① $\frac{dx}{dt} = x - y$ $\quad \frac{dy}{dt} = y - x$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow (1-\lambda)^2 - 1 = 0$$

$$\lambda = 0, 2$$

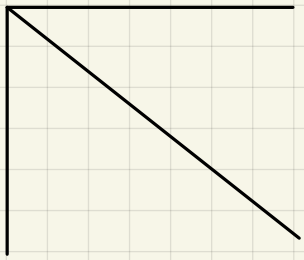
$(A - \lambda I)\, v = 0 \quad (1\ 1) \quad (A - 2I) = (-1\ 1)$

$c_1 + c_2 e^{2t}$

$c_1 - c_2 e^{2t}$

$c_1 + c_2 = 1 \quad c_1 - c_2 = 1$

$x(t) = 1 \quad y(t) = 1$ thus 0 is an equilibrium point



$X = -Y$

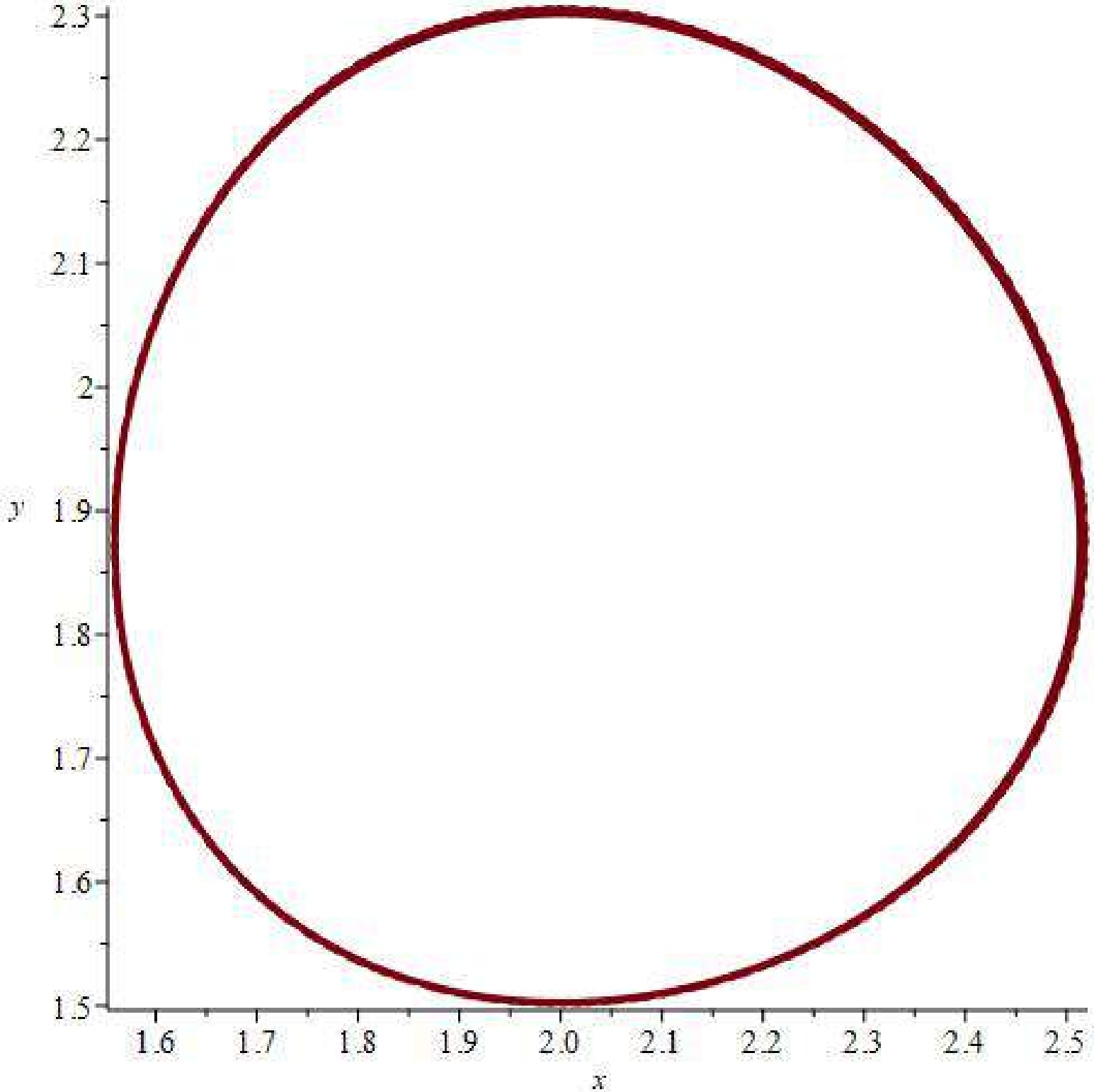
② attached

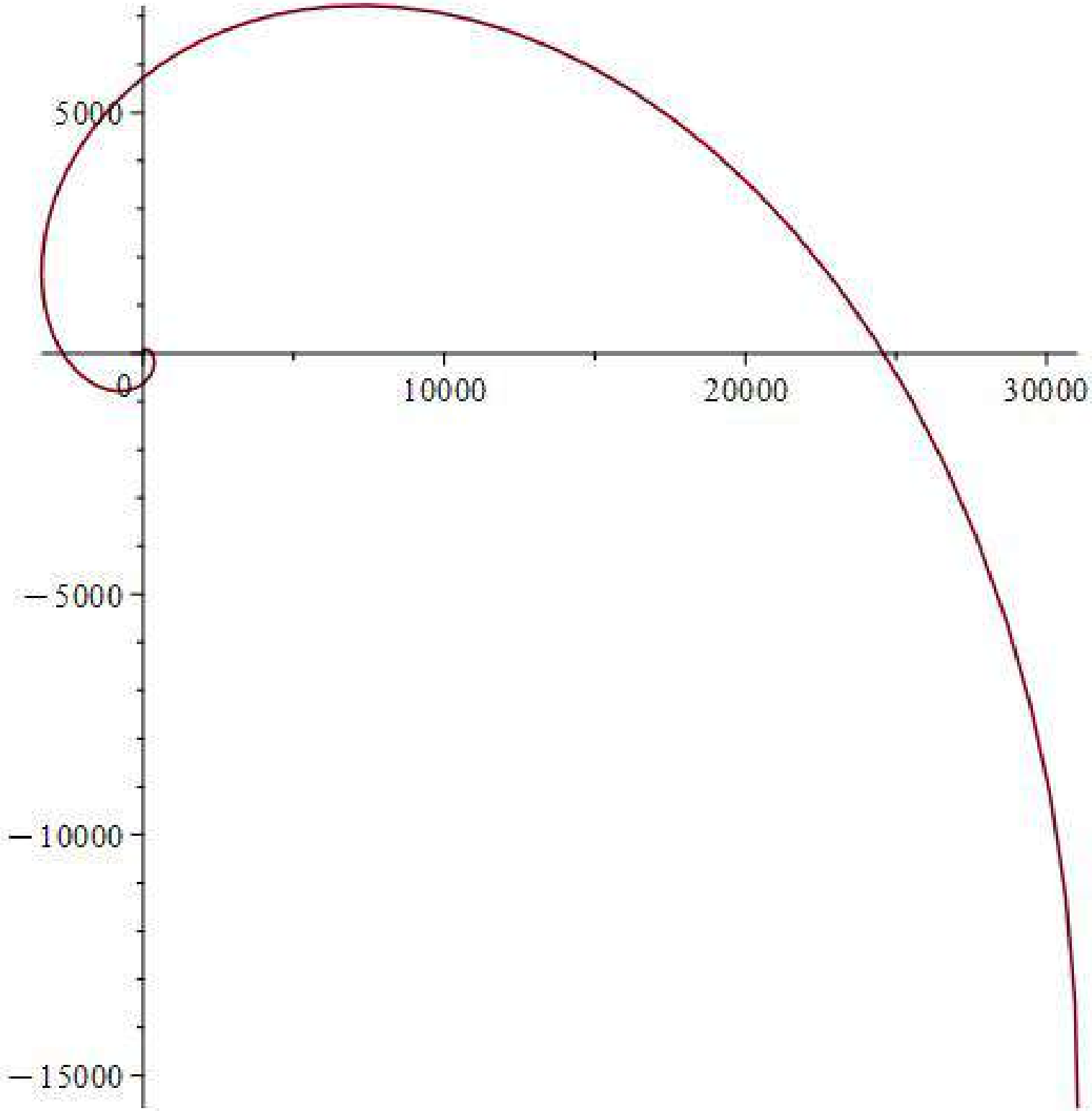
③ attached

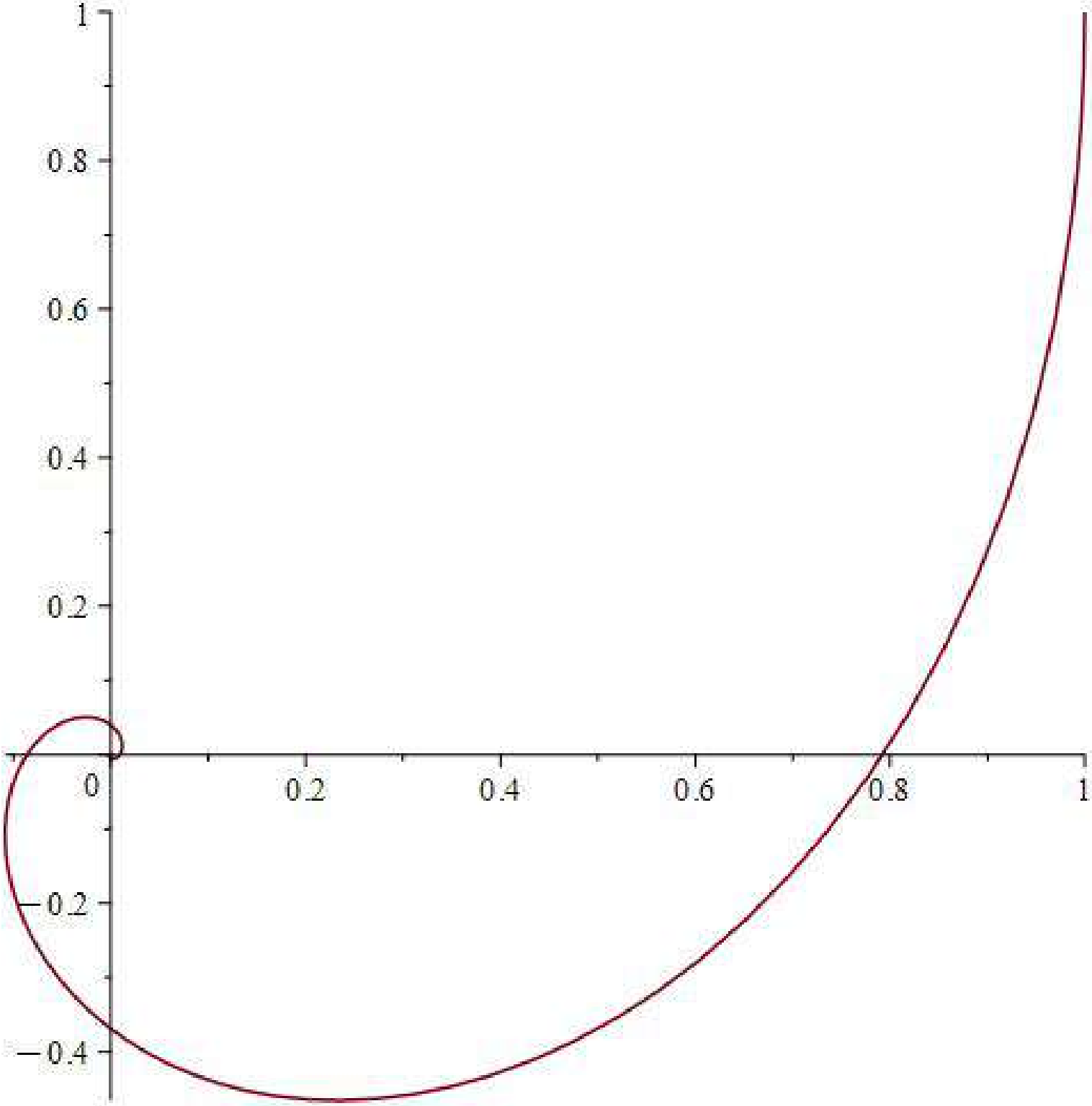
