## Homework for Lecture 21 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file and/or .txt file) to

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by 8:00pm Monday, Dec. 1,, 2025.

Subject: hw21

with an attachment hw21FirstLast.pdf and/or hw21FirstLast.txt

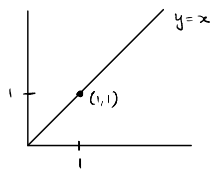
$$\frac{d}{dx}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

**1.** By hand solve the system

$$\frac{dx}{dt} = 1-1 = 0 \qquad \frac{dy}{dt} = 1-1 = 0$$

$$x(t) = 1 \qquad y(t) = 1$$

$$\frac{dx}{dt} = x - y \quad , \frac{dy}{dt} = y - x \quad , \quad x(0) = 1 \quad , \quad y(0) = 1 \quad .$$

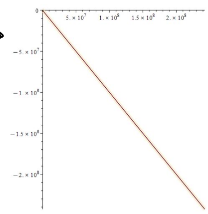



Plot, by hand, the phase-plane diagram.

**2.** Now use Maple with the command

```
S:=dsolve({diff(x(t),t)=x(t)-y(t),diff(y(t),t)=y(t)-x(t),x(0)=1,y(0)=0},{x(t),y(t)});
plot([subs(S,x(t)),subs(S,y(t)),t=0..10]);
```

Maple image →



did you get the same thing?

No because $y(0)=0$ in the command ; $y(0)=1$ in #1

**3.** Use Maple to solve and then plot the phase-plane diagram for the system

$$\frac{dx}{dt} = a_{11}x + a_{12}y \quad , \frac{dy}{dt} = a_{21}x + a_{22}y \quad , \quad x(0) = 1 \quad , \quad y(0) = 1 \quad ,$$

for three randomly chosen matrices

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad .$$

**4.** Carefully read, and understand, the Maple code for the following procedures (type Help(ProcedureName); for instructions)

`Lotka, Volterra, VolterraM`

in the Maple package

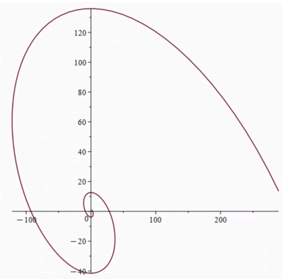
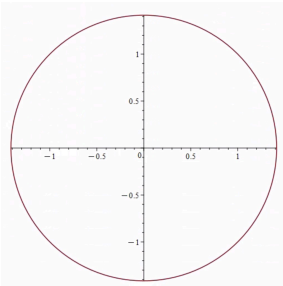
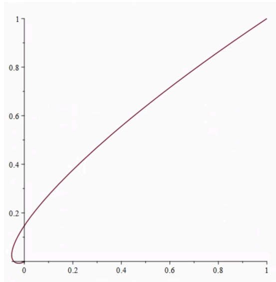
`https://sites.math.rutgers.edu/~zeilberg/Bio25/DMB.txt` ,

For **each of them**, experiment with **three** randmom choices of parameters, and random initial conditions, using `Dis` (with $h = 0.01$), of *each* of the quantities in question.

Send me these nice plots.

Confirm the numerics by using `SEquP`.

3. $A_1 = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$ $A_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $A_3 = \begin{pmatrix} -1 & -2 \\ 1 & -3 \end{pmatrix}$

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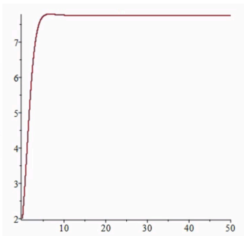
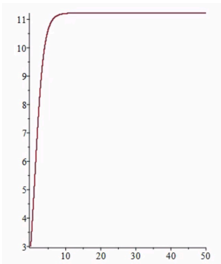
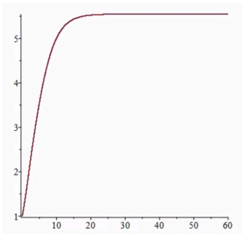
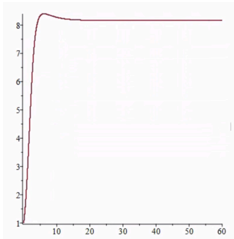
4.

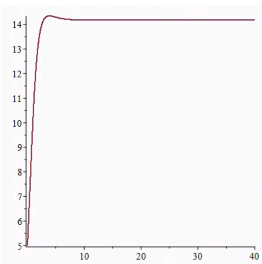
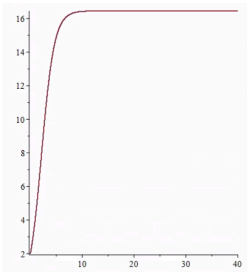
```
> r1 := 1.0 : k1 := 10.0 :
  r2 := 0.8 : k2 := 12.0 :
  b12 := 0.2 : b21 := 0.1 :
  F1 := Lotka(r1, k1, r2, k2, b12, b21, N1, N2) :
  TimeSeries(F1, [N1, N2], [2.0, 3.0], 0.01, 50.0, 1); # N1(t)
  TimeSeries(F1, [N1, N2], [2.0, 3.0], 0.01, 50.0, 2); # N2(t)
  SEquP(F1, [N1, N2]);
```

```
> F2 := Lotka(0.5, 8.0, 1.2, 9.0, 0.3, 0.15, N1, N2) :
  TimeSeries(F2, [N1, N2], [1.0, 1.0], 0.01, 60.0, 1);
  TimeSeries(F2, [N1, N2], [1.0, 1.0], 0.01, 60.0, 2);
  SEquP(F2, [N1, N2]);
```

```
> F3 := Lotka(1.5, 15.0, 1.0, 20.0, 0.05, 0.25, N1, N2) :
  TimeSeries(F3, [N1, N2], [5.0, 2.0], 0.01, 40.0, 1);
  TimeSeries(F3, [N1, N2], [5.0, 2.0], 0.01, 40.0, 2);
  SEquP(F3, [N1, N2]);
```

