HW 21

1] $\frac{dx}{dt} = x - y \qquad \frac{dy}{dt} = y - x$

$\frac{d^2x}{dt^2} = \frac{dx}{dt} - \frac{dy}{dt}$

$\frac{d^2x}{dt^2} = \frac{dx}{dt} - y + x$

$y = x - \frac{dx}{dt}$

$\frac{d^2x}{dt^2} = \frac{dx}{dt} - x + \frac{dx}{dt} + x$

$\lambda^2 = 2\lambda$

$\lambda = 0, 2$

$x = C_1 e^{0t} + C_2 e^{2t}$

$1 = C_1 + C_2$

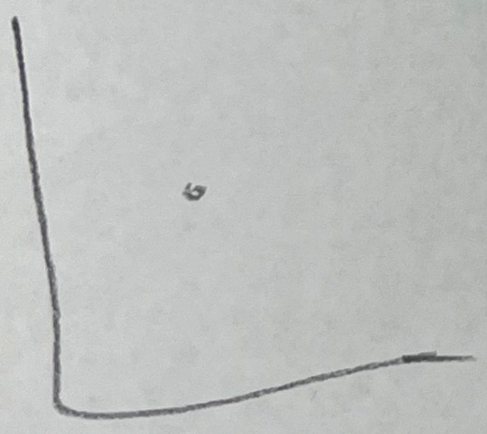
$y = C_1 + C_2 e^{2t} - 2C_2 e^{2t}$

$y = C_1 - C_2 e^{2t}$

$1 = C_1 - C_2$

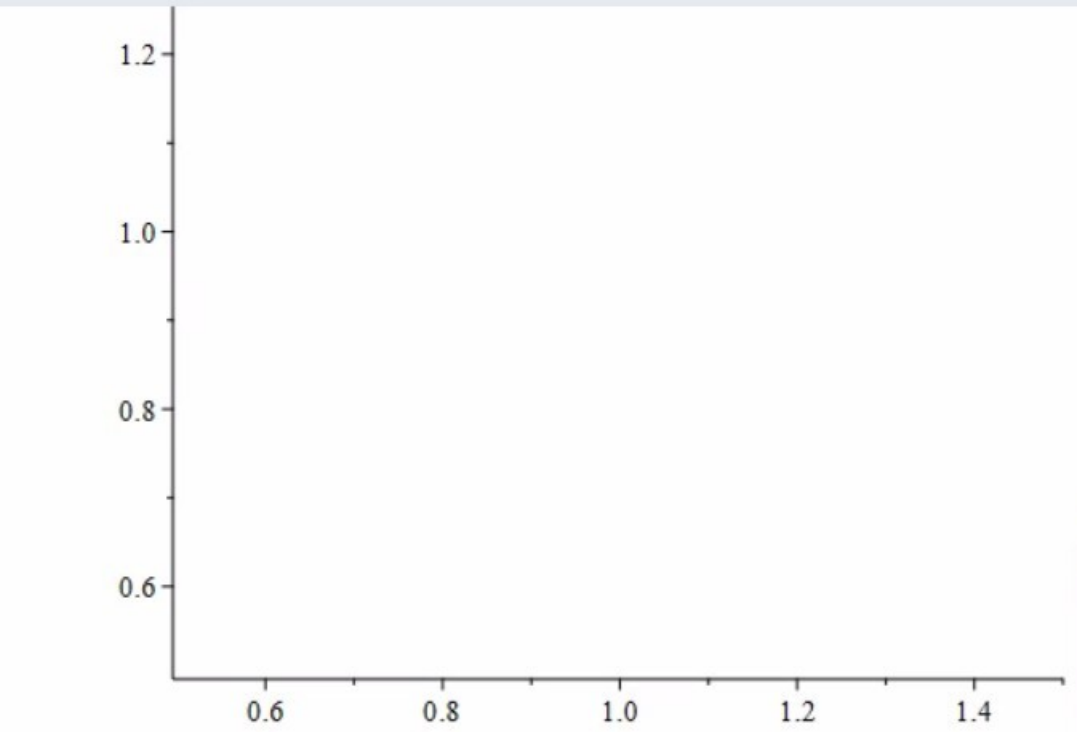
$C_1 = 1, C_2 = 0$

$x(t) = 1, y(t) = 1$



$x' = 0$

$y' = 0$

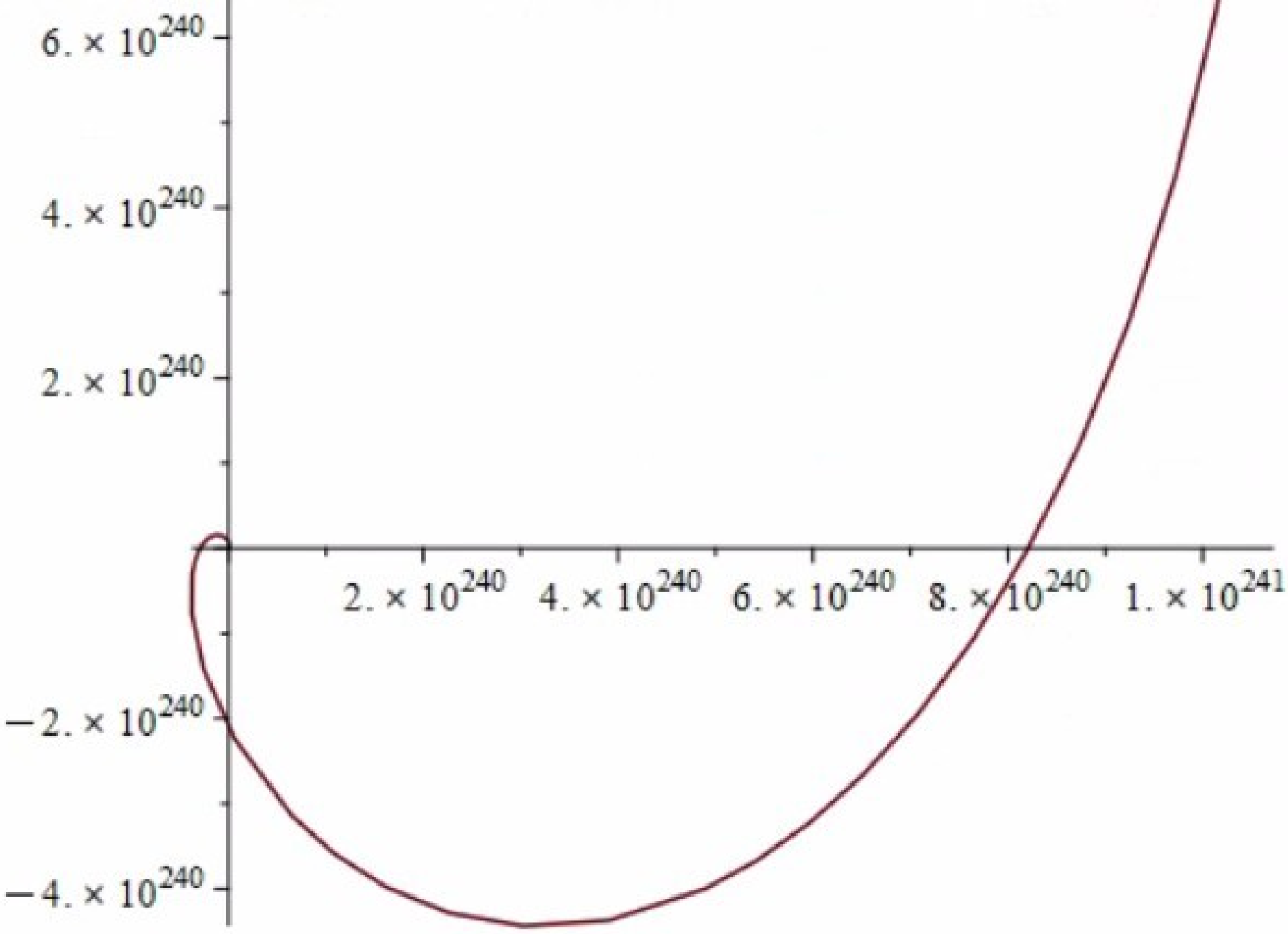

1.2
1.0
0.8
0.6
0.6
0.8
1.0
1.2
1.4

```
with(LinearAlgebra) :
L := RandomMatrix(2, 2)
```

$$L := \begin{bmatrix} 44 & -31 \\ 92 & 67 \end{bmatrix}$$

```
S := dsolve({diff(x(t), t) = L(1, 1) · x(t) + L(1, 2) · y(t), diff(y(t), t) = L(2, 2) · y(t) + L(2, 1) · x(t), x(0) = 1, y(0) = 1}, {x(t), y(t)});
plot([subs(S, x(t)), subs(S, y(t)), t = 0 ..10])
```

$$S := \left\{ x(t) = e^{\frac{111t}{2}} \left(-\frac{85\sqrt{10879}\,\sin\left(\frac{\sqrt{10879}\,t}{2}\right)}{10879} + \cos\left(\frac{\sqrt{10879}\,t}{2}\right) \right), y(t) = \frac{e^{\frac{111t}{2}} \left(\frac{558\sqrt{10879}\,\sin\left(\frac{\sqrt{10879}\,t}{2}\right)}{473} + 62\cos\left(\frac{\sqrt{10879}\,t}{2}\right) \right)}{62} \right\}$$