④ attached

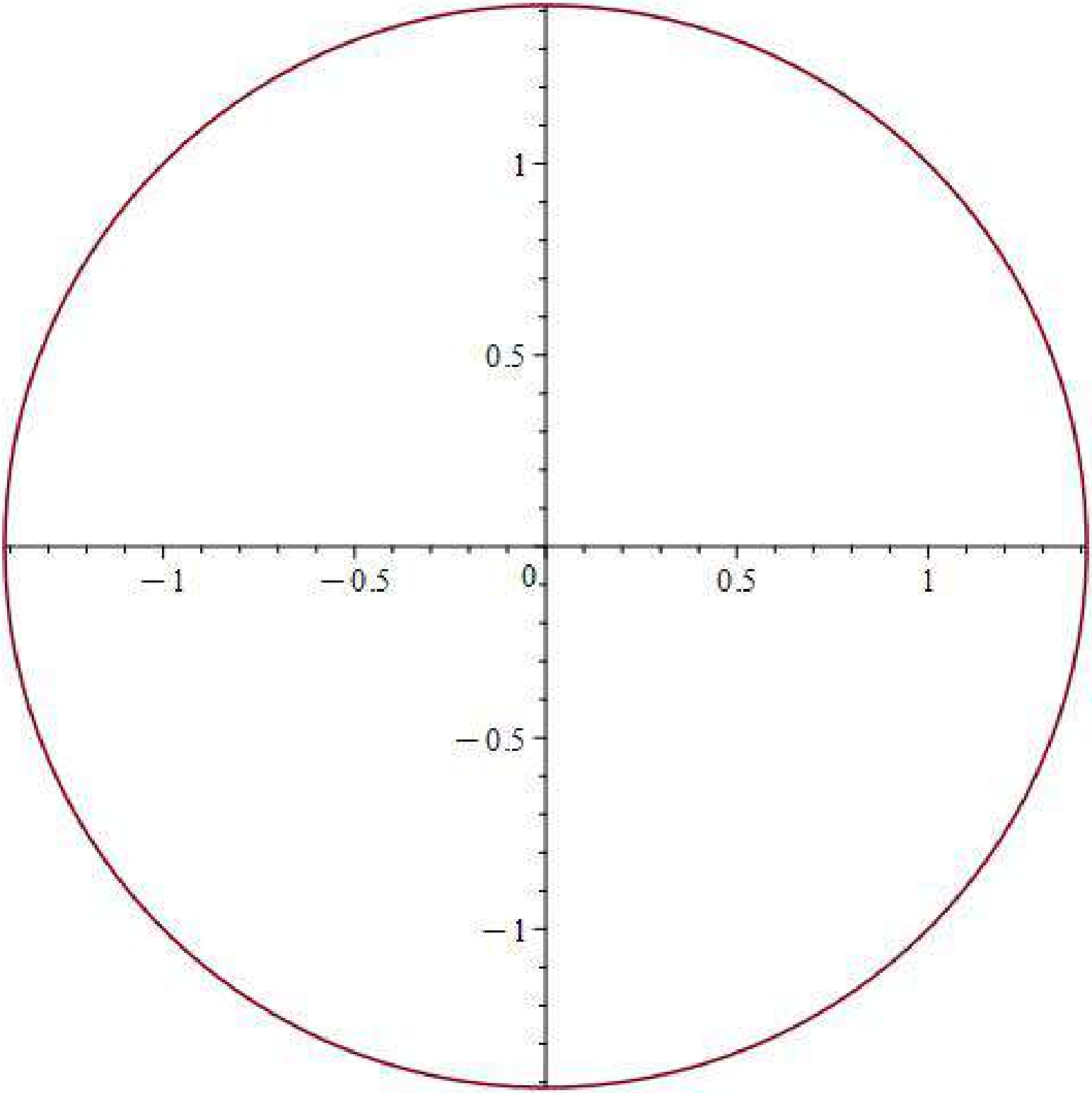
```
S1 := dsolve({diff(x(t), t) = x(t) + 2*y(t), diff(y(t), t) = y(t) - x(t), x(0) = 1,
y(0) = 1}, {x(t), y(t)});
plot([subs(S1, x(t)), subs(S1, y(t)), t = 0 .. 10]);
S2 := dsolve({diff(x(t), t) = y(t) - x(t), diff(y(t), t) = -2*x(t) - y(t), x(0) = 1,
y(0) = 1}, {x(t), y(t)});
plot([subs(S2, x(t)), subs(S2, y(t)), t = 0 .. 10]);
S3 := dsolve({diff(x(t), t) = 0*x(t) - y(t), diff(y(t), t) = x(t) + 0*y(t), x(0) = 1,
y(0) = 1}, {x(t), y(t)});
plot([subs(S3, x(t)), subs(S3, y(t)), t = 0 .. 10]);
```

