MATH 396 - HW #21

1. $\begin{cases} \frac{dx}{dt} = x - y \\ \frac{dy}{dt} = y - x \end{cases}$, $x(0) = y(0) = 1$

$\underbrace{\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}_{M} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix}$; $\det[M - \lambda I] = (1-\lambda)^2 - (-1)(-1) = \lambda^2 - 2\lambda = \lambda(\lambda - 2)$

$\lambda_1 = 0$, $\lambda_2 = 2$

for $\lambda_1 = 0$, $\begin{cases} x - y = 0 \\ y - x = 0 \end{cases}$) $x = y$ $\vec{e}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for $\lambda = 2$, $\begin{cases} -x - y = 0 \\ -y - x = 0 \end{cases}$) $y = -x$ $\vec{e}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

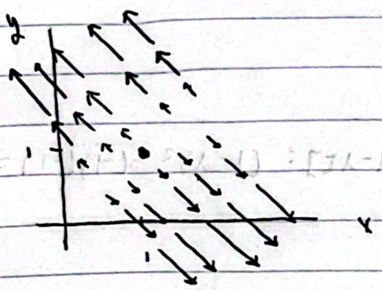
general solution: $\underset{\downarrow \begin{bmatrix} x \\ y \end{bmatrix}}{\vec{z}(t)} = C_1 e^{0 \cdot t} \cdot \vec{e}_0 + C_2 e^{2 \cdot t} \cdot \vec{e}_2 = C_1 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{2t} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ using initial conditions...

$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{cases} 1 = C_1 + C_2 \\ 1 = C_1 - C_2 \end{cases}$; $C_1 = 1$, $C_2 = 0$

$\begin{cases} x(t) = 1 + 0 \cdot e^{2t} = 1 \\ y(t) = 1 - 0 \cdot e^{2t} = 1 \end{cases}$) final solution, $x(t) = y(t) = 1$

Phase plane:



2. Yes, solution and phase plane from Maple matched those of question #1

3. See txt file attached

4. See attached txt file

> **read** `DMB.txt`;

For a list of the Main procedures type: Help(); for help with a specific procedure type: Help (ProcedureName); for example Help(Feig);

For a list of the Continuous Dynamical Models procedures type: HelpC(); for help with a specific procedure type: Help(ProcedureName); for example Help(Feig); **(1)**

>

> $Mat1 := RandMat(2, 5);$

$$Mat1 := [[3, 4], [4, 5]] \quad \textbf{(2)}$$

> $Mat2 := RandMat(2, 5);$

$$Mat2 := [[3, 1], [5, 2]] \quad \textbf{(3)}$$

> $Mat3 := RandMat(2, 5);$

$$Mat3 := [[3, 2], [2, 4]] \quad \textbf{(4)}$$

> $eqn1 := diff(x(t), t) = 3 \cdot x(t) + 4 \cdot y(t), diff(y(t), t) = 4 \cdot x(t) + 5 \cdot y(t)$

$$eqn1 := \frac{d}{dt}\, x(t) = 3\, x(t) + 4\, y(t),\ \frac{d}{dt}\, y(t) = 4\, x(t) + 5\, y(t) \quad \textbf{(5)}$$

> $eqn2 := diff(x(t), t) = 3 \cdot x(t) + y(t), diff(y(t), t) = 5 \cdot x(t) + 2 \cdot y(t)$

$$eqn2 := \frac{d}{dt}\, x(t) = 3\, x(t) + y(t),\ \frac{d}{dt}\, y(t) = 5\, x(t) + 2\, y(t) \quad \textbf{(6)}$$

> $eqn3 := diff(x(t), t) = 3 \cdot x(t) + 2 \cdot y(t), diff(y(t), t) = 2 \cdot x(t) + 4 \cdot y(t)$

$$eqn3 := \frac{d}{dt}\, x(t) = 3\, x(t) + 2\, y(t),\ \frac{d}{dt}\, y(t) = 2\, x(t) + 4\, y(t) \quad \textbf{(7)}$$

