HW21

$\frac{dx}{dt} = x - y \quad \frac{dy}{dt} = y - x \qquad x(0) = 1 \quad y(0) = 1$

$\vec{x}'(t) = A\vec{x}(t)$

where $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

$(1-\lambda)^2 - 1$

$\lambda^2 - 2\lambda = 0$

$\lambda(\lambda - 2) = 0$

$\lambda = 0, 2$

$\lambda_1 = 0 \Rightarrow x - y = 0$

$\vec{a}_1 = (1, 1)$

$\lambda_2 = 2 \Rightarrow -x - y = 0$

$\vec{a}_2 = (1, -1)$

$\vec{x}_1(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^0 \qquad \vec{x}_2(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t}$

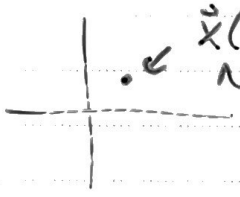
$\vec{x}(t) = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\vec{x}(0) = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$c_1 = 1, \; c_2 = 0$

$\vec{x}(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Phase-plane



$\vec{x}(t)$ is const. Not depending on t

Maple code has incorrect initial value of y

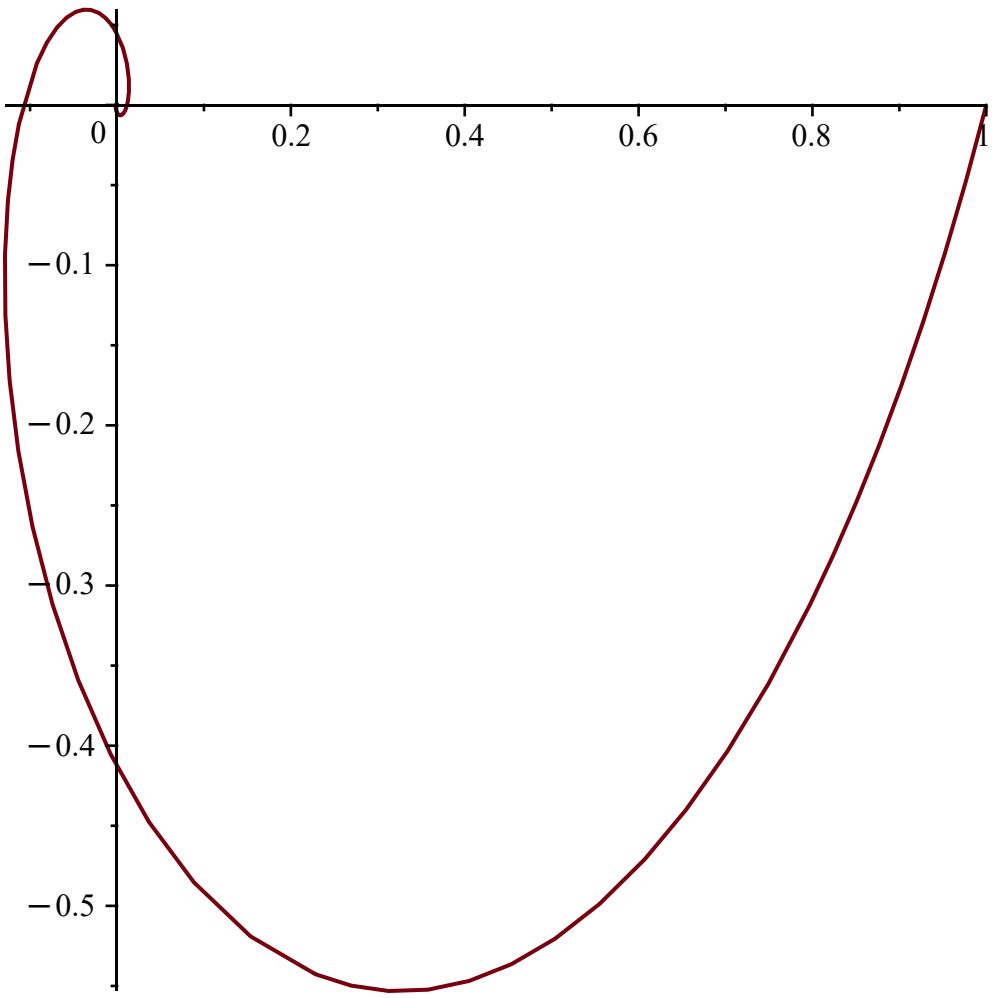
```
> read `DMB.txt`
```

For a list of the Main procedures type: Help(); for help with a specific procedure type: Help (ProcedureName); for example Help(Feig);

For a list of the Continuous Dynamical Models procedures type: HelpC(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig); **(1)**

```
> S := dsolve( {diff(x(t), t) = rand(-10.01..10.01)( ) · x(t) + rand(-10.01..10.01)( )
      · y(t), diff(y(t), t) = rand(-10.01..10.01)( ) · x(t) + rand(-10.01..10.01)( ) · y(t),
      x(0) = 1, y(0) = 0}, {x(t), y(t)});
  plot([subs(S, x(t)), subs(S, y(t)), t = 0..10]);
```

$$S := \left\{ x(t) = e^{-\frac{788578901\, t}{200000000}} \left(\frac{12446139 \sqrt{1236159959798097383}\, \sin\left(\frac{\sqrt{1236159959798097383}\, t}{200000000} \right)}{176594279971156769} + \cos\left(\frac{\sqrt{1236159959798097383}\, t}{200000000} \right) \right), y(t) = -\frac{1483726172\, e^{-\frac{788578901\, t}{200000000}} \sqrt{1236159959798097383}\, \sin\left(\frac{\sqrt{1236159959798097383}\, t}{200000000} \right)}{1236159959798097383} \right\}$$




```
> S := dsolve( {diff(x(t),t) = rand(-10.01 .. 10.01)( ) * x(t) + rand(-10.01 .. 10.01)( )
      * y(t), diff(y(t),t) = rand(-10.01 .. 10.01)( ) * x(t) + rand(-10.01 .. 10.01)( )
      * y(t), x(0) = 1, y(0) = 0}, {x(t), y(t)});
  plot( [subs(S, x(t)), subs(S, y(t)), t = 0 .. 10]);
```

$$S := \left\{ x(t) = \left(\frac{1}{2} + \frac{109833733\sqrt{48197385646939529}}{68853408067056470} \right) e^{\frac{\left(-1159117611 + 5\sqrt{48197385646939529}\right) t}{200000000}} + \left(\frac{1}{2} \right.$$

$$\left. - \frac{109833733\sqrt{48197385646939529}}{68853408067056470} \right) e^{-\frac{\left(1159117611 + 5\sqrt{48197385646939529}\right) t}{200000000}}, y(t)$$

$$= \frac{1}{383811148} \left(5 \left(\frac{1}{2} \right. \right.$$

$$+\frac{109833733\sqrt{48197385646939529}}{68853408067056470}\Bigg)$$

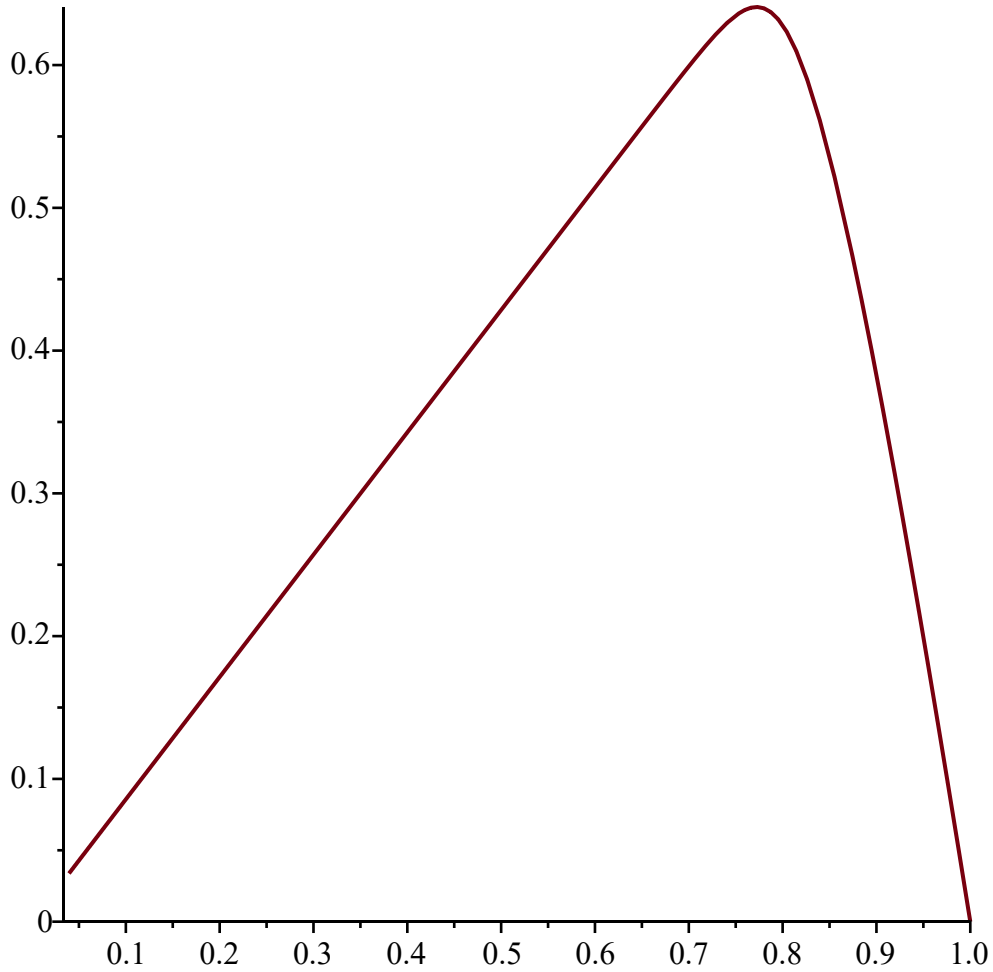
$$e^{\frac{\left(-1159117611+5\sqrt{48197385646939529}\right)t}{200000000}}\sqrt{48197385646939529}\Bigg)-\frac{1}{383811148}\Bigg(5\Bigg(\frac{1}{2}$$

