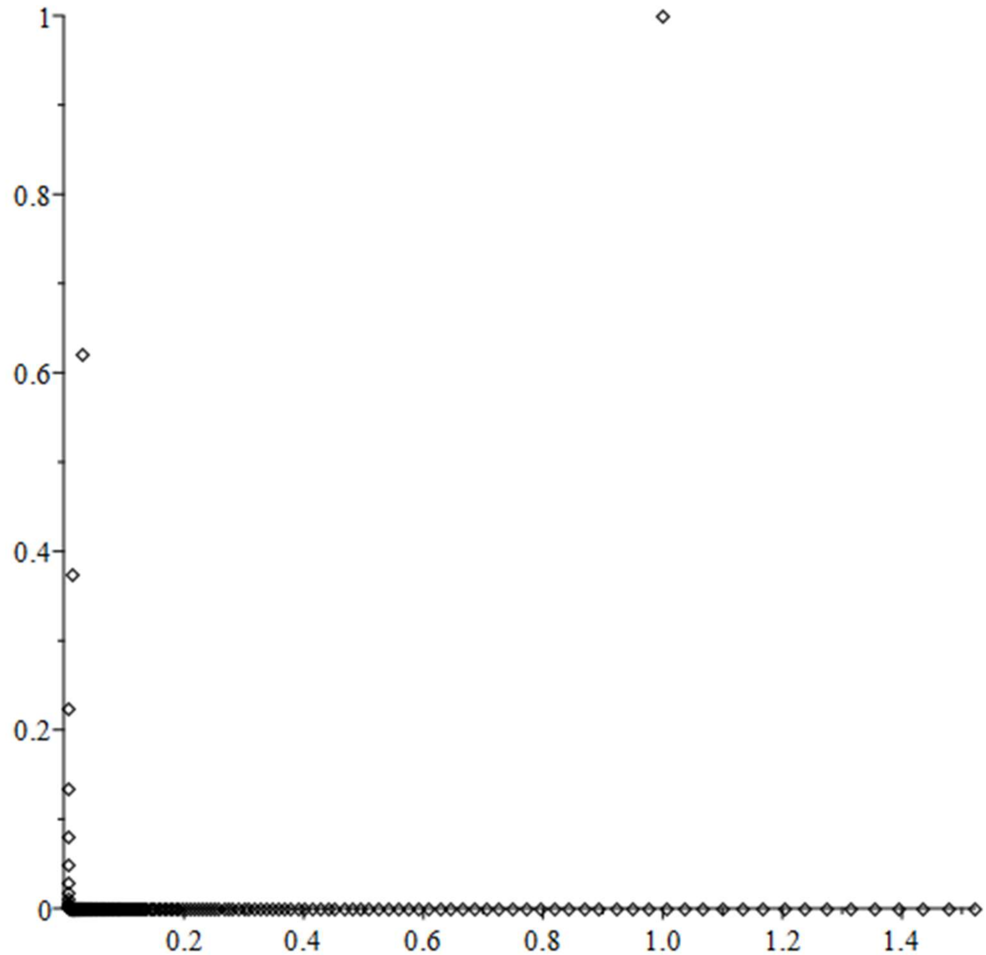


1.015
1.010
1.005
1
0.995
0.990
0.985
0.985
0.990
0.995
1
1.005
1.010
1.015

(a,b,c,d)=(0.1,8,0.1,8)