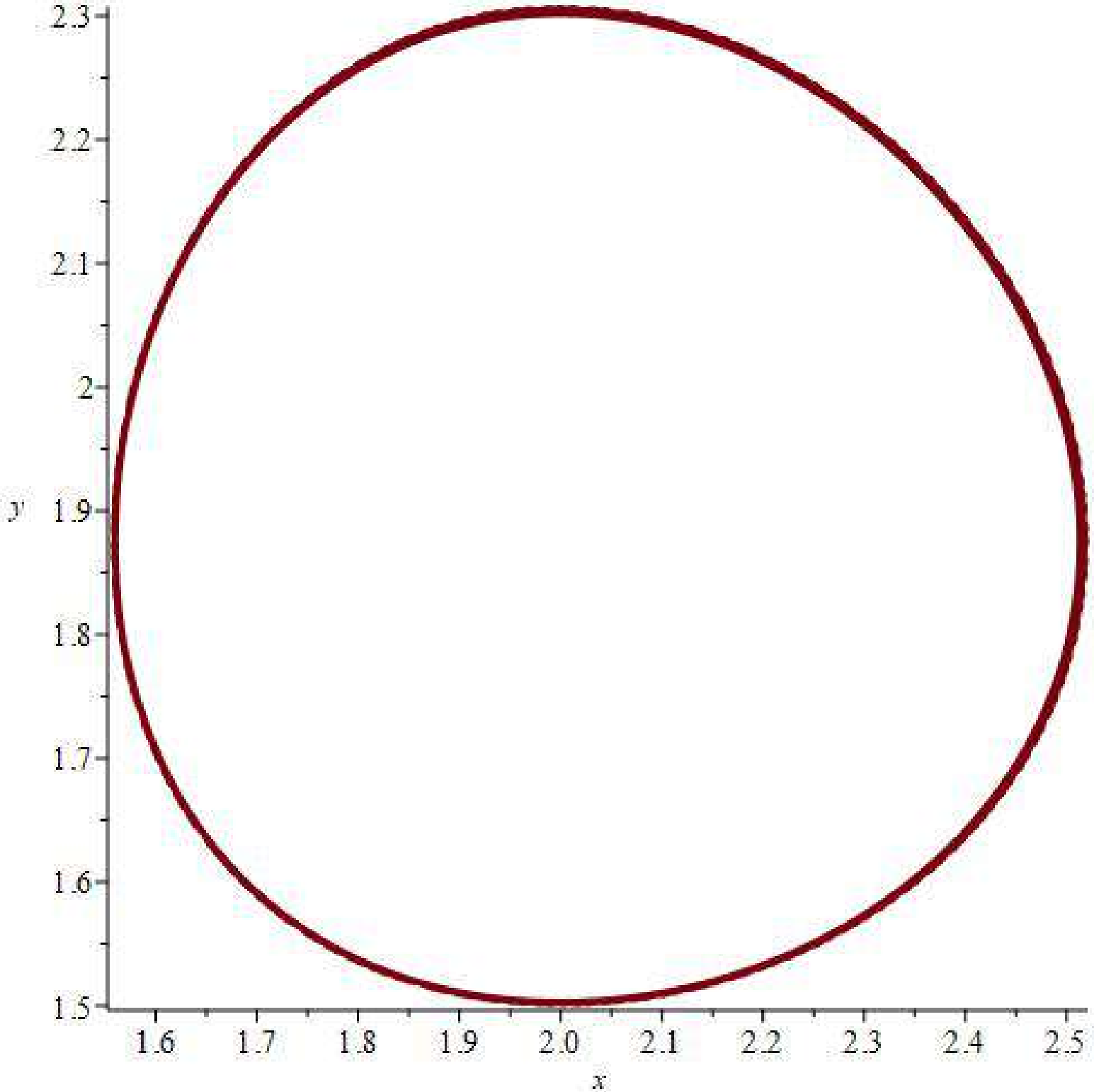
```
# ========================================================================
# VOLTERRA MODELS
# ========================================================================

# Volterra Model - Set 1: a=1.5, b=0.8, c=1.2, d=0.6, x(0)=2, y(0)=1.5
S1:=dsolve({diff(x(t),t)=1.5*x(t)-0.8*x(t)*y(t),diff(y(t),t)=-1.2*y(t)+0.6*x(t)*y(t),x(0)=2,y(0)=1.5},{x(t),y(t)},numeric):
odeplot(S1,[[x(t),y(t)]],0..50);

# Volterra Model - Set 2: a=2.0, b=1.0, c=1.5, d=0.8, x(0)=3, y(0)=2
S2:=dsolve({diff(x(t),t)=2.0*x(t)-1.0*x(t)*y(t),diff(y(t),t)=-1.5*y(t)+0.8*x(t)*y(t),x(0)=3,y(0)=2},{x(t),y(t)},numeric):
odeplot(S2,[[x(t),y(t)]],0..50);

# Volterra Model - Set 3: a=1.2, b=0.5, c=0.8, d=0.4, x(0)=1.5, y(0)=1
S3:=dsolve({diff(x(t),t)=1.2*x(t)-0.5*x(t)*y(t),diff(y(t),t)=-0.8*y(t)+0.4*x(t)*y(t),x(0)=1.5,y(0)=1},{x(t),y(t)},numeric):
odeplot(S3,[[x(t),y(t)]],0..50);

# Volterra Stable Equilibrium Points
print(`====================================`);
print(`Volterra Set 1 Stable Equilibria:`);
print(`Parameters: a=1.5, b=0.8, c=1.2, d=0.6`);
eq1:=SEquP([1.5*x-0.8*x*y, -1.2*y+0.6*x*y], [x,y]);
print(eq1);

print(`====================================`);
print(`Volterra Set 2 Stable Equilibria:`);
print(`Parameters: a=2.0, b=1.0, c=1.5, d=0.8`);
eq2:=SEquP([2.0*x-1.0*x*y, -1.5*y+0.8*x*y], [x,y]);
print(eq2);

print(`====================================`);
print(`Volterra Set 3 Stable Equilibria:`);
print(`Parameters: a=1.2, b=0.5, c=0.8, d=0.4`);
eq3:=SEquP([1.2*x-0.5*x*y, -0.8*y+0.4*x*y], [x,y]);
print(eq3);

# ========================================================================
# MODIFIED VOLTERRA MODELS
# ========================================================================

# Modified Volterra - Set 1: a=1.5, b=0.8, c=1.2, d=0.6, K=10, x(0)=2, y(0)=1.5
SM1:=dsolve({diff(x(t),t)=1.5*x(t)*(1-x(t)/10)-0.8*x(t)*y(t),diff(y(t),t)=-1.2*y(t)+0.6*x(t)*y(t),x(0)=2,y(0)=1.5},{x(t),y(t)},numeric):
odeplot(SM1,[[x(t),y(t)]],0..50);

# Modified Volterra - Set 2: a=2.0, b=1.0, c=1.5, d=0.8, K=15, x(0)=3, y(0)=2
SM2:=dsolve({diff(x(t),t)=2.0*x(t)*(1-x(t)/15)-1.0*x(t)*y(t),diff(y(t),t)=-1.5*y(t)+0.8*x(t)*y(t),x(0)=3,y(0)=2},{x(t),y(t)},numeric):
odeplot(SM2,[[x(t),y(t)]],0..50);

# Modified Volterra - Set 3: a=1.8, b=0.7, c=1.0, d=0.5, K=12, x(0)=4, y(0)=1.5
SM3:=dsolve({diff(x(t),t)=1.8*x(t)*(1-x(t)/12)-0.7*x(t)*y(t),diff(y(t),t)=-1.0*y(t)+0.5*x(t)*y(t),x(0)=4,y(0)=1.5},{x(t),y(t)},numeric):
odeplot(SM3,[[x(t),y(t)]],0..50);