> $S1 = dsolve(\{eqn1, x(0) = 1, y(0) = 1\}, \{x(t), y(t)\});$

$$S1 = \left\{ x(t) = \left(\frac{1}{2} + \frac{3\sqrt{17}}{34}\right) e^{(4+\sqrt{17})\,t} + \left(\frac{1}{2} - \frac{3\sqrt{17}}{34}\right) e^{-(-4+\sqrt{17})\,t}, y(t) = \frac{\left(\frac{1}{2} + \frac{3\sqrt{17}}{34}\right) e^{(4+\sqrt{17})\,t}\sqrt{17}}{4} - \frac{\left(\frac{1}{2} - \frac{3\sqrt{17}}{34}\right) e^{-(-4+\sqrt{17})\,t}\sqrt{17}}{4} + \frac{\left(\frac{1}{2} + \frac{3\sqrt{17}}{34}\right) e^{(4+\sqrt{17})\,t}}{4} + \frac{\left(\frac{1}{2} - \frac{3\sqrt{17}}{34}\right) e^{-(-4+\sqrt{17})\,t}}{4} \right\} \quad \textbf{(8)}$$

> $S2 = dsolve(\{eqn2, x(0) = 1, y(0) = 1\}, \{x(t), y(t)\});$

(9)

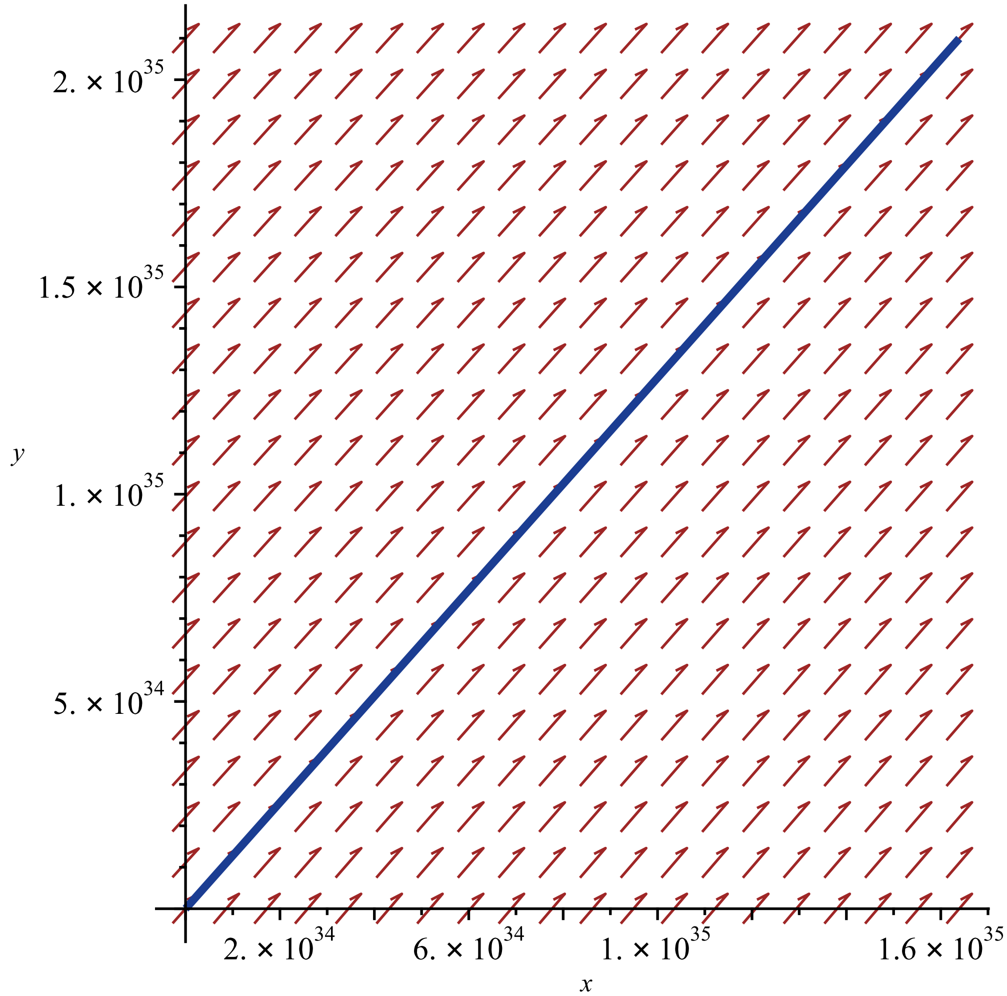
$$S2 = \left\{ x(t) = \left(\frac{1}{2} + \frac{\sqrt{21}}{14}\right) e^{\frac{(5+\sqrt{21})t}{2}} + \left(\frac{1}{2} - \frac{\sqrt{21}}{14}\right) e^{-\frac{(-5+\sqrt{21})t}{2}}, y(t) = \frac{\left(\frac{1}{2} + \frac{\sqrt{21}}{14}\right) e^{\frac{(5+\sqrt{21})t}{2}} \sqrt{21}}{2} - \frac{\left(\frac{1}{2} - \frac{\sqrt{21}}{14}\right) e^{-\frac{(-5+\sqrt{21})t}{2}} \sqrt{21}}{2} - \frac{\left(\frac{1}{2} + \frac{\sqrt{21}}{14}\right) e^{\frac{(5+\sqrt{21})t}{2}}}{2} - \frac{\left(\frac{1}{2} - \frac{\sqrt{21}}{14}\right) e^{-\frac{(-5+\sqrt{21})t}{2}}}{2} \right\} \quad \textbf{(9)}$$