$$-\frac{109833733\sqrt{48197385646939529}}{68853408067056470}\Bigg)$$

$$e^{-\frac{\left(1159117611+5\sqrt{48197385646939529}\right)t}{200000000}}\sqrt{48197385646939529}\Bigg)$$

$$-\frac{109833733\left(\frac{1}{2}+\frac{109833733\sqrt{48197385646939529}}{68853408067056470}\right)e^{\frac{\left(-1159117611+5\sqrt{48197385646939529}\right)t}{200000000}}}{54830164}$$

$$-\frac{109833733\left(\frac{1}{2}-\frac{109833733\sqrt{48197385646939529}}{68853408067056470}\right)e^{-\frac{\left(1159117611+5\sqrt{48197385646939529}\right)t}{200000000}}}{54830164}\Bigg\}$$




```
> S := dsolve( {diff(x(t),t) = rand(-10.01 .. 10.01)( ) * x(t) + rand(-10.01 .. 10.01)( )
      * y(t), diff(y(t),t) = rand(-10.01 .. 10.01)( ) * x(t) + rand(-10.01 .. 10.01)( )
      * y(t), x(0) = 1, y(0) = 0}, {x(t), y(t)});
  plot( [subs(S, x(t)), subs(S, y(t)), t = 0 .. 10]);
```

$$S := \left\{ x(t) = \left(\frac{1}{2} - \frac{33163951\sqrt{41320555693776997}}{82641111387553994} \right) e^{\frac{\left(-1958198147+\sqrt{41320555693776997}\right)t}{200000000}} + \left(\frac{1}{2} + \frac{33163951\sqrt{41320555693776997}}{82641111387553994} \right) e^{-\frac{\left(1958198147+\sqrt{41320555693776997}\right)t}{200000000}}, y(t) \right.$$

$$= \frac{1}{648113982} \left(\left(\frac{1}{2} \right. \right.$$

$$-\frac{33163951\sqrt{41320555693776997}}{82641111387553994}\Bigg)$$

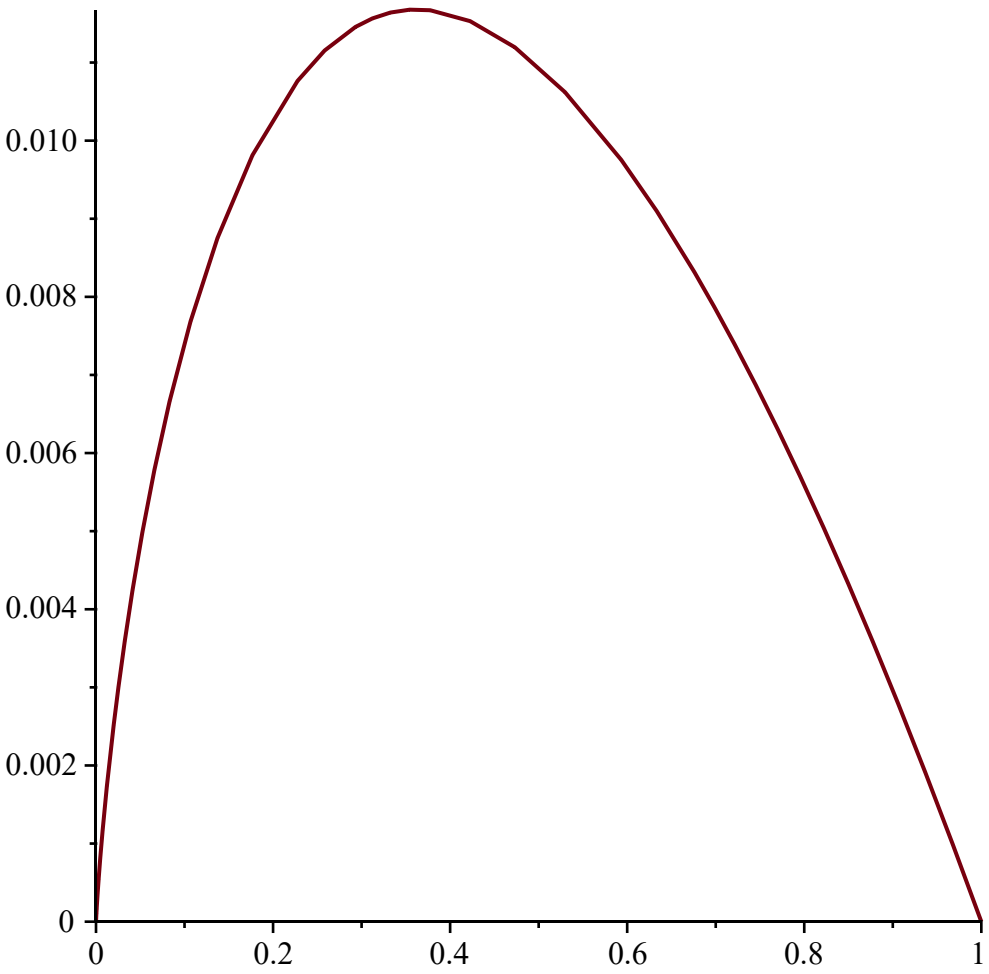
$$e^{\frac{\left(-1958198147+\sqrt{41320555693776997}\right)t}{200000000}}\sqrt{41320555693776997}\Bigg)-\frac{1}{648113982}\Bigg(\Bigg(\frac{1}{2}$$

$$+\frac{33163951\sqrt{41320555693776997}}{82641111387553994}\Bigg)$$

$$e^{-\frac{\left(1958198147+\sqrt{41320555693776997}\right)t}{200000000}}\sqrt{41320555693776997}\Bigg)$$

$$+\frac{33163951\left(\frac{1}{2}-\frac{33163951\sqrt{41320555693776997}}{82641111387553994}\right)e^{\frac{\left(-1958198147+\sqrt{41320555693776997}\right)t}{200000000}}}{648113982}$$

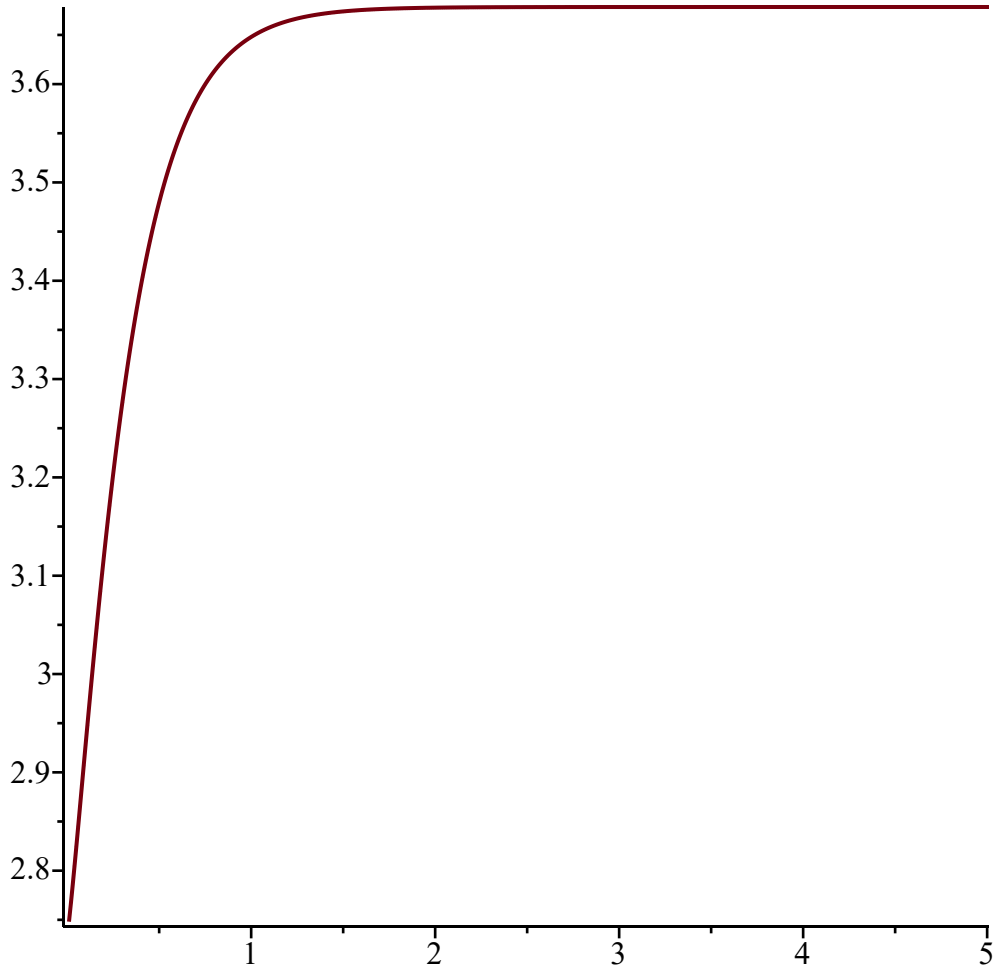
$$+\frac{33163951\left(\frac{1}{2}+\frac{33163951\sqrt{41320555693776997}}{82641111387553994}\right)e^{-\frac{\left(1958198147+\sqrt{41320555693776997}\right)t}{200000000}}}{648113982}\Bigg\}$$