# VolterraM Stable Equilibrium Points
print(`====================================`);
print(`VolterraM Set 1 Stable Equilibria:`);
print(`Parameters: a=1.5, b=0.8, c=1.2, d=0.6, K=10`);
eqm1:=SEquP([1.5*x*(1-x/10)-0.8*x*y, -1.2*y+0.6*x*y], [x,y]);
print(eqm1);

print(`====================================`);
print(`VolterraM Set 2 Stable Equilibria:`);
print(`Parameters: a=2.0, b=1.0, c=1.5, d=0.8, K=15`);
eqm2:=SEquP([2.0*x*(1-x/15)-1.0*x*y, -1.5*y+0.8*x*y], [x,y]);
print(eqm2);

print(`====================================`);
print(`VolterraM Set 3 Stable Equilibria:`);
print(`Parameters: a=1.8, b=0.7, c=1.0, d=0.5, K=12`);
eqm3:=SEquP([1.8*x*(1-x/12)-0.7*x*y, -1.0*y+0.5*x*y], [x,y]);
print(eqm3);

# ========================================================================
# LOTKA COMPETITION MODELS
# ========================================================================

# Lotka - Set 1: r1=1.2, k1=10, r2=1.5, k2=12, b12=0.8, b21=0.6, N1(0)=3, N2(0)=4
SL1:=dsolve({diff(N1(t),t)=1.2*N1(t)*(10-N1(t)-0.8*N2(t))/10,diff(N2(t),t)=1.5*N2(t)*(12-N2(t)-0.6*N1(t))/12,N1(0)=3,N2(0)=4},
{N1(t),N2(t)},numeric):
odeplot(SL1,[[N1(t),N2(t)]],0..50);

# Lotka - Set 2: r1=1.5, k1=15, r2=1.8, k2=14, b12=1.2, b21=1.0, N1(0)=5, N2(0)=6
SL2:=dsolve({diff(N1(t),t)=1.5*N1(t)*(15-N1(t)-1.2*N2(t))/15,diff(N2(t),t)=1.8*N2(t)*(14-N2(t)-1.0*N1(t))/14,N1(0)=5,N2(0)=6},
{N1(t),N2(t)},numeric):
odeplot(SL2,[[N1(t),N2(t)]],0..50);

# Lotka - Set 3: r1=2.0, k1=20, r2=1.6, k2=18, b12=0.5, b21=0.7, N1(0)=8, N2(0)=7
SL3:=dsolve({diff(N1(t),t)=2.0*N1(t)*(20-N1(t)-0.5*N2(t))/20,diff(N2(t),t)=1.6*N2(t)*(18-N2(t)-0.7*N1(t))/18,N1(0)=8,N2(0)=7},
{N1(t),N2(t)},numeric):
odeplot(SL3,[[N1(t),N2(t)]],0..50);

# Lotka Stable Equilibrium Points
print(`====================================`);
print(`Lotka Set 1 Stable Equilibria:`);
print(`Parameters: r1=1.2, k1=10, r2=1.5, k2=12, b12=0.8, b21=0.6`);
eql1:=SEquP([1.2*N1*(10-N1-0.8*N2)/10, 1.5*N2*(12-N2-0.6*N1)/12], [N1,N2]);
print(eql1);

print(`====================================`);
print(`Lotka Set 2 Stable Equilibria:`);
print(`Parameters: r1=1.5, k1=15, r2=1.8, k2=14, b12=1.2, b21=1.0`);
eql2:=SEquP([1.5*N1*(15-N1-1.2*N2)/15, 1.8*N2*(14-N2-1.0*N1)/14], [N1,N2]);
print(eql2);

print(`====================================`);
print(`Lotka Set 3 Stable Equilibria:`);
print(`Parameters: r1=2.0, k1=20, r2=1.6, k2=18, b12=0.5, b21=0.7`);
eql3:=SEquP([2.0*N1*(20-N1-0.5*N2)/20, 1.6*N2*(18-N2-0.7*N1)/18], [N1,N2]);
print(eql3);
```