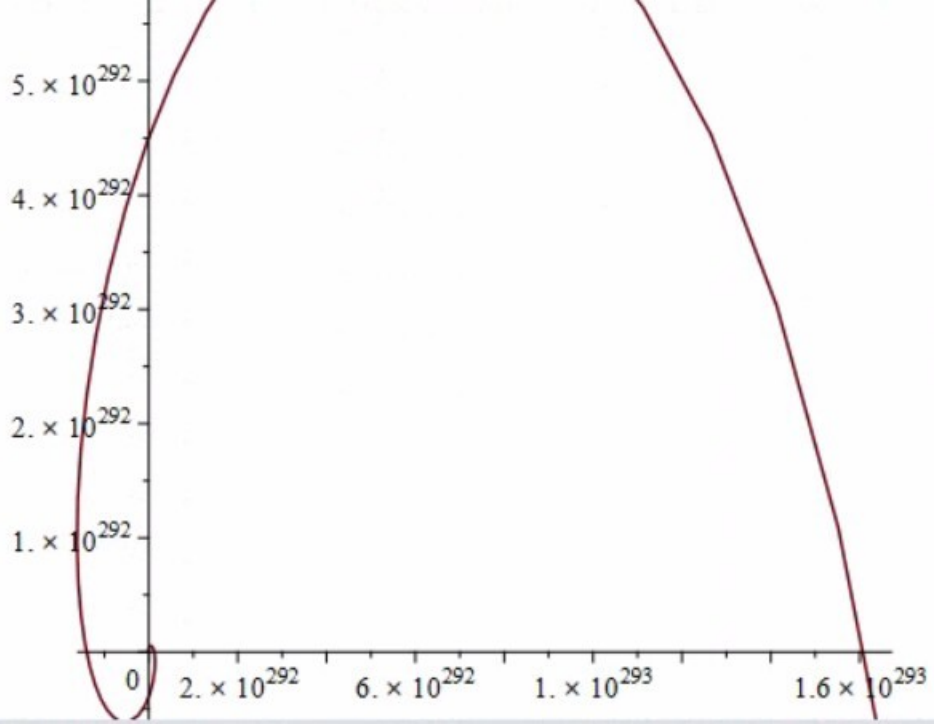

6. × 10^240
4. × 10^240
2. × 10^240
2. × 10^240
4. × 10^240
6. × 10^240
8. × 10^240
1. × 10^241
−2. × 10^240
−4. × 10^240

```
with(LinearAlgebra) :
L := RandomMatrix(2, 2)
```

$$L := \begin{bmatrix} 50 & 80 \\ -98 & 85 \end{bmatrix}$$

```
S := dsolve({diff(x(t), t) = L(1, 1) · x(t) + L(1, 2) · y(t), diff(y(t), t) = L(2, 2) · y(t) + L(2, 1) · x(t), x(0) = 1, y(0) = 1}, {x(t), y(t)});
plot([subs(S, x(t)), subs(S, y(t)), t = 0 ..10])
```

$$S := \left\{ x(t) = e^{\frac{135t}{2}} \left(\frac{25\sqrt{615}\,\sin\left(\frac{7\sqrt{615}\,t}{2}\right)}{861} + \cos\left(\frac{7\sqrt{615}\,t}{2}\right) \right), y(t) = -\frac{7\,e^{\frac{135t}{2}} \left(\frac{736\sqrt{615}\,\sin\left(\frac{7\sqrt{615}\,t}{2}\right)}{861} - \frac{160\cos\left(\frac{7\sqrt{615}\,t}{2}\right)}{7} \right)}{160} \right\}$$

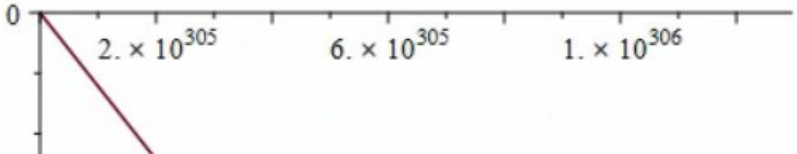

5. × 10^292
4. × 10^292
3. × 10^292
2. × 10^292
1. × 10^292
0
2. × 10^292
6. × 10^292
1. × 10^293
1.6 × 10^293