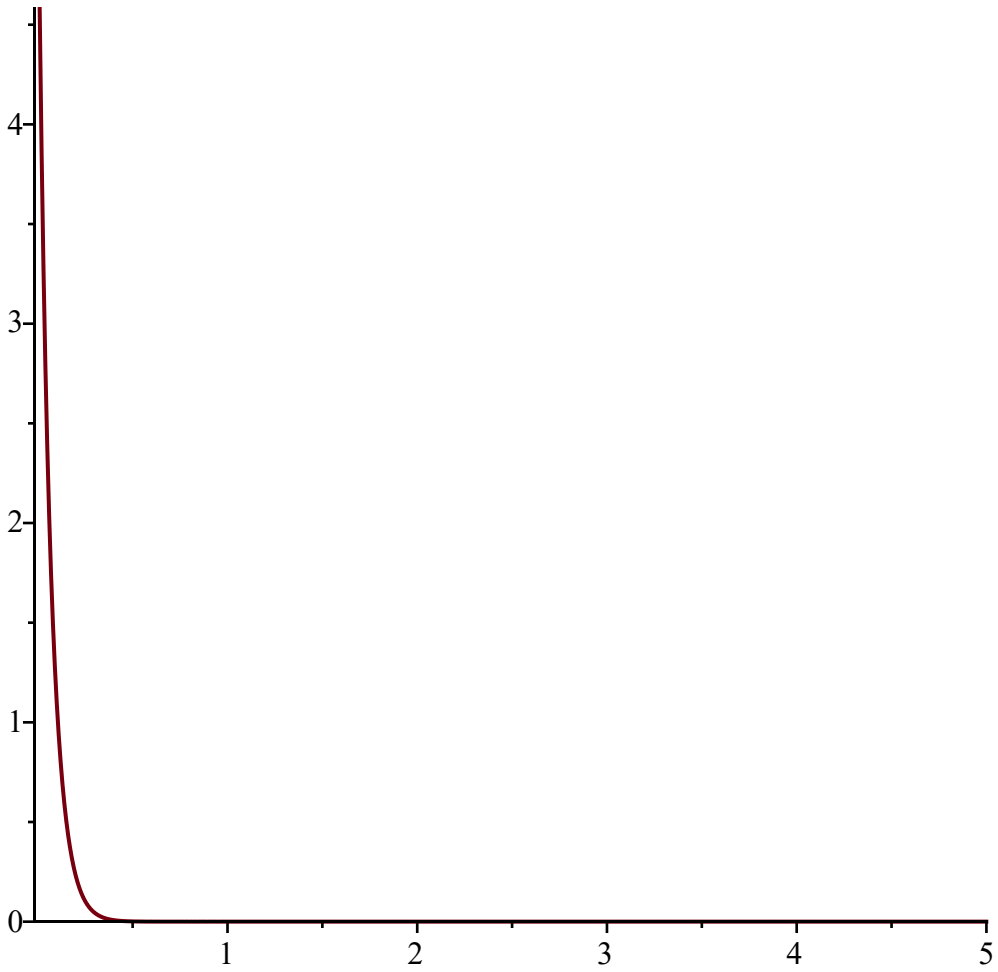


>

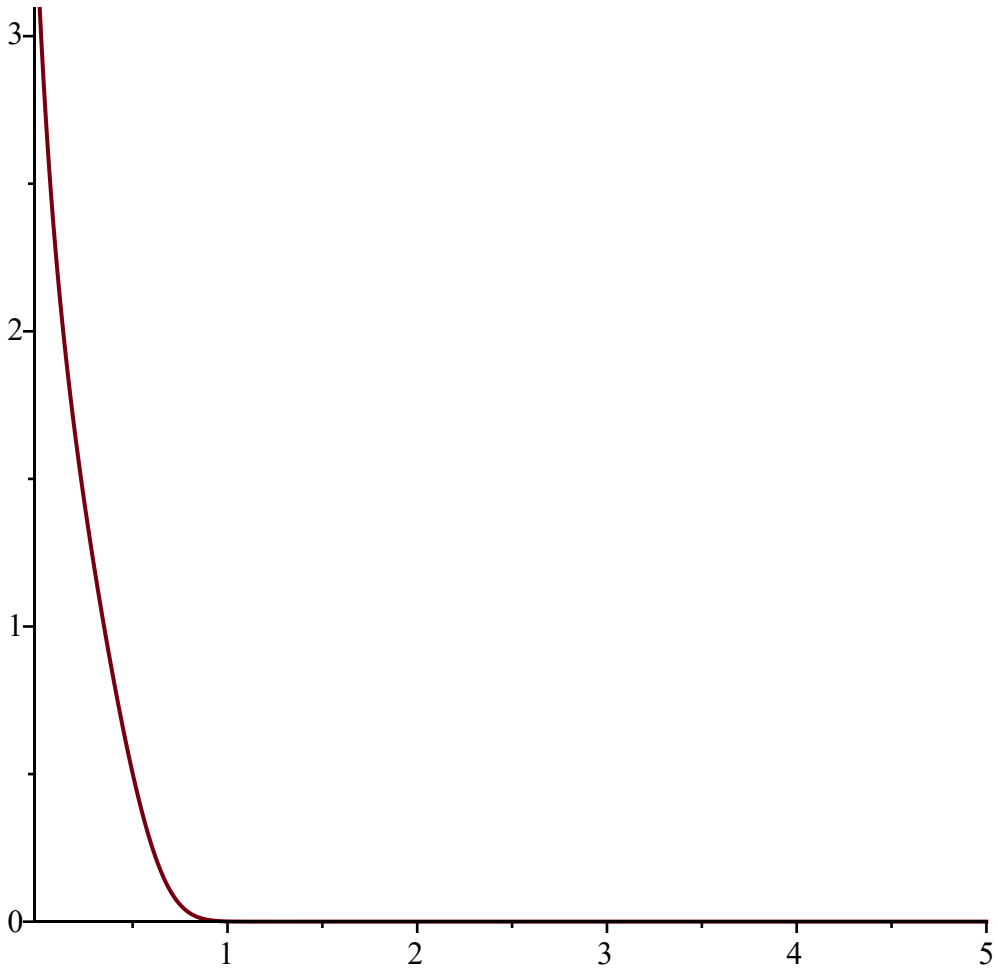
(2)

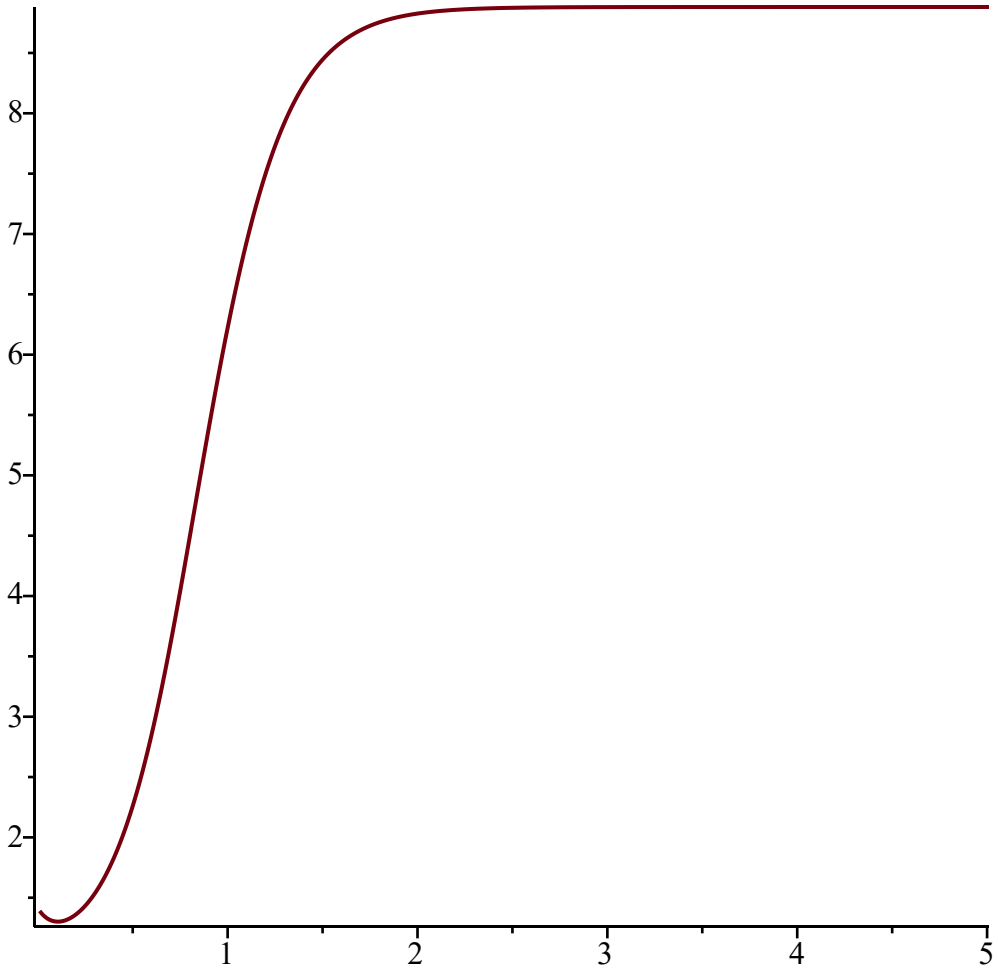
```
> L := Dis(Lotka( rand(0..10.01)( ), rand(0..10.01)( ), rand(0..10.01)( ), rand(0
      ..10.01)( ), rand(0..10.01)( ), rand(0..10.01)( ), x, y), [x, y], [ rand(0..10.01)( ),
       rand(0..10.01)( ) ], 0.01, 5) :
  plot( [seq( [L[i1][1], L[i1][2][1]], i1 = 1 .. nops(L) ) ]);
  plot( [seq( [L[i1][1], L[i1][2][2]], i1 = 1 .. nops(L) ) ]);
```