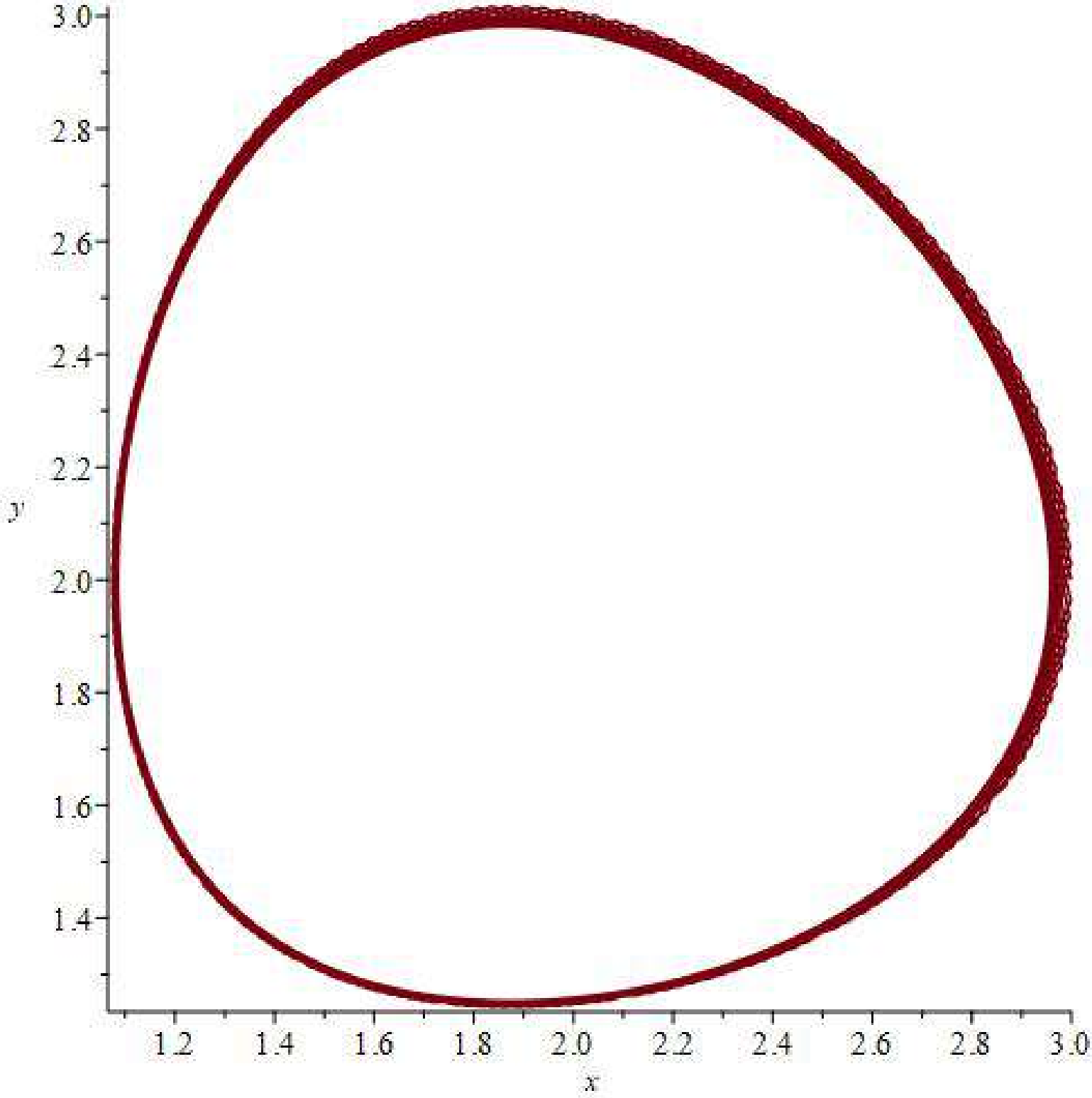

2.3
2.2
2.1
2
y
1.9
1.8
1.7
1.6
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
x