Lotka

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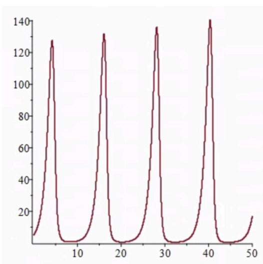
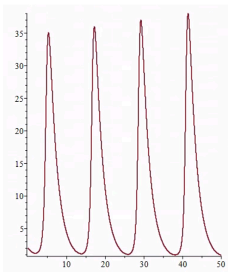
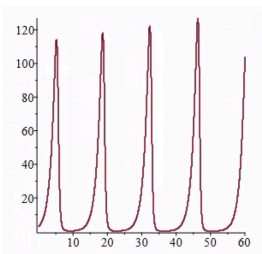
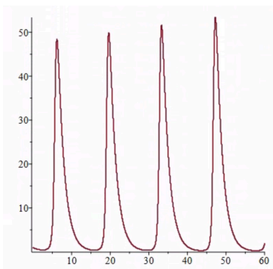
```
> Fv1 := Volterra(1.0, 0.1, 0.5, 0.02, x, y) :
  TimeSeries(Fv1, [x, y], [5.0, 2.0], 0.01, 50.0, 1);
  TimeSeries(Fv1, [x, y], [5.0, 2.0], 0.01, 50.0, 2);
  SEquP(Fv1, [x, y]);
```

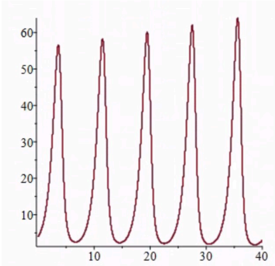



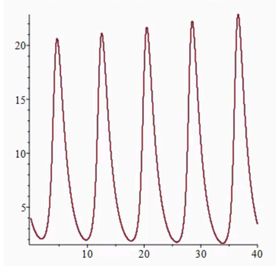
```
> Fv2 := Volterra(0.8, 0.08, 0.6, 0.03, x, y) :
  TimeSeries(Fv2, [x, y], [3.0, 1.0], 0.01, 60.0, 1);
  TimeSeries(Fv2, [x, y], [3.0, 1.0], 0.01, 60.0, 2);
  SEquP(Fv2, [x, y]);
```

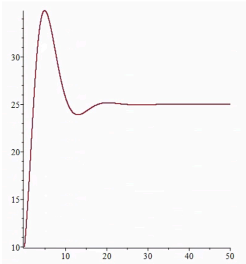
Volterra

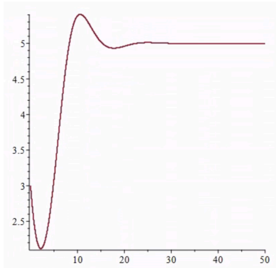
```
> Fv3 := Volterra(1.2, 0.15, 0.7, 0.04, x, y) :
  TimeSeries(Fv3, [x, y], [4.0, 4.0], 0.01, 40.0, 1);
  TimeSeries(Fv3, [x, y], [4.0, 4.0], 0.01, 40.0, 2);
  SEquP(Fv3, [x, y]);
```



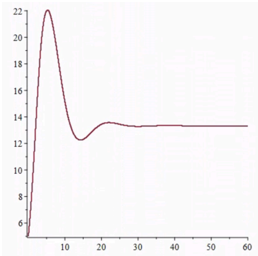



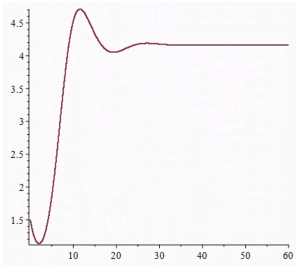

```
> Fvm1 := VolterraM(1.0, 0.1, 0.5, 50.0, 0.02, x, y) :
  TimeSeries(Fvm1, [x, y], [10.0, 3.0], 0.01, 50.0, 1);
  TimeSeries(Fvm1, [x, y], [10.0, 3.0], 0.01, 50.0, 2);
  SEquP(Fvm1, [x, y]);
```



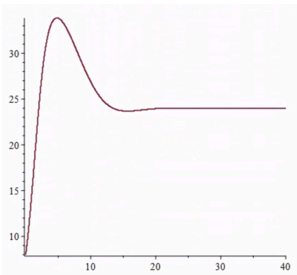



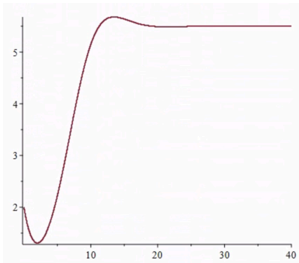

```
> Fvm2 := VolterraM(0.9, 0.12, 0.4, 30.0, 0.03, x, y) :
  TimeSeries(Fvm2, [x, y], [5.0, 1.5], 0.01, 60.0, 1);
  TimeSeries(Fvm2, [x, y], [5.0, 1.5], 0.01, 60.0, 2);
  SEquP(Fvm2, [x, y]);
```




```
> Fvm3 := VolterraM(1.1, 0.08, 0.6, 40.0, 0.025, x, y) :
  TimeSeries(Fvm3, [x, y], [8.0, 2.0], 0.01, 40.0, 1);
  TimeSeries(Fvm3, [x, y], [8.0, 2.0], 0.01, 40.0, 2);
  SEquP(Fvm3, [x, y]);
```




Volterra M