3.6
3.5
3.4
3.3
3.2
3.1
3
2.9
2.8
1
2
3
4
5

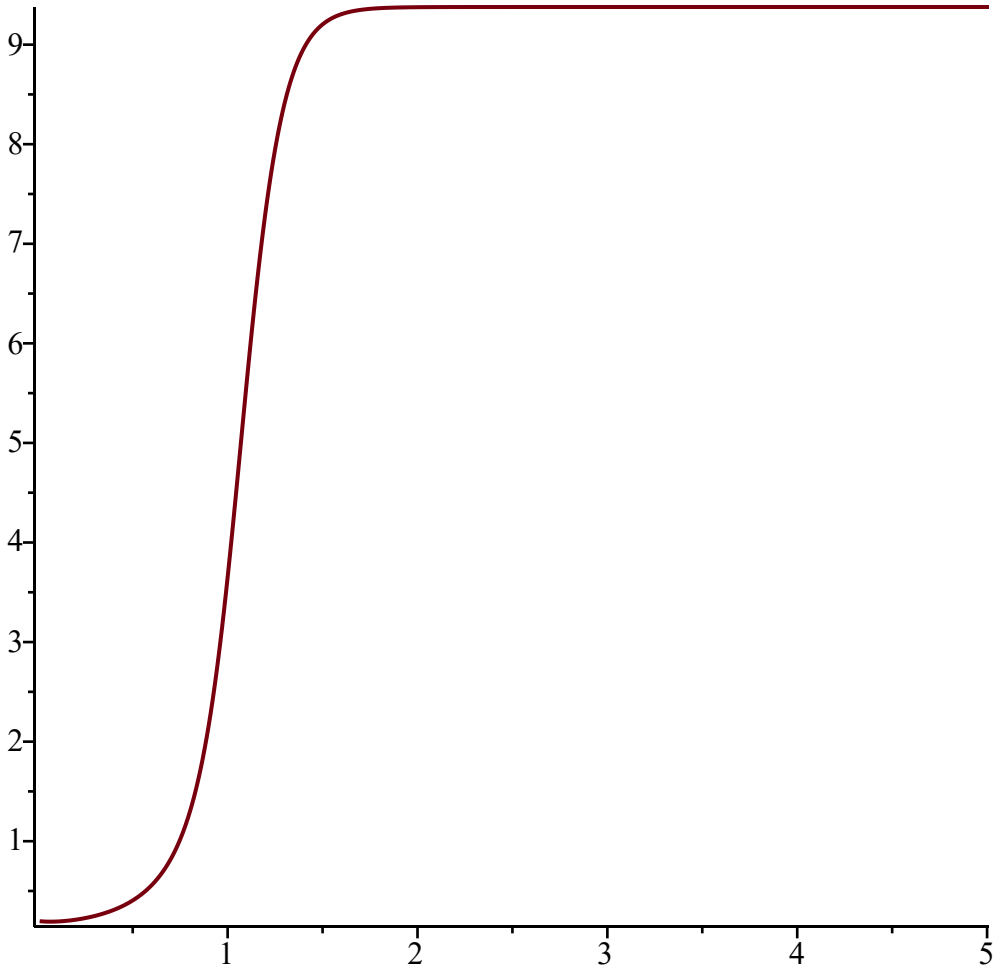


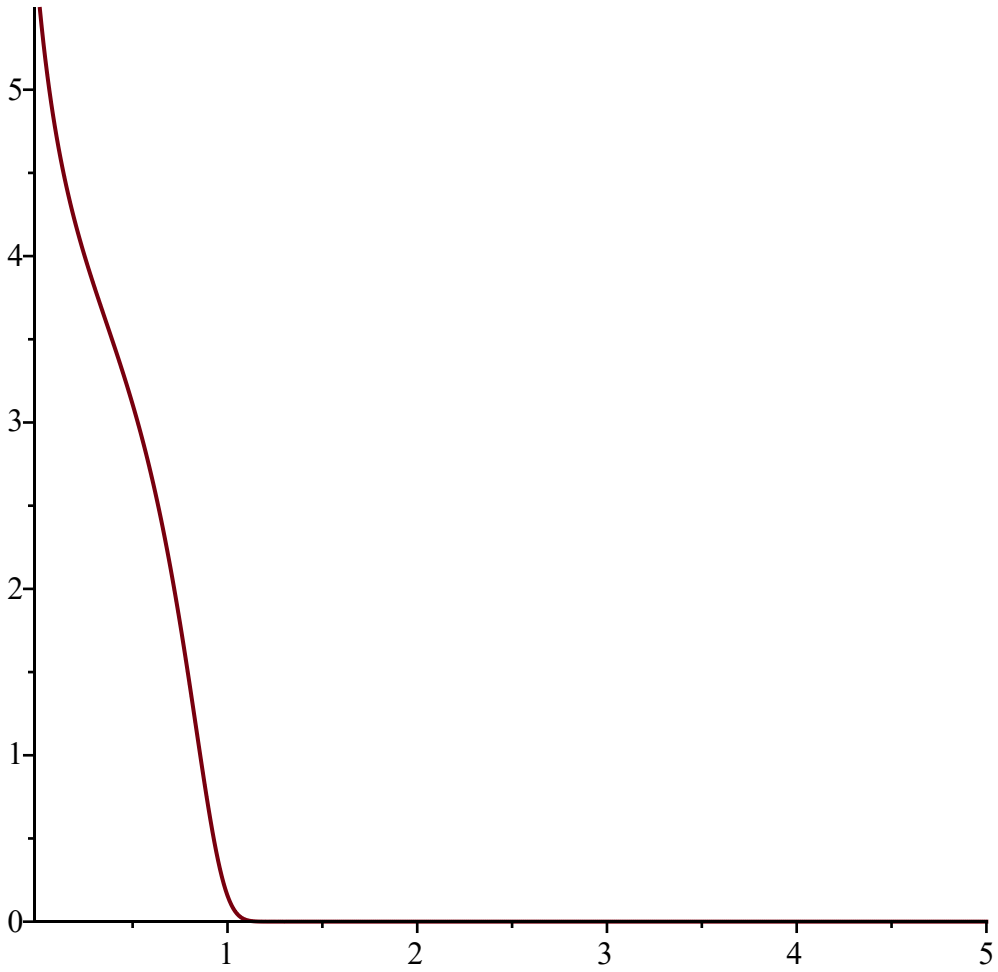

```
> L := Dis(Lotka(rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0
      .. 10.01)( ), rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), x, y), [x, y], [rand(0 .. 10.01)( ),
       rand(0 .. 10.01)( )], 0.01, 5) :
  plot([seq([L[i1][1], L[i1][2][1]], i1 = 1 .. nops(L))]);
  plot([seq([L[i1][1], L[i1][2][2]], i1 = 1 .. nops(L))]);
```


3
2
1
0
1
2
3
4
5




```
> L := Dis(Lotka(rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0
      .. 10.01)( ), rand(0 .. 10.01)( ), x, y), [x, y], [rand(0 .. 10.01)( ),
      rand(0 .. 10.01)( )], 0.01, 5) :
  plot([seq([L[i1][1], L[i1][2][1]], i1 = 1 .. nops(L))]);
  plot([seq([L[i1][1], L[i1][2][2]], i1 = 1 .. nops(L))]);
```


9
8
7
6
5
4
3
2
1
1
2
3
4
5




```
> Help(Volterra)
```

Volterra(a,b,c,d,x,y): The (simple, original) Volterra predator-prey continuous-time dynamical system with parameters a,b,c,d

Eqs. (7a) (7b) in Edelstein-Keshet p. 219 (section 6.2)