> $S3 = dsolve(\{eqn3, x(0) = 1, y(0) = 1\}, \{x(t), y(t)\});$

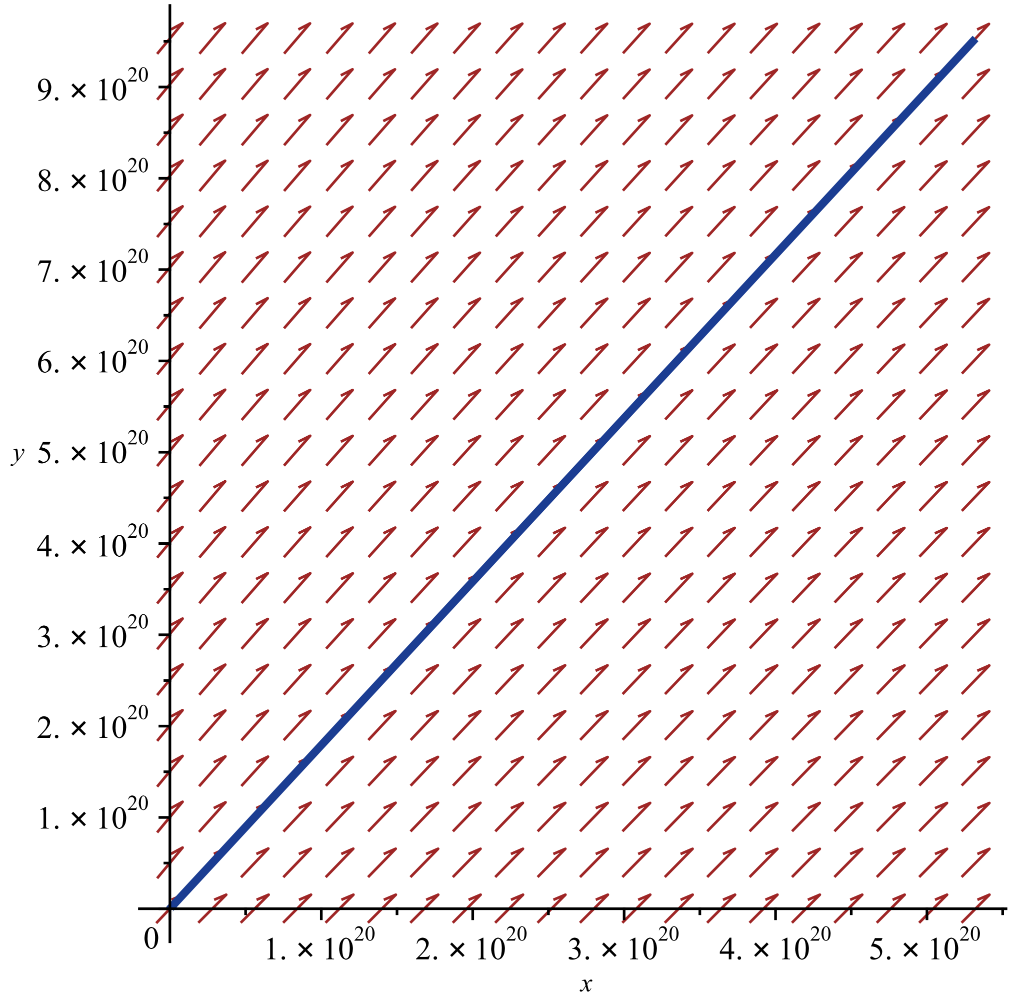
$$S3 = \left\{ x(t) = \left(\frac{1}{2} + \frac{3\sqrt{17}}{34}\right) e^{\frac{(7+\sqrt{17})t}{2}} + \left(\frac{1}{2} - \frac{3\sqrt{17}}{34}\right) e^{-\frac{(-7+\sqrt{17})t}{2}}, y(t) = \frac{\left(\frac{1}{2} + \frac{3\sqrt{17}}{34}\right) e^{\frac{(7+\sqrt{17})t}{2}} \sqrt{17}}{4} - \frac{\left(\frac{1}{2} - \frac{3\sqrt{17}}{34}\right) e^{-\frac{(-7+\sqrt{17})t}{2}} \sqrt{17}}{4} + \frac{\left(\frac{1}{2} + \frac{3\sqrt{17}}{34}\right) e^{\frac{(7+\sqrt{17})t}{2}}}{4} + \frac{\left(\frac{1}{2} - \frac{3\sqrt{17}}{34}\right) e^{-\frac{(-7+\sqrt{17})t}{2}}}{4} \right\} \quad \textbf{(10)}$$