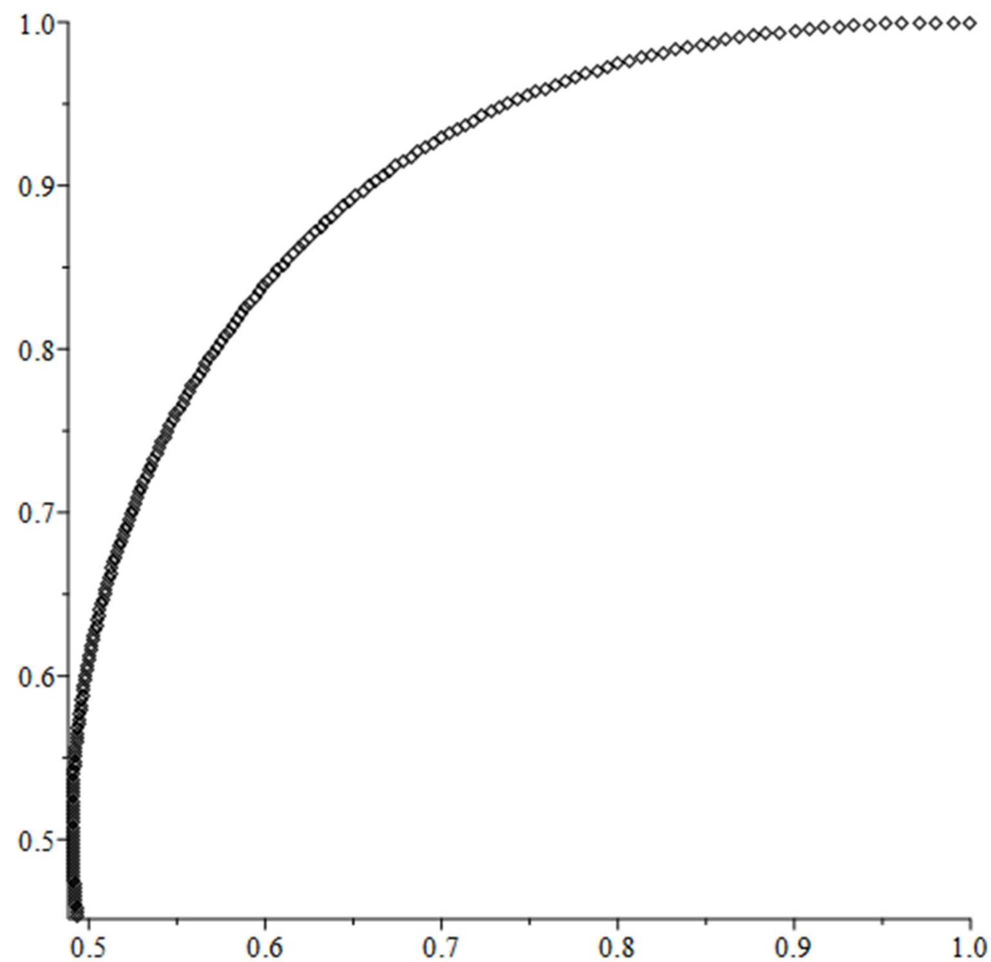


(a,b,c,d)=(3, 100, 40, 2)