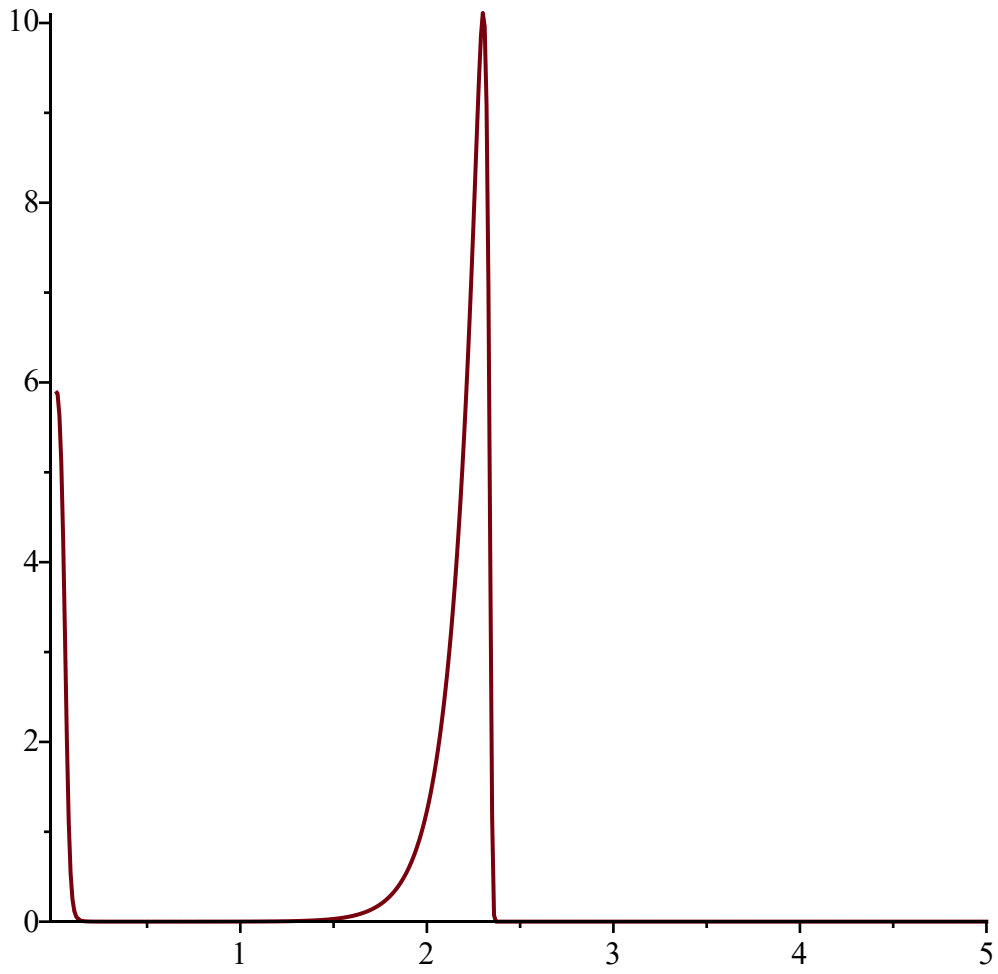
a,b,c,d may be symbolic or numeric

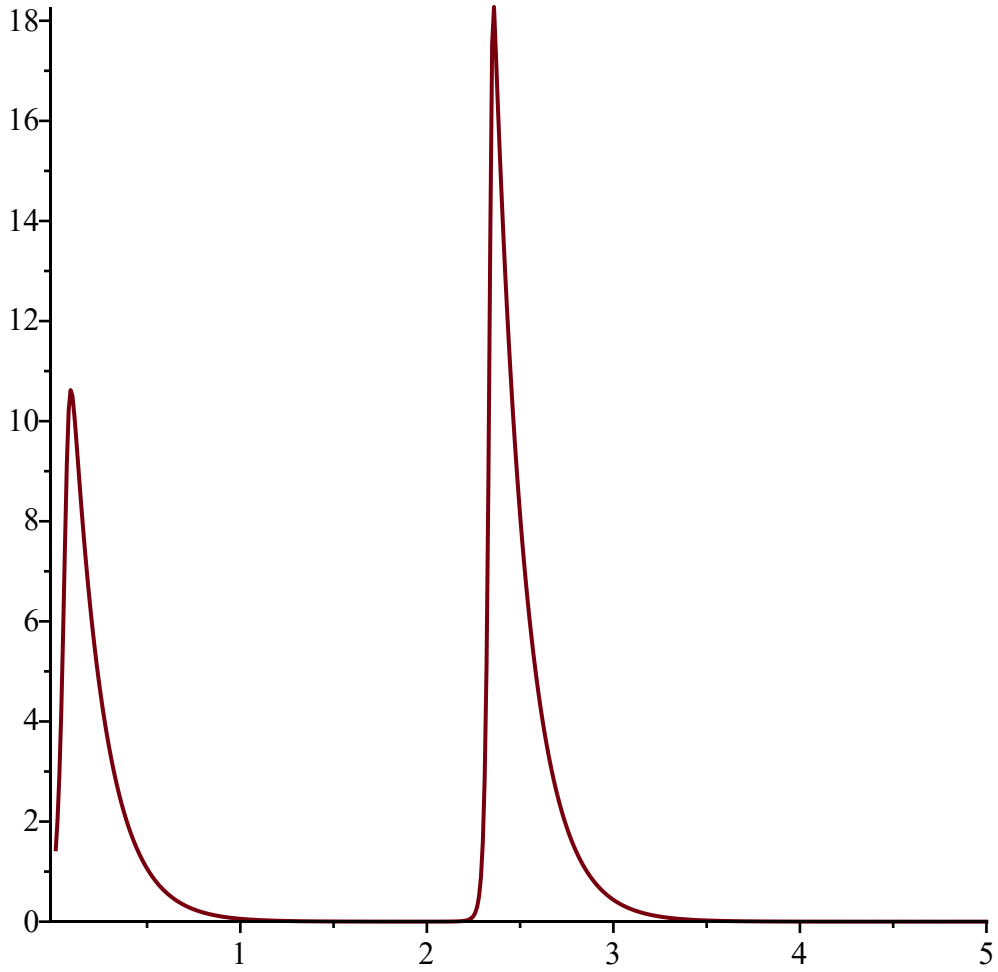
Try:

Volterra(a,b,c,d,x,y);

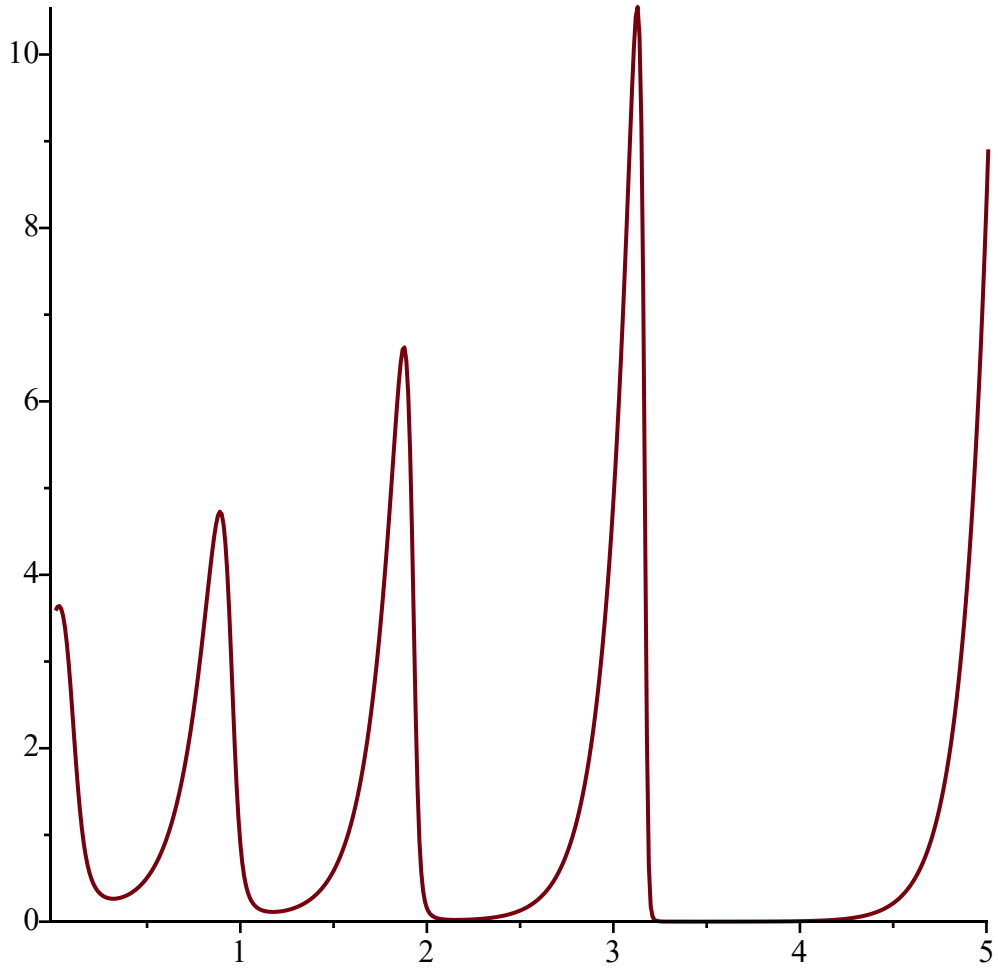
Volterra(1,2,3,4,x,y); **(3)**

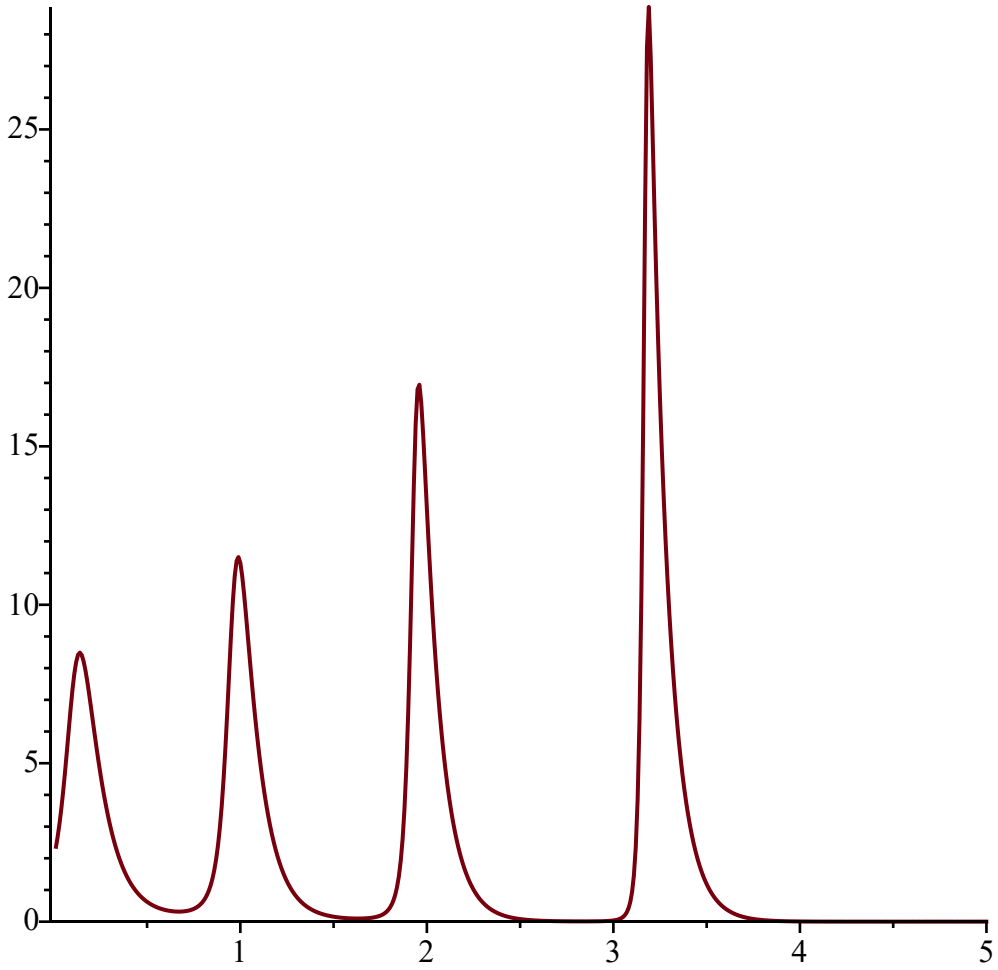
```
> L := Dis(Volterra( rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0
      .. 10.01)( ), x, y), [x, y], [rand(0 .. 10.01)( ), rand(0 .. 10.01)( )], 0.01, 5) :
  plot( [seq( [L[i1][1], L[i1][2][1]], i1 = 1 .. nops(L) )]);
  plot( [seq( [L[i1][1], L[i1][2][2]], i1 = 1 .. nops(L) )]);
```