> $with(DEtools):$

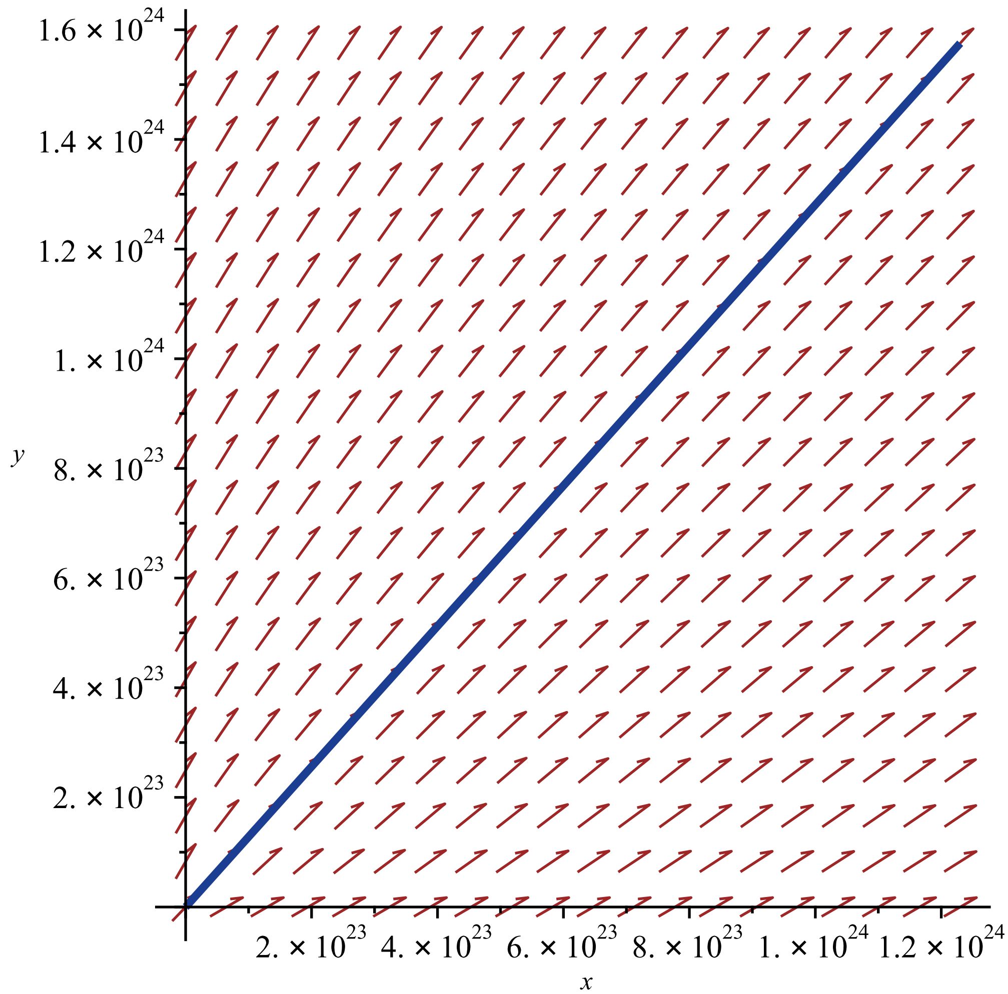
> $phaseportrait([eqn1], [x(t), y(t)], t = 0..10, [[x(0) = 1, y(0) = 1]]);$