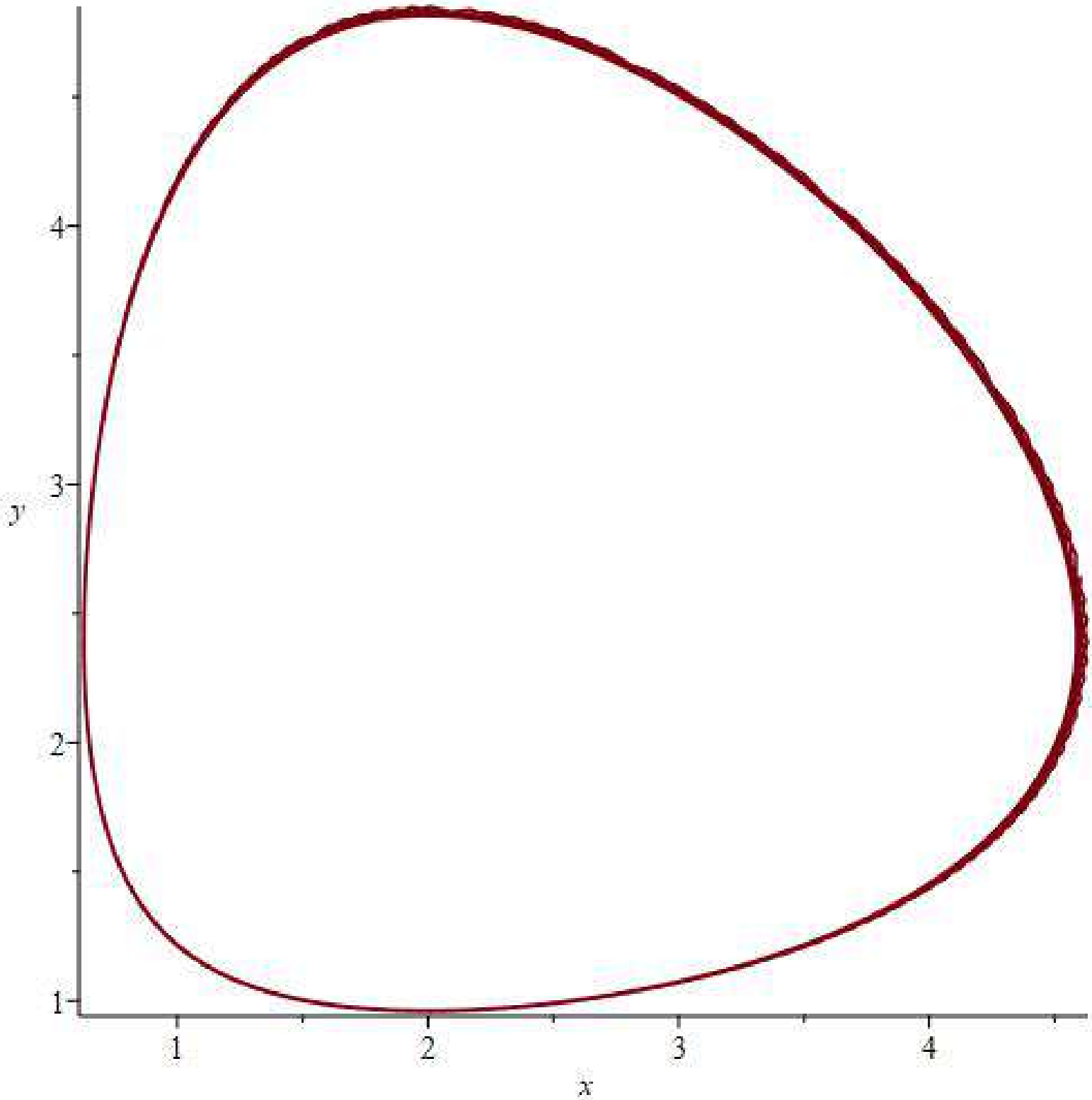

5000
0
10000
20000
30000
−5000
−10000
−15000

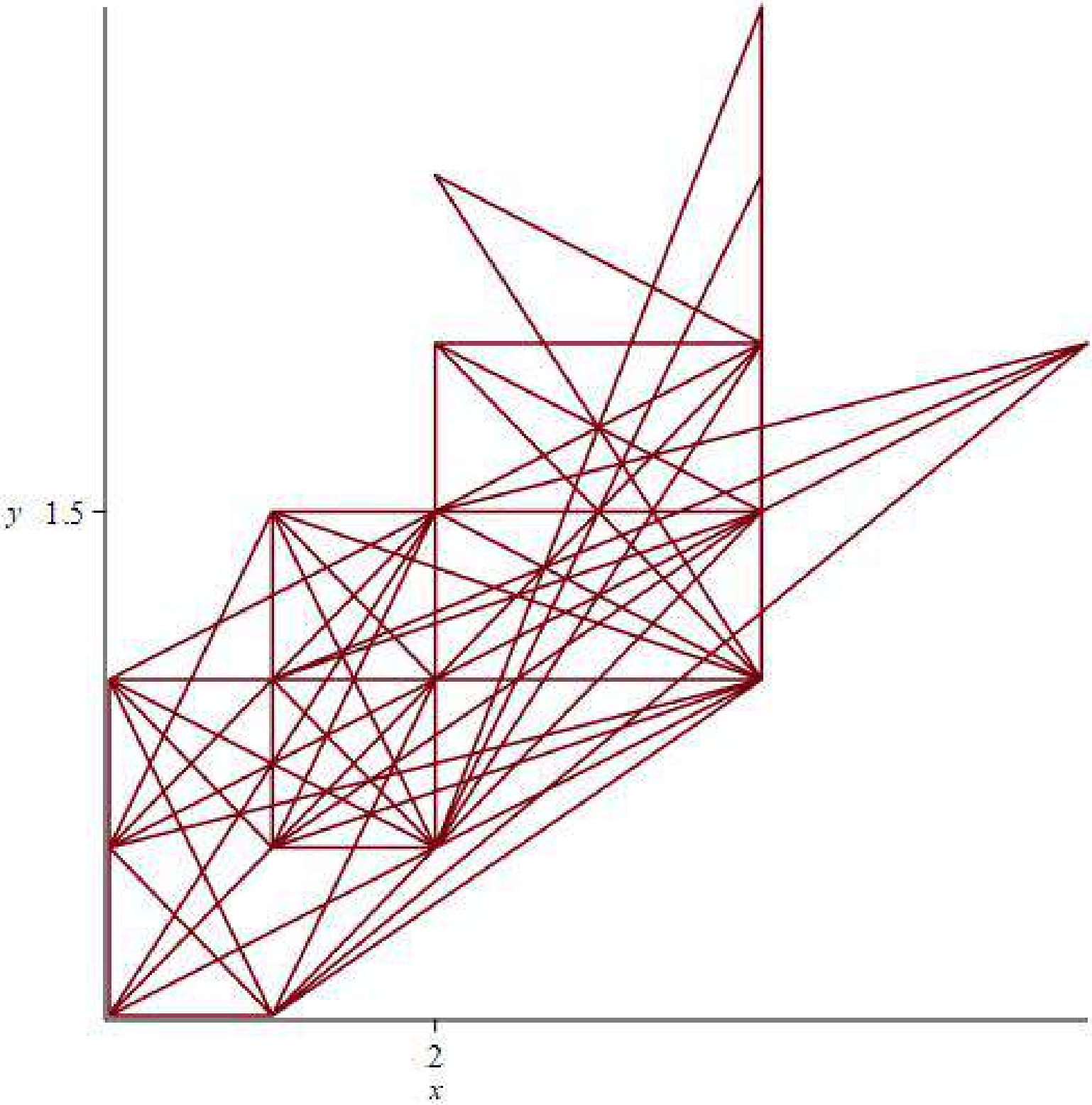

1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
−0.2
−0.4

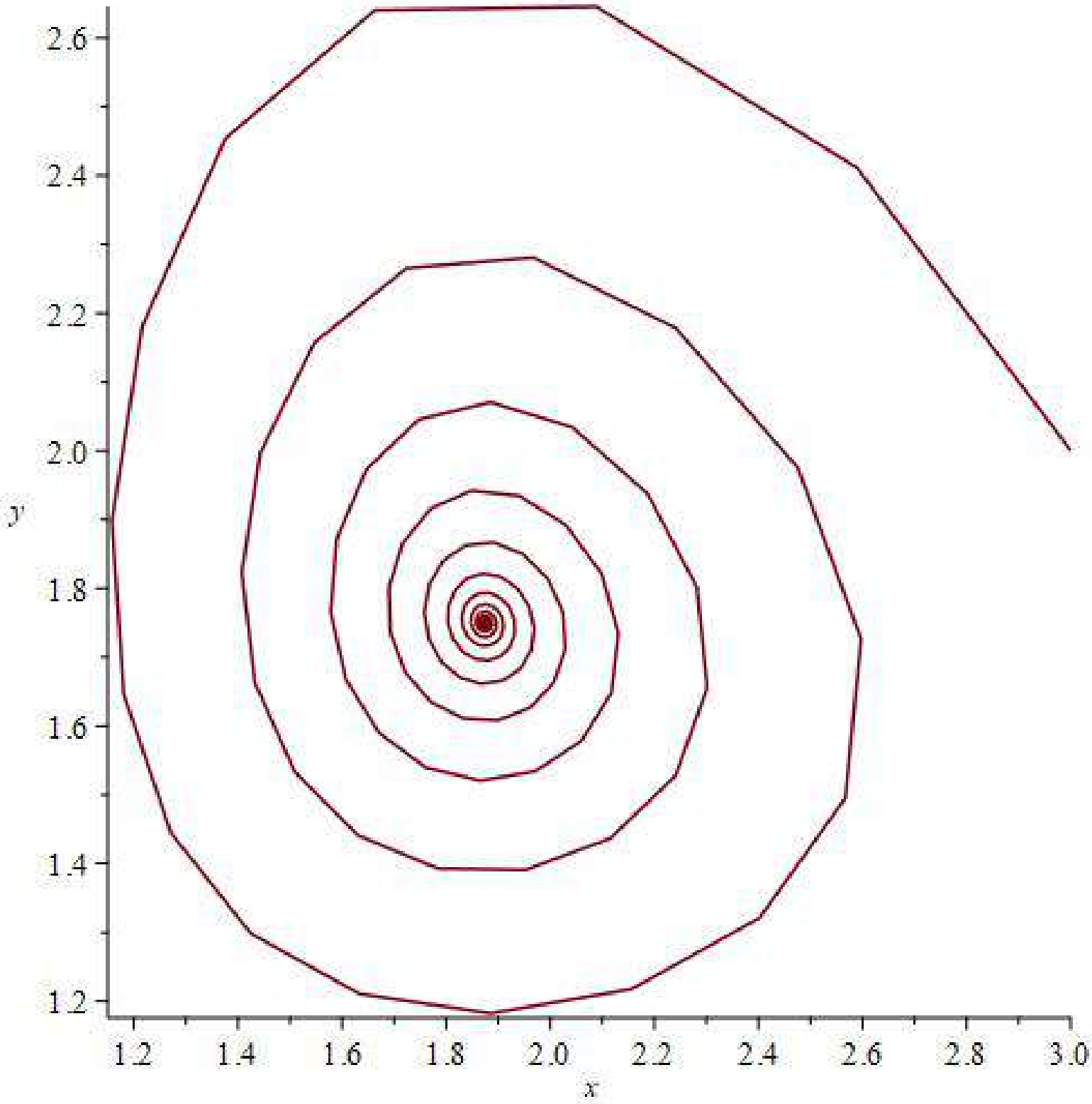

1
0.5
−1
−0.5
0
0.5
1
−0.5
−1