10
8
6
4
2
0
1
2
3
4
5

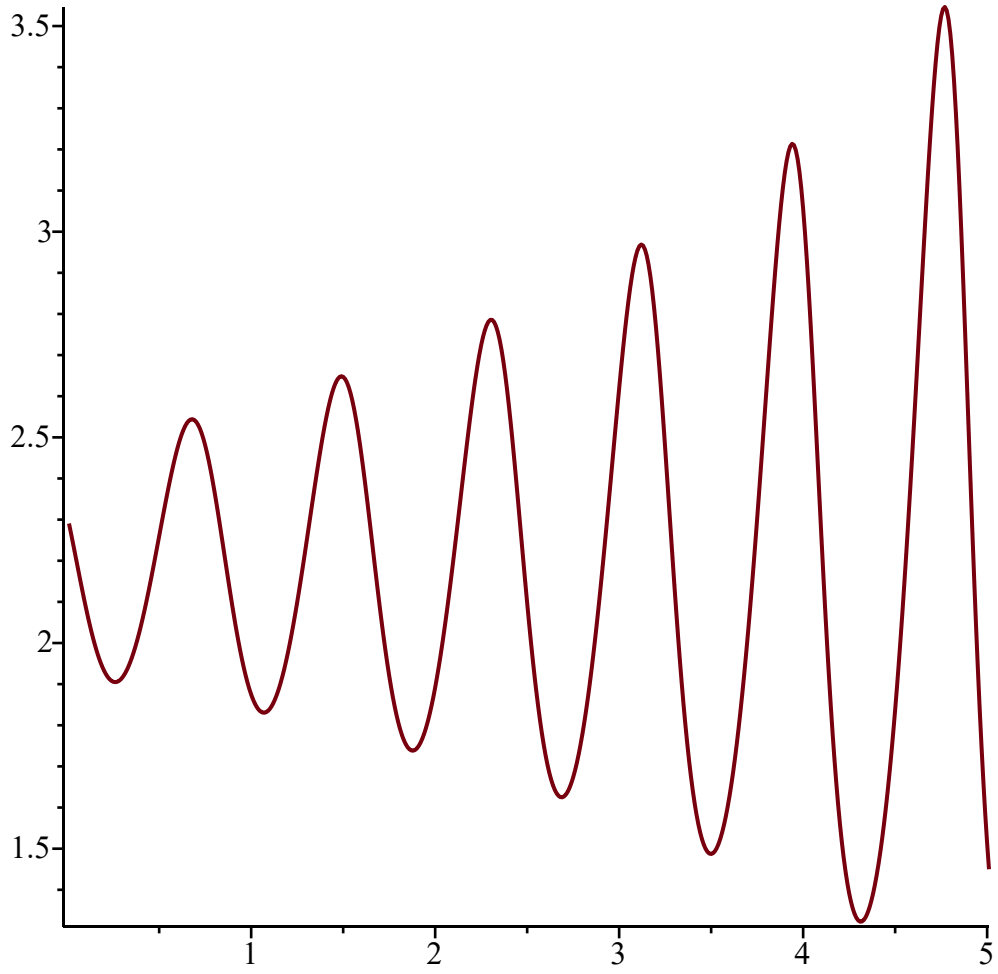


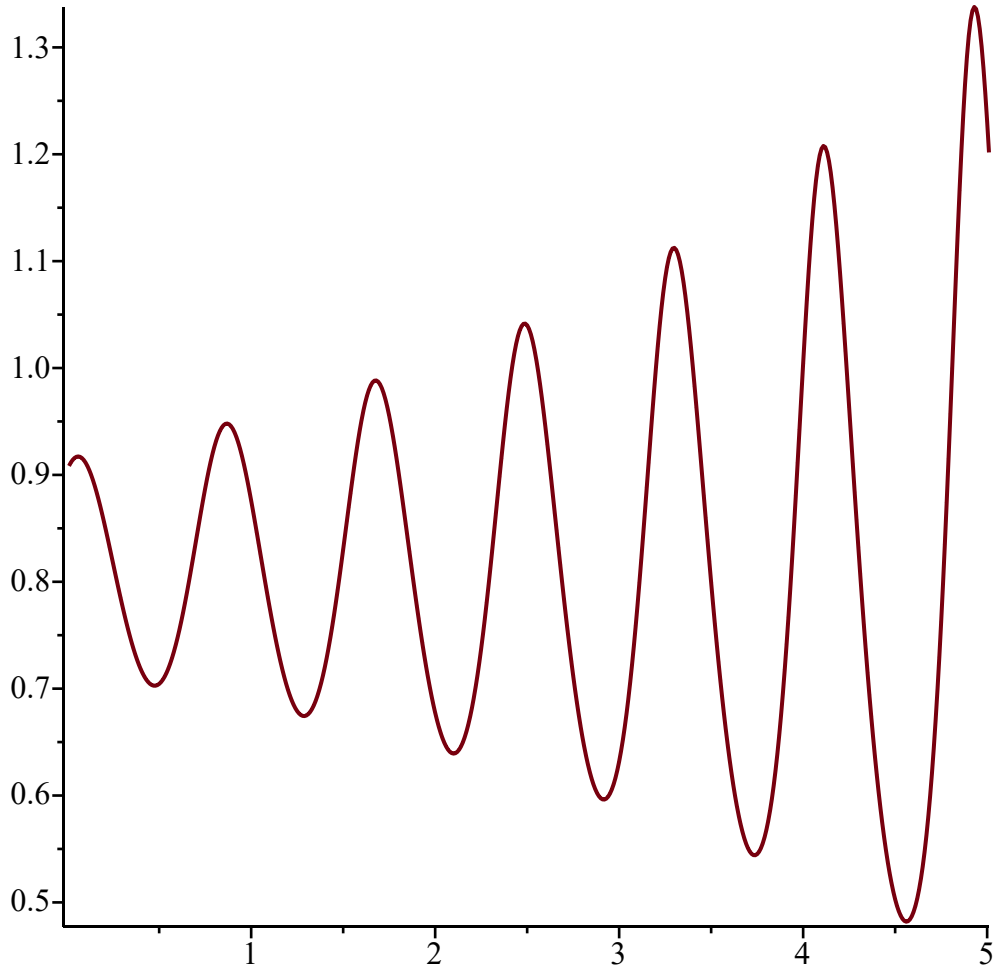

```
> L := Dis(Volterra(rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0
      .. 10.01)( ), x, y), [x, y], [rand(0 .. 10.01)( ), rand(0 .. 10.01)( )], 0.01, 5) :
  plot([seq([L[i1][1], L[i1][2][1]], i1 = 1 .. nops(L))]);
  plot([seq([L[i1][1], L[i1][2][2]], i1 = 1 .. nops(L))]);
```


10
8
6
4
2
0
1
2
3
4
5




```
> L := Dis(Volterra(rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0
      .. 10.01)( ), x, y), [x, y], [rand(0 .. 10.01)( ), rand(0 .. 10.01)( )], 0.01, 5) :
  plot([seq([L[i1][1], L[i1][2][1]], i1 = 1 .. nops(L))]);
  plot([seq([L[i1][1], L[i1][2][2]], i1 = 1 .. nops(L))]);
```


3.5
3
2.5
2
1.5
1
2
3
4
5




```
> Help(VolterraM)
```

VolterraM(a,b,c,d,K,x,y): The modified Volterra predator-prey continuous-time dynamical system with parameters a,b,c,d,K

Eqs. (8a) (8b) in Edelstein-Keshet p. 220 (section 6.2)