> *phaseportrait*([*eqn2*], [$x(t)$, $y(t)$], $t = 0..10$, [[$x(0) = 1$, $y(0) = 1$]]);



> *phaseportrait*$([eqn3], [x(t), y(t)], t = 0..10, [[x(0) = 1, y(0) = 1]]);$



1.6×10^{24}
1.4×10^{24}
1.2×10^{24}
$1.\times10^{24}$
y
$8.\times10^{23}$
$6.\times10^{23}$
$4.\times10^{23}$
$2.\times10^{23}$
$2.\times10^{23}$ $4.\times10^{23}$ $6.\times10^{23}$ $8.\times10^{23}$ $1.\times10^{24}$ 1.2×10^{24}
x

> $L1 := Lotka(1, 2, 3, 4, 5, 6, N1, N2);$

$$L1 := \left[\frac{N1\ (2 - N1 - 5\ N2)}{2}, \frac{3\ N2\ (4 - N2 - 6\ N1)}{4} \right] \tag{11}$$

>

> $Dis(L1, [N1, N2], [0.5, 0.5], 0.01, 10)[1 ..10];$

[[0.01, [0.5, 0.5]], [0.02, [0.4975000000, 0.5018750000]], [0.03, [0.4949953984, 0.5038064346]], [0.04, [0.4924857035, 0.5057947867]], [0.05, [0.4899704322, 0.5078405660]], [0.06, [0.4874491099, 0.5099443089]], [0.07, [0.4849212703, 0.5121065787]], [0.08, [0.4823864555, 0.5143279657]], [0.09, [0.4798442155, 0.5166090873]], [0.10, [0.4772941082, 0.5189505881]]] **(12)**

> $L2 := Lotka(2, 2, 2, 2, 2, 2, N1, N2);$

$$L2 := [N1\ (2 - N1 - 2\ N2), N2\ (2 - N2 - 2\ N1)] \tag{13}$$

> $Dis(L2, [N1, N2], [0.5, 0.5], 0.01, 10)[1 ..10];$

[[0.01, [0.5, 0.5]], [0.02, [0.5025, 0.5025]], [0.03, [0.5049748125, 0.5049748125]], [0.04, [0.5074243219, 0.5074243219]], [0.05, [0.5098484251, 0.5098484251]], [0.06, [0.5122470311, 0.5122470311]], [0.07, [0.5146200611, 0.5146200611]], [0.08, [0.5169674481, 0.5169674481]], [0.09, [0.5192891368, 0.5192891368]], [0.10, [0.5215850833, 0.5215850833]]] **(14)**

> $L3 := Lotka(6, 5, 4, 3, 2, 1, N1, N2);$

$$L3 := \left[\frac{6\ N1\ (5 - N1 - 2\ N2)}{5}, \frac{4\ N2\ (3 - N2 - N1)}{3} \right] \tag{15}$$

> $Dis(L3, [N1, N2], [0.5, 0.5], 0.01, 10)[1 ..10];$

[[0.01, [0.5, 0.5]], [0.02, [0.5210000000, 0.5133333333]], [0.03, [0.5425839880, 0.5267872296]], [0.04, [0.5647464271, 0.5403476374]], [0.05, [0.5874801249, 0.5539997433]], [0.06, [0.6107762055, 0.5677280056]], [0.07, [0.6346240928, 0.5815161945]], [0.08, [0.6590115050, 0.5953474387]], [0.09, [0.6839244619, 0.6092042778]], [0.10, [0.7093473046, 0.6230687214]]] **(16)**

> $V1 := Volterra(1, 2, 3, 4, x, y);$

$$V1 := [-2\ x\ y + x, 4\ x\ y - 3\ y] \tag{17}$$

> $Dis(V1, [x, y], [0.5, 0.5], 0.01, 10)[1 ..10];$

[[0.01, [0.5, 0.5]], [0.02, [0.5000, 0.4950]], [0.03, [0.5000500000, 0.4900500000]], [0.04, [0.5001495100, 0.4851504801]], [0.05, [0.5002980496, 0.4803018767]], [0.06, [0.5004951483, 0.4755045841]], [0.07, [0.5007403451, 0.4707589561]], [0.08, [0.5010331886, 0.4660653075]], [0.09, [0.5013732368, 0.4614239158]], [0.10, **(18)**