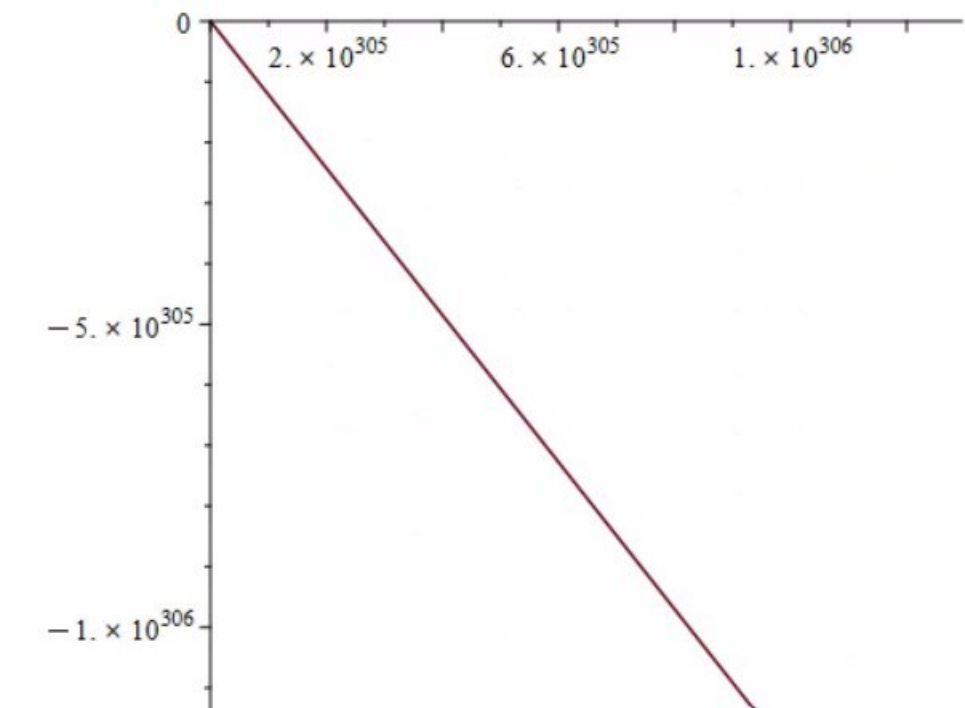
```
with(LinearAlgebra) :
L := RandomMatrix(2, 2)
```

$$L := \begin{bmatrix} 10 & -50 \\ -99 & -11 \end{bmatrix}$$

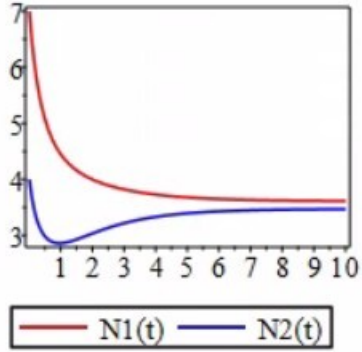
```
S := dsolve({diff(x(t), t) = L(1, 1) · x(t) + L(1, 2) · y(t), diff(y(t), t) = L(2, 2) · y(t) + L(2, 1) · x(t), x(0) = 1, y(0) = 1}, {x(t), y(t)});
plot([subs(S, x(t)), subs(S, y(t)), t = 0 ..10])
```

$$:= \left\{ x(t) = \left(\frac{1}{2} - \frac{79\sqrt{2249}}{13494}\right) e^{\frac{(-1+3\sqrt{2249})\,t}{2}} + \left(\frac{1}{2} + \frac{79\sqrt{2249}}{13494}\right) e^{-\frac{(1+3\sqrt{2249})\,t}{2}}, y(t) = -\frac{3\left(\frac{1}{2} - \frac{79\sqrt{2249}}{13494}\right) e^{\frac{(-1+3\sqrt{2249})\,t}{2}} \sqrt{2249}}{100} + \frac{3\left(\frac{1}{2} + \frac{79\sqrt{2249}}{13494}\right) e^{-\frac{(1+3\sqrt{2249})\,t}{2}} \sqrt{2249}}{100} + \frac{21\left(\frac{1}{2} - \frac{79\sqrt{2249}}{13494}\right) e^{\frac{(-1+3\sqrt{2249})\,t}{2}}}{100} + \frac{21\left(\frac{1}{2} + \frac{79\sqrt{2249}}{13494}\right) e^{-\frac{(1+3\sqrt{2249})\,t}{2}}}{100} \right\}$$