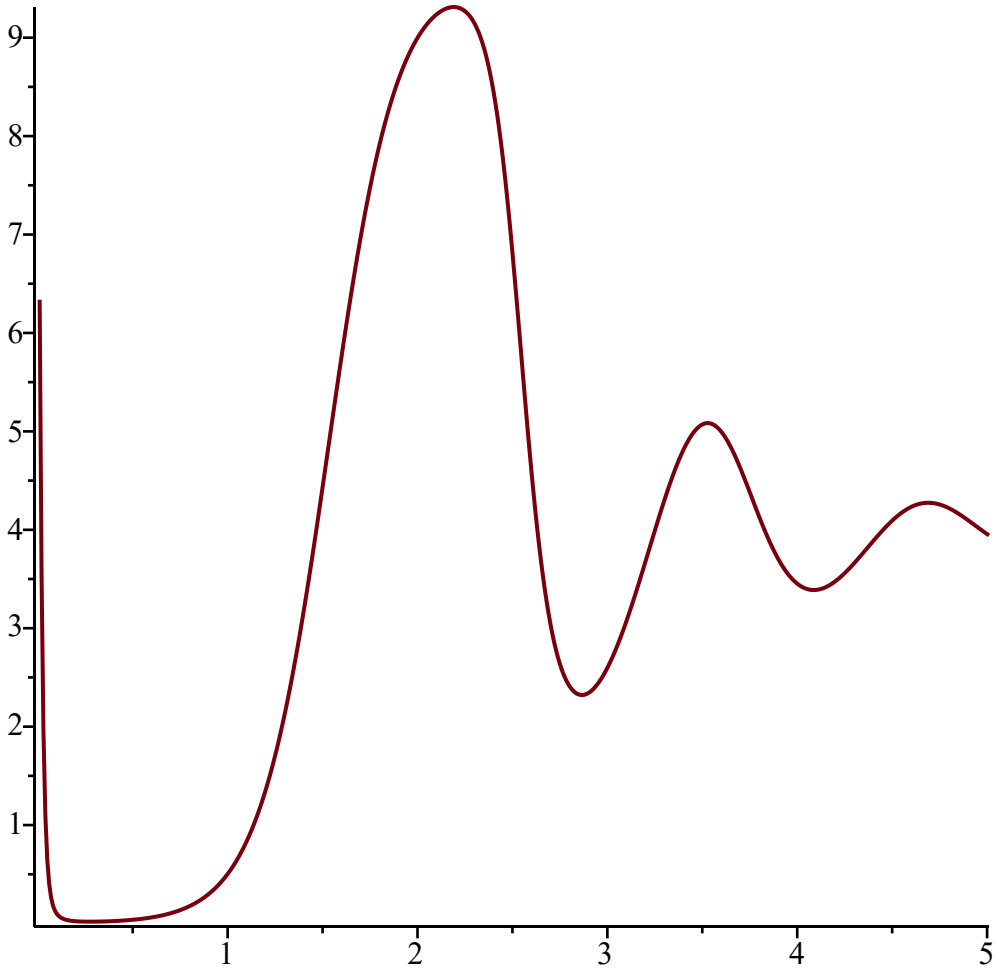
a,b,c,d,K may be symbolic or numeric

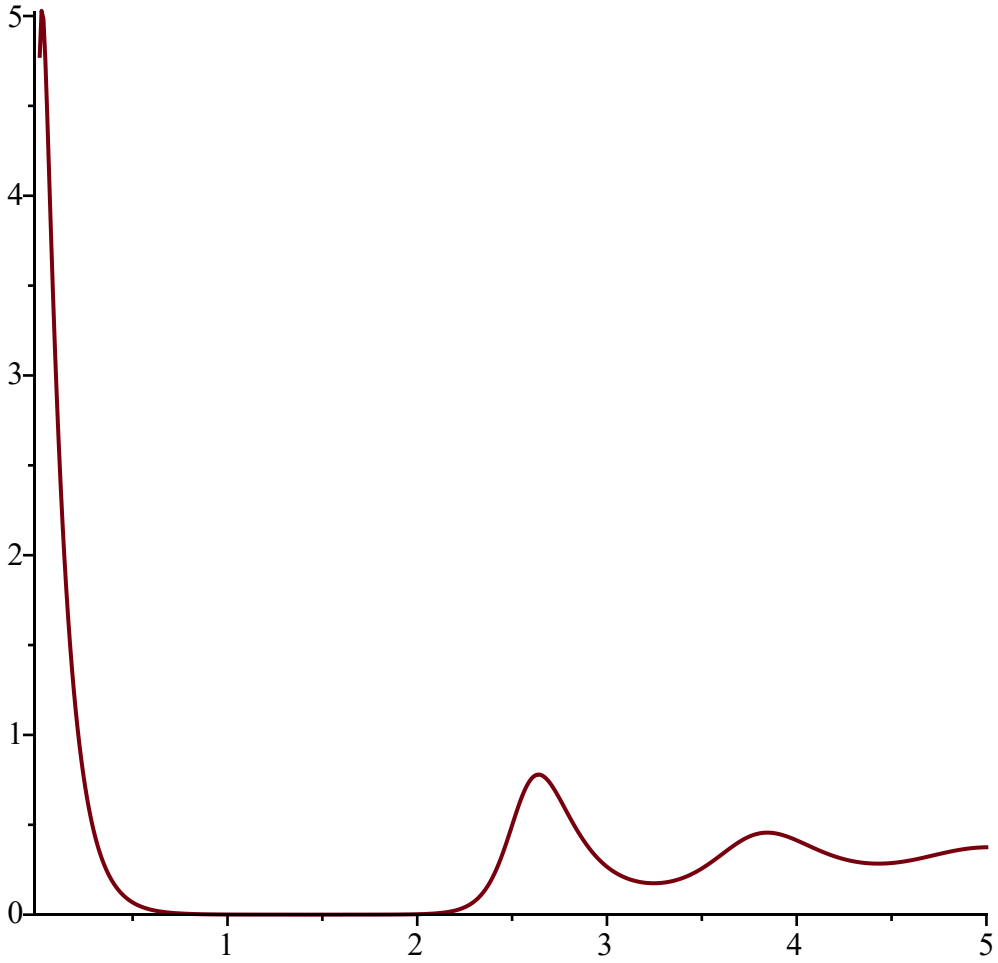
Try:

VolterraM(a,b,c,d,K,x,y);

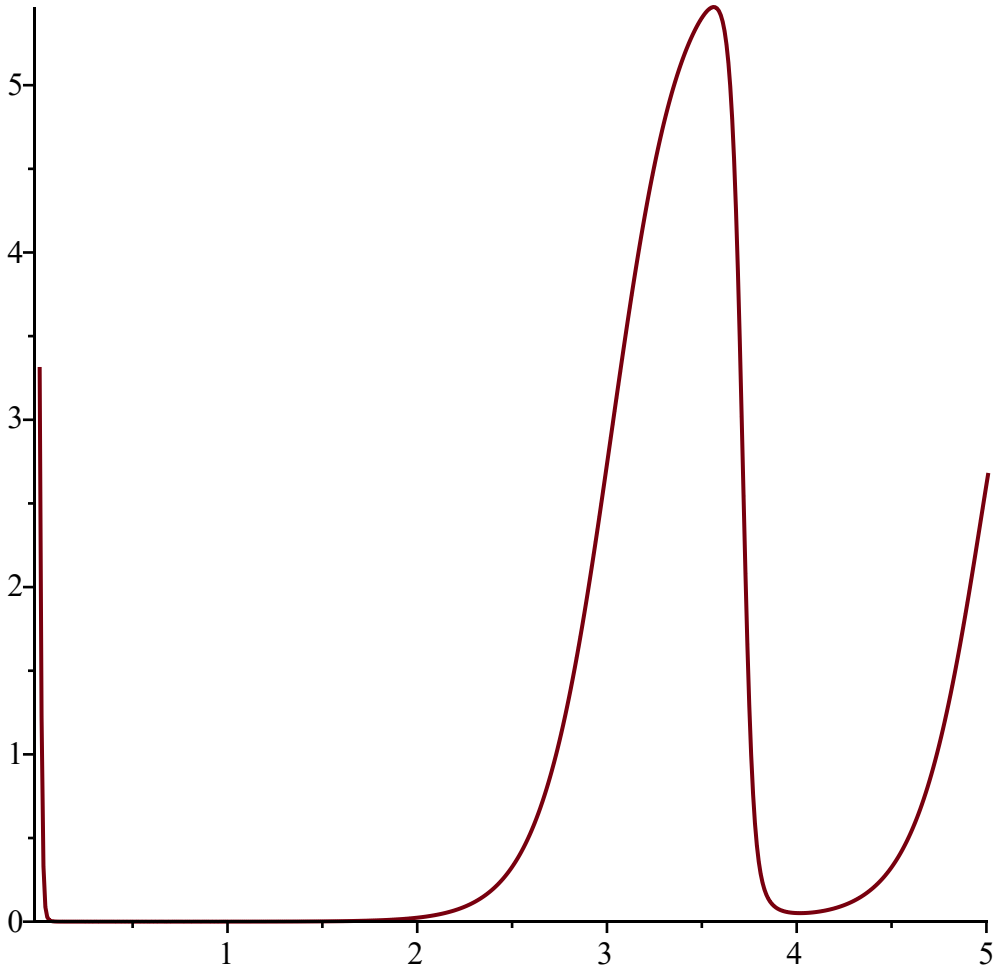
VolterraM(1,2,3,4,2,x,y); **(4)**

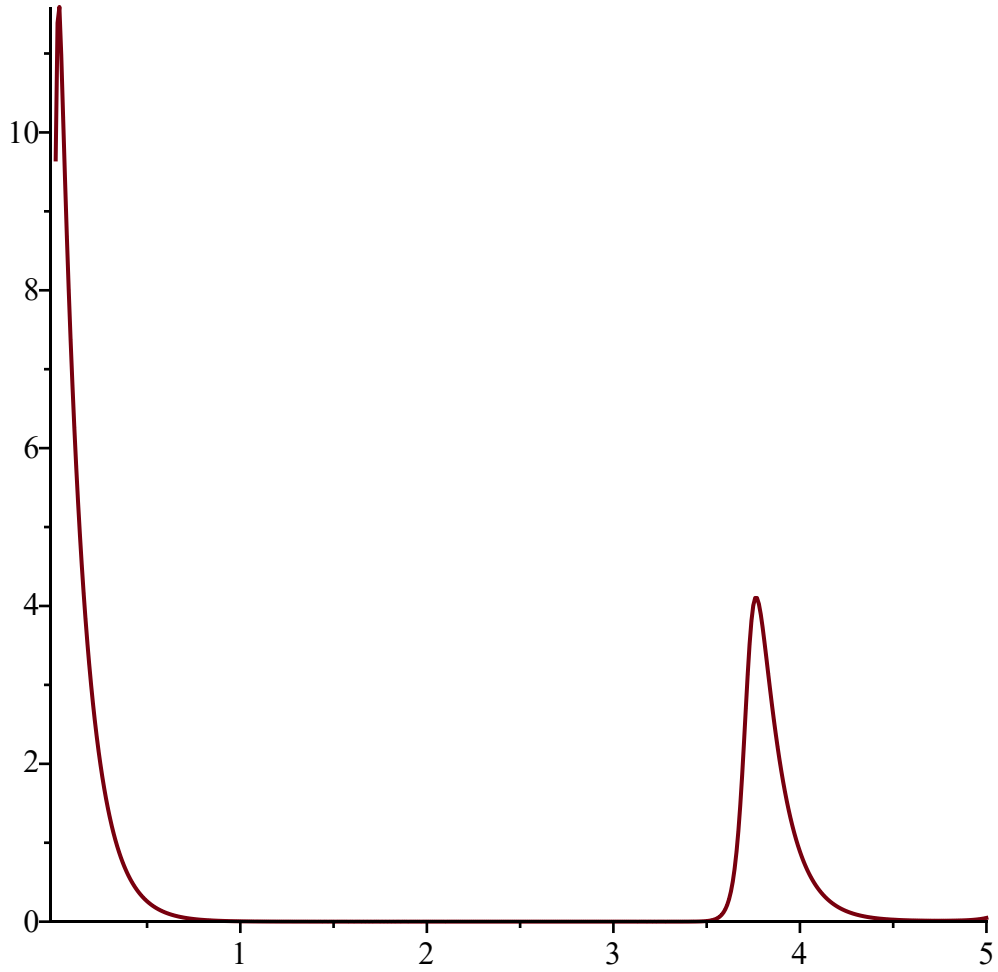
```
> L := Dis(VolterraM(rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0
      .. 10.01)( ), rand(0 .. 10.01)( ), x, y), [x, y], [rand(0 .. 10.01)( ), rand(0 .. 10.01)( )],
      0.01, 5) :
  plot([seq([L[i1][1], L[i1][2][1]], i1 = 1 .. nops(L))]);
  plot([seq([L[i1][1], L[i1][2][2]], i1 = 1 .. nops(L))]);
```