$[0.5017600572, 0.4568350224]]]$

> $V2 := Volterra(1, 1, 1, 1, x, y);$

$$V2 := [-x\,y + x, x\,y - y] \tag{19}$$

> $Dis(V2, [x, y], [0.5, 0.5], 0.01, 10)[1..10];$

$$[[0.01, [0.5, 0.5]], [0.02, [0.5025, 0.4975]], [0.03, [0.5050250625, 0.4950249375]], [0.04, [0.5075753131, 0.4925746881]], [0.05, [0.5101508787, 0.4901491287]], [0.06, [0.5127518874, 0.4877481375]], [0.07, [0.5153784685, 0.4853715939]], [0.08, [0.5180307525, 0.4830193787]], [0.09, [0.5207088711, 0.4806913738]], [0.10, [0.5234129572, 0.4783874627]]] \tag{20}$$

> $V3 := Volterra(4, 3, 2, 1, x, y);$

$$V3 := [-3\,x\,y + 4\,x, x\,y - 2\,y] \tag{21}$$

> $Dis(V3, [x, y], [0.5, 0.5], 0.01, 10)[1..10];$

$$[[0.01, [0.5, 0.5]], [0.02, [0.5125, 0.4925]], [0.03, [0.5254278125, 0.4851740625]], [0.04, [0.5387972066, 0.4780198207]], [0.05, [0.5526224226, 0.4710349817]], [0.06, [0.5669181847, 0.4642173270]], [0.07, [0.5816997148, 0.4575647129]], [0.08, [0.5969827455, 0.4510750712]], [0.09, [0.6127835343, 0.4447464101]], [0.10, [0.6291188774, 0.4385768147]]] \tag{22}$$

> $VM1 := VolterraM(1, 2, 3, 4, 2, x, y);$

$$VM1 := \left[x\left(1 - \frac{x}{4}\right) - 2\,x\,y, 2\,x\,y - 3\,y\right] \tag{23}$$

> $Dis(VM1, [x, y], [0.5, 0.5], 0.01, 10)[1..10];$

$$[[0.01, [0.5, 0.5]], [0.02, [0.4993750000, 0.4900]], [0.03, [0.4988514365, 0.4801938750]], [0.04, [0.4984269109, 0.4705789669]], [0.05, [0.4980991221, 0.4611525823]], [0.06, [0.4978658626, 0.4519119987]], [0.07, [0.4977250141, 0.4428544698]], [0.08, [0.4976745439, 0.4339772306]], [0.09, [0.4977125011, 0.4252775021]], [0.10, [0.4978370132, 0.4167524956]]] \tag{24}$$

> $VM2 := VolterraM(1, 1, 1, 1, 4, x, y);$

$$VM2 := [x\,(1 - x) - x\,y, 4\,x\,y - y] \tag{25}$$

> $Dis(VM2, [x, y], [0.5, 0.5], 0.01, 10)[1..10];$

$$[[0.01, [0.5, 0.5]], [0.02, [0.5000, 0.5050]], [0.03, [0.4999750000, 0.5100500000]], [0.04, [0.4999248775, 0.5151499900]], [0.05, [0.4998495144, 0.5202999419]], [0.06, [0.4997487975, 0.5254998094]], [0.07, [0.4996226179, 0.5307495272]], [0.08, [0.4994708718, 0.5360490106]], [0.09, [0.4992934603, 0.5413981552]], [0.10, [0.4990902897, 0.5467968359]]] \tag{26}$$

> $VM3 := VolterraM(4, 3, 2, 1, 1, x, y);$

$$VM3 := [4\,x\,(1-x) - 3\,x\,y,\, x\,y - 2\,y] \tag{27}$$

> $Dis(VM3, [x, y], [0.5, 0.5], 0.01, 10)[1..10];$

$$[[0.01, [0.5, 0.5]], [0.02, [0.5025, 0.4925]], [0.03, [0.5050753125, 0.4851248125]], [0.04, [0.5077235451, 0.4778725619]], [0.05, [0.5104423445, 0.4707413822]], [0.06, [0.5132293928, 0.4637294179]], [0.07, [0.5160824051, 0.4568348252]], [0.08, [0.5189991268, 0.4500557729]], [0.09, [0.5219773315, 0.4433904429]], [0.10, [0.5250148186, 0.4368370316]]] \tag{28}$$

>