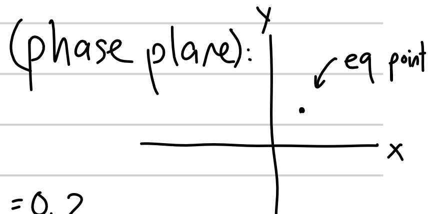
Daniyal Chaudhry HW 21



1. $x' = x - y, \quad y' = y - x \rightarrow \underbrace{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}\begin{bmatrix} x \\ y \end{bmatrix}$

$$(1-\lambda)^2 - 1 \rightarrow \lambda^2 - 2\lambda \rightarrow \lambda = 0, 2$$

for $\lambda = 0$, $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow a = b$, so $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ works

$\lambda = 2$, $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow a = -b$, so $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ works

The solution can be expressed as:

$$\left.\begin{aligned} x &= c_1 e^{0t}(1) + c_2 e^{2t}(1) = c_1 + c_2 e^{2t} \\ y &= c_1 e^{0t}(1) + c_2 e^{2t}(-1) = c_1 - c_2 e^{2t} \end{aligned}\right\} \left.\begin{aligned} x(0) &= 1 \\ y(0) &= 1 \end{aligned}\right\} \text{ so } c_2 e^{2t} = 0, \text{ and } c_1 = 1 \ldots \boxed{x = y = 1}$$

if you intended $y(0) = 0$, then $c_1 = 0.5$ and $c_2 = 0.5$ so $x = 0.5 + 0.5e^{2t}, y = 0.5 - 0.5e^{2t}$

2. I did not get the same thing as 1, as the initial conditions were different. However, I got the same thing as my adjusted #1, where I swapped $y(0) = 1 \rightarrow 0$.

3. All the solutions are very complicated. I'll just write λ and c_1, c_2...

$\begin{bmatrix} 2 & -4 \\ 3 & -2 \end{bmatrix} \rightarrow \lambda = \frac{\pm 5 + \sqrt{33}}{2}, \quad c_{1,2} = \frac{1}{2} \mp \frac{3 - \sqrt{33}}{22}$

$\begin{bmatrix} 6 & -3 \\ 2 & -1 \end{bmatrix} \rightarrow \lambda = \pm 4 + \sqrt{7}, \quad c_{1,2} = \frac{1}{2} \mp \frac{\sqrt{7}}{14}$ (seems like these matrices result in complex λ)

$\begin{bmatrix} 3 & -7 \\ 8 & -2 \end{bmatrix} \rightarrow$ simpler: $x = -\frac{5}{9}e^{10t} + \frac{14}{9}e^{t}, \quad y(t) = \frac{5}{9}e^{10t} + \frac{4}{9}e^{t}$

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4. Lotka(1, 2, 0.2, 3, 0.1, 0.2, N1, N2) has SEquP for N1, N2: [1.7346, 2.653]
I could not get the plot to work using Dis, but I did get Dis to work with conditions [3,5], that converged to the mentioned Equ. P.
Lotka(4, 1, 0.4, 6, 0.7, 0.8, N1, N2) has SEquP [0, 6]. Similar to above, Dis converged from [3,5] to [0,6].
Lotka(5, 2, 0.6, 2, 0.1, 0.1, N1, N2) has SEquP [1.81, 1.81]. Similarly, Dis converged from [3,5] to [1.81, 1.81], but it took much longer.

For the rest I will write the function, SEquP, init. conditions, and Dis convergence.
→ Volterra(1, 0.02, 0.5, 0.01, x, y) → failed SEquP → [3,5] went to [12.88, 1.97]
→ VolterraM(0.7, 0.03, 0.8, 0.02, 5, x, y) → SEquP [0,0] → [3,5] went to [−∞, ∞]?
→ Volterra(0.8, 0.03, 0.6, 0.02, x, y) → [30, 26.67] → [3,5] went to [4, 2.32]
→ VolterraM(1, 0.02, 0.5, 0.01, 10, x, y) → [0,0] → [3,5] went to [−∞, ∞]
→ Volterra(1.5, 0.01, 0.4, 0.03, x, y) → [13.33, 150] → (3,5) went to [30, 2.14]
→ VolterraM(1.2, 0.01, 0.6, 0.02, 12, x, y) → [0,0] → [3,5] went to [−∞, ∞]

I used the same init. conditions everywhere, but I also feel like I may have done some wrong inputs, and I don't know why plot() won't accept the sequences I get.