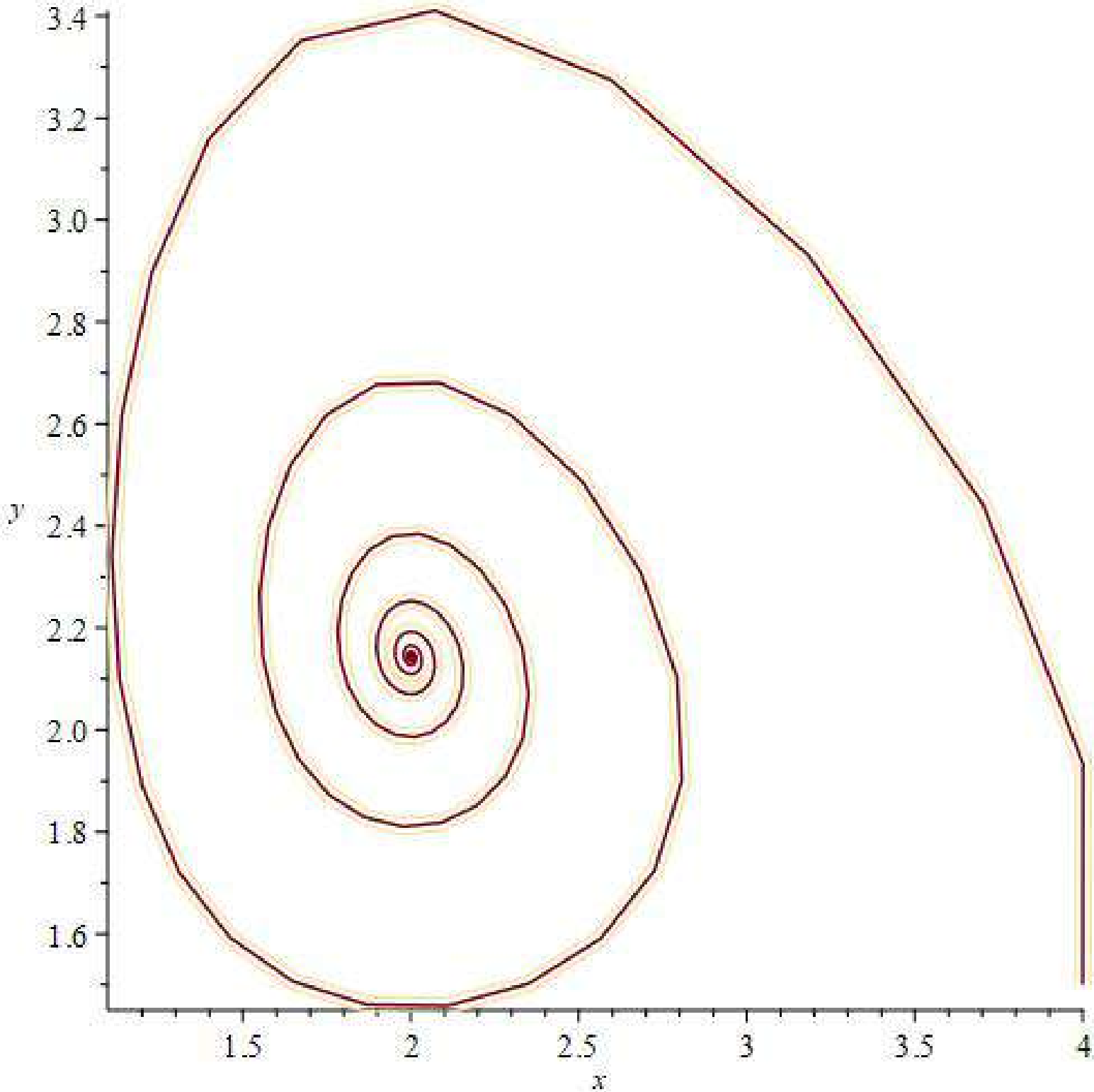

2.3
2.2
2.1
2
y
1.9
1.8
1.7
1.6
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
x

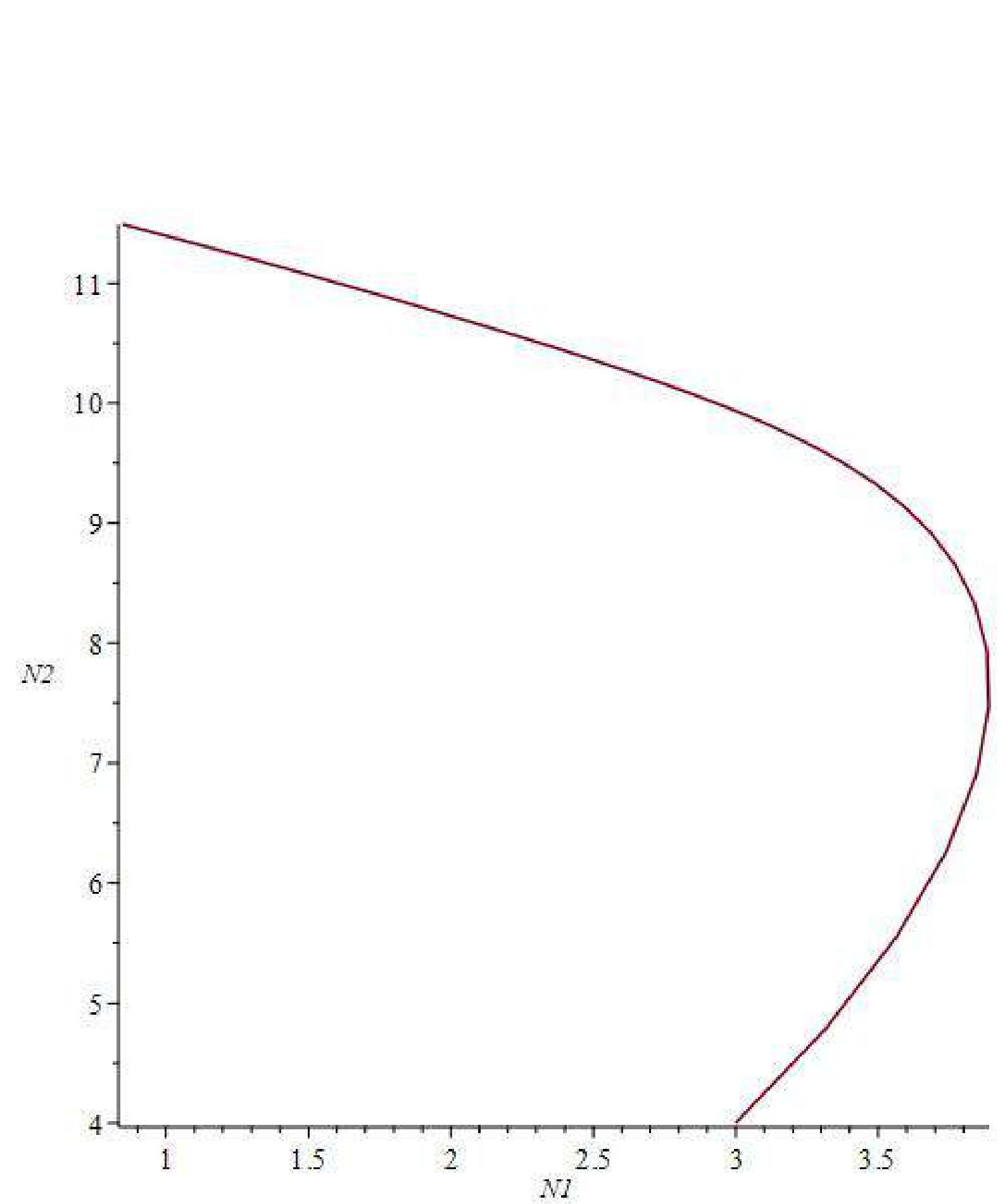

3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
y
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
x