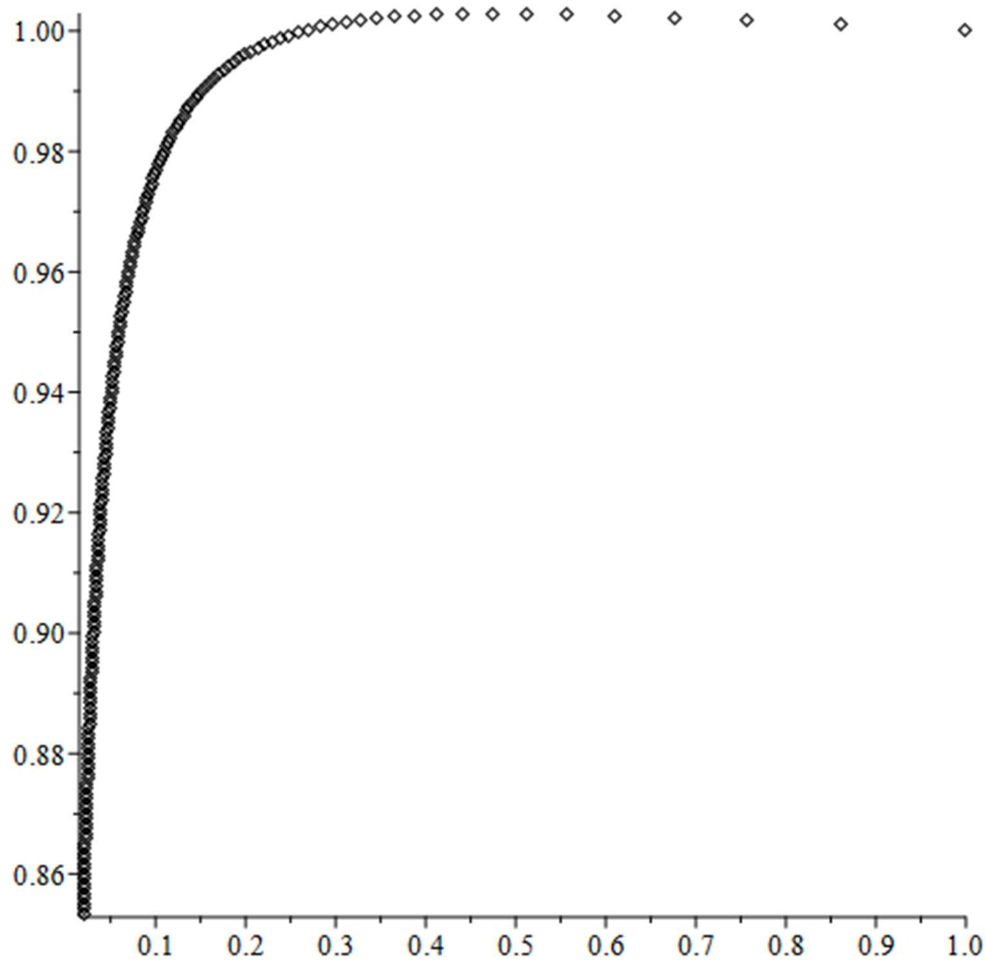


VolterraM

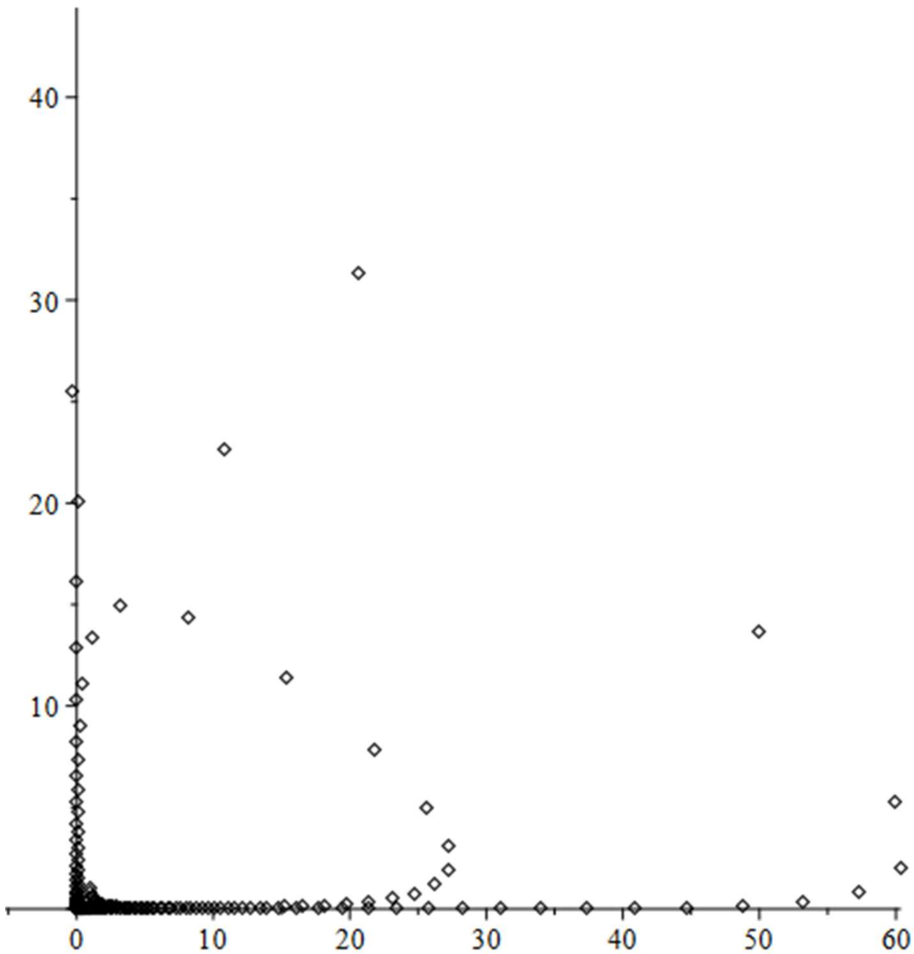
(a,b,c,d,K)=(1,1,1,1,1)



(a,b,c,d,K)=(0.4, 1, 0.1, 0.03, 0.2)



(a,b,c,d,K)=(10, 5, 20, 1000, 3)



40
30
20
10
0
10
20
30
40
50
60