9
8
7
6
5
4
3
2
1
1
2
3
4
5

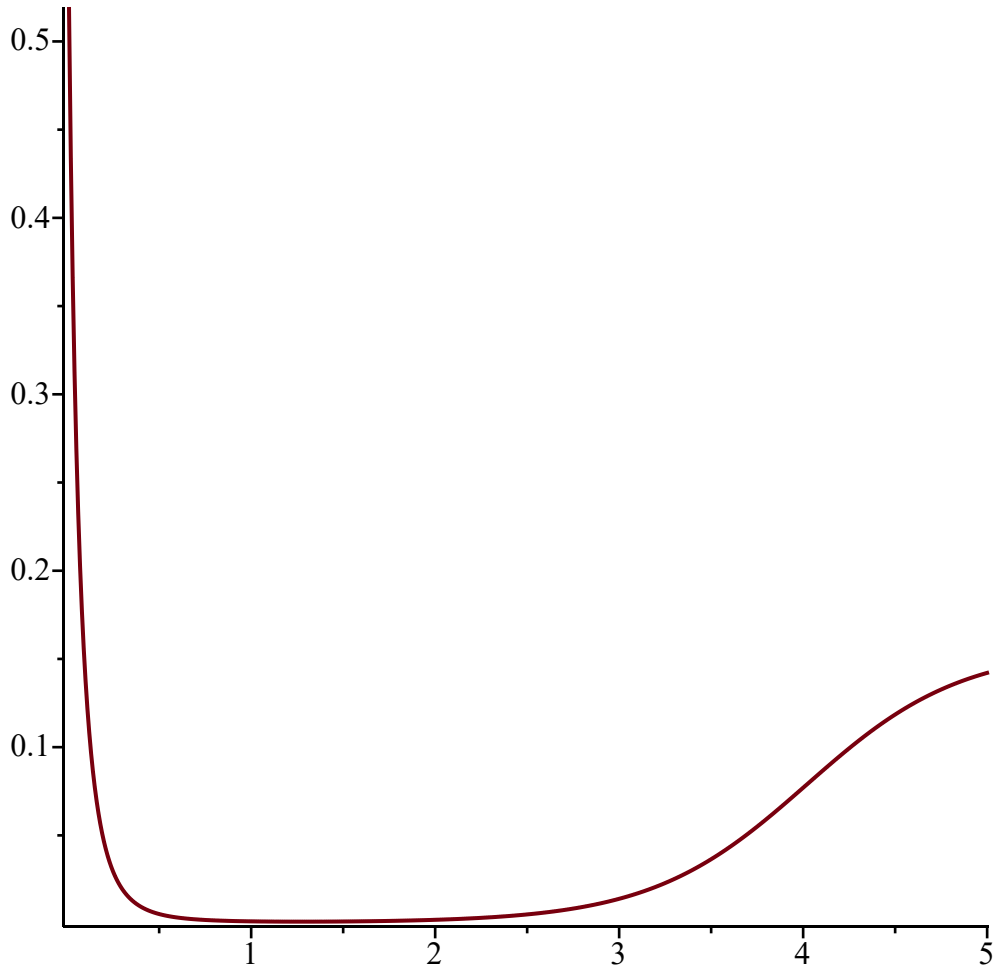


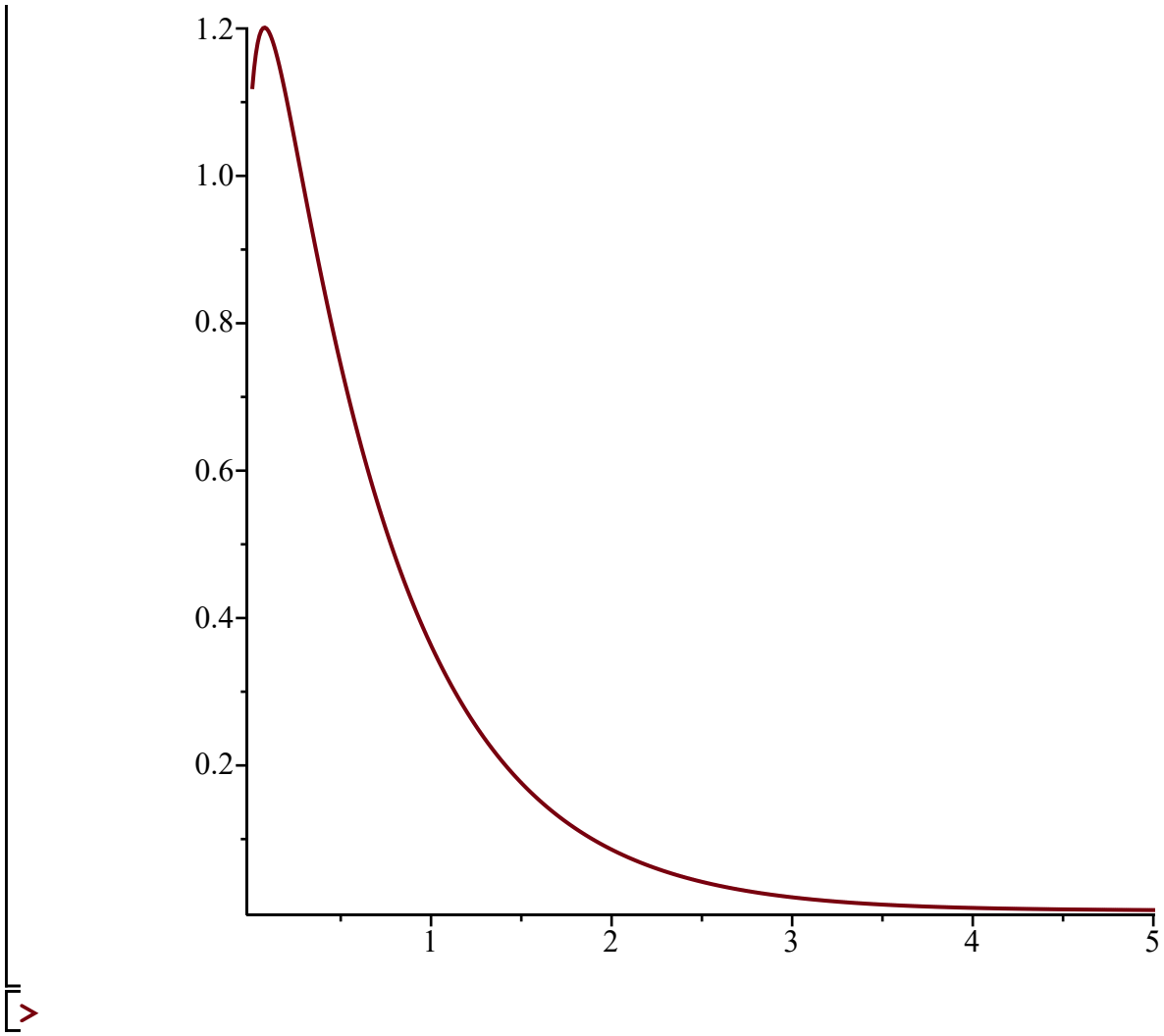

```
> L := Dis(VolterraM(rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0
      .. 10.01)( ), rand(0 .. 10.01)( ), x, y), [x, y], [rand(0 .. 10.01)( ), rand(0 .. 10.01)( )],
      0.01, 5) :
  plot([seq([L[i1][1], L[i1][2][1]], i1 = 1 .. nops(L))]);
  plot([seq([L[i1][1], L[i1][2][2]], i1 = 1 .. nops(L))]);
```


5
4
3
2
1
0
1
2
3
4
5




```
> L := Dis(VolterraM(rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0 .. 10.01)( ), rand(0
      .. 10.01)( ), rand(0 .. 10.01)( ), x, y), [x, y], [rand(0 .. 10.01)( ), rand(0 .. 10.01)( )],
      0.01, 5) :
  plot([seq([L[i1][1], L[i1][2][1]], i1 = 1 .. nops(L))]);
  plot([seq([L[i1][1], L[i1][2][2]], i1 = 1 .. nops(L))]);
```


0.5
0.4
0.3
0.2
0.1
1
2
3
4
5


1.2
1.0
0.8
0.6
0.4
0.2
1
2
3
4
5