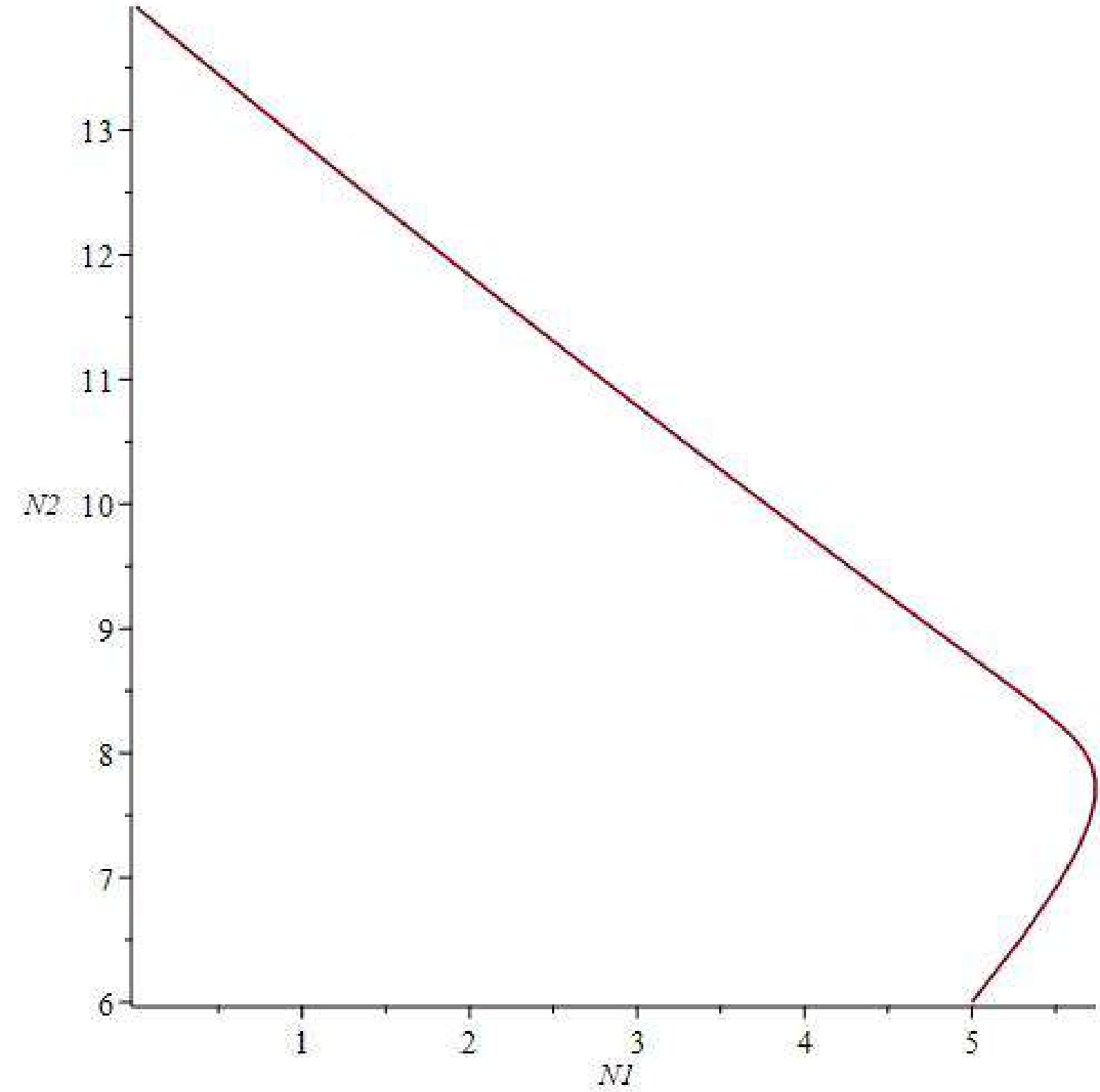

4
3
y
2
1
1
2
3
4
x

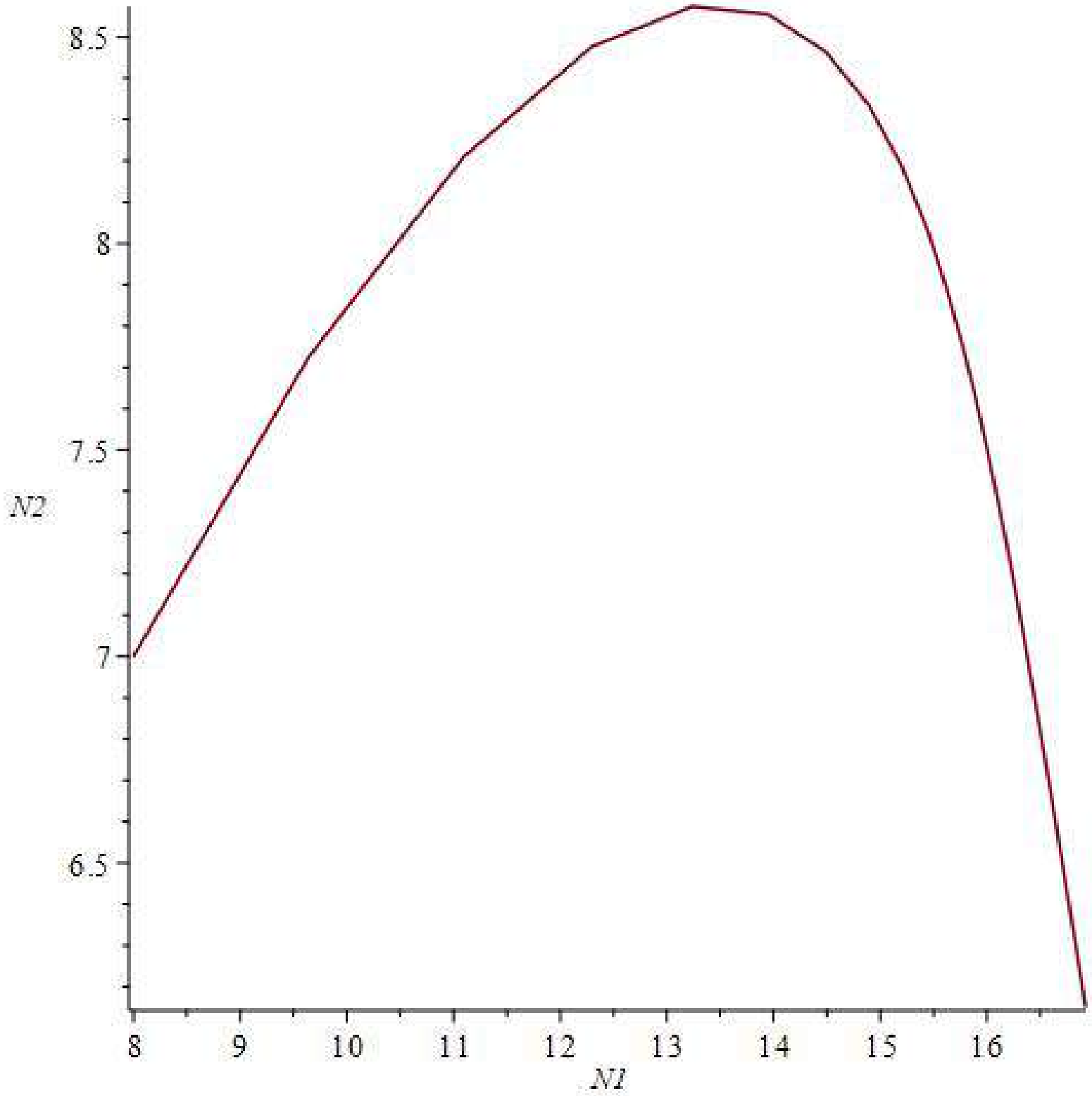

y
1.5
2
x


2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
y
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
x


3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
y
1.5
2
2.5
3
3.5
4
x


11
10
9
8
N2
7
6
5
4
1
1.5
2
2.5
3
3.5
N1


13
12
11
N2 10
9
8
7
6
1
2
3
4
5
N1


8.5
8
7.5
7
6.5
N2
8
9
10
11
12
13
14
15
16
N1

==========

Volterra Set 1 Stable Equilibria:

Parameters: a=1.5, b=0.8, c=1.2, d=0.6

$eq1 := \varnothing$

$\varnothing$

==========

Volterra Set 2 Stable Equilibria:

Parameters: a=2.0, b=1.0, c=1.5, d=0.8

$eq2 := \varnothing$

$\varnothing$

==========

Volterra Set 3 Stable Equilibria:

Parameters: a=1.2, b=0.5, c=0.8, d=0.4

$eq3 := \varnothing$

$\varnothing$

==========

VolterraM Set 1 Stable Equilibria:

Parameters: a=1.5, b=0.8, c=1.2, d=0.6, K=10

eqm1 := {[2., 1.500000000]}

{[2., 1.500000000]}

==========

VolterraM Set 2 Stable Equilibria:

Parameters: a=2.0, b=1.0, c=1.5, d=0.8, K=15

eqm2 := {[1.875000000, 1.750000000]}

{[1.875000000, 1.750000000]}

==========

VolterraM Set 3 Stable Equilibria:

Parameters: a=1.8, b=0.7, c=1.0, d=0.5, K=12

eqm3 := {[2., 2.142857143]}

{[2., 2.142857143]}

==========

Lotka Set 1 Stable Equilibria:

Parameters: r1=1.2, k1=10, r2=1.5, k2=12, b12=0.8, b21=0.6

eql1 := {[0.7692307692, 11.53846154]}

{[0.7692307692, 11.53846154]}

==========

Lotka Set 2 Stable Equilibria:

Parameters: r1=1.5, k1=15, r2=1.8, k2=14, b12=1.2, b21=1.0

eql2 := {[0., 14.], [15., 0.]}

{[0., 14.], [15., 0.]}

==========

Lotka Set 3 Stable Equilibria:

Parameters: r1=2.0, k1=20, r2=1.6, k2=18, b12=0.5, b21=0.7

eql3 := {[16.92307692, 6.153846154]}

{[16.92307692, 6.153846154]}