0
2. × 10^305
6. × 10^305
1. × 10^306
−5. × 10^305
−1. × 10^306

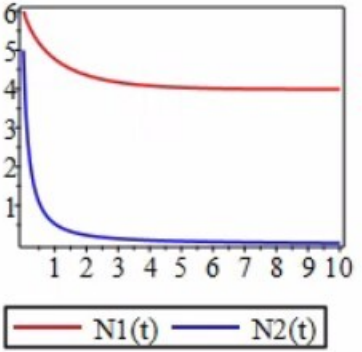
```
F1 := evalf(Lotka(1.5, 5, 2.5, 6, 0.4, 0.7, N1, N2)) :
X := Dis(F1, [N1, N2], [7.0, 4.0], 0.01, 10) :
N1curve := [ seq( [ X[i][1], X[i][2][1] ], i = 1 ..nops(X) ) ] :
N2curve := [ seq( [ X[i][1], X[i][2][2] ], i = 1 ..nops(X) ) ] :
plot([N1curve, N2curve], color = [red, blue], legend = ["N1(t)", "N2(t)"], axes = boxed, size = [200, 200]);
```




```
SEquP(F1, [N1, N2]);
```

{[3.611111111, 3.472222222]}

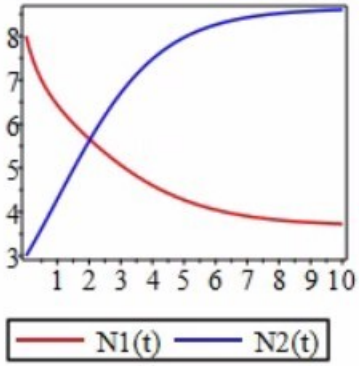
```
F2 := evalf(Lotka(0.6, 4, 2.5, 3, 0.2, 0.8, N1, N2)) :
X := Dis(F2, [N1, N2], [6.0, 5.0], 0.01, 10) :
N1curve := [ seq( [ X[i][1], X[i][2][1] ], i = 1 ..nops(X) ) ] :
N2curve := [ seq( [ X[i][1], X[i][2][2] ], i = 1 ..nops(X) ) ] :
plot([N1curve, N2curve], color = [red, blue], legend = ["N1(t)", "N2(t)"], axes = boxed, size = [200, 200]);
```




```
SEquP(F2, [N1, N2]);
```

{[4., 0.]}

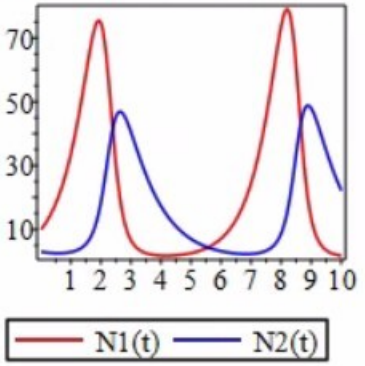
```
F3 := evalf(Lotka(2.0, 8, 0.7, 9, 0.5, 0.1, N1, N2)) :
X := Dis(F3, [N1, N2], [8.0, 3.0], 0.01, 10) :
N1curve := [ seq( [ X[i][1], X[i][2][1] ], i = 1 ..nops(X) ) ] :
N2curve := [ seq( [ X[i][1], X[i][2][2] ], i = 1 ..nops(X) ) ] :
plot([N1curve, N2curve], color = [red, blue], legend = ["N1(t)", "N2(t)"], axes = boxed, size = [200, 200]);
```




```
SEquP(F3, [N1, N2]);
```

{[3.684210526, 8.631578947]}

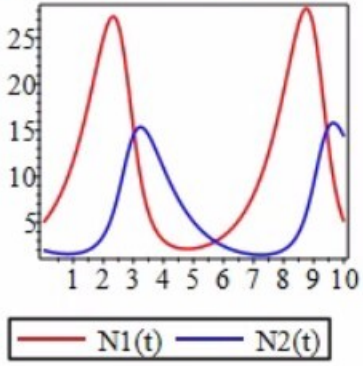
```
F := evalf( Volterra(1.5, 0.1, 1.0, 0.05, x, y) ) :
X := Dis(F, [x, y], [10.0, 3.0], 0.01, 10) :
N1curve := [ seq( [ X[i][1], X[i][2][1] ], i = 1 ..nops(X) ) ] :
N2curve := [ seq( [ X[i][1], X[i][2][2] ], i = 1 ..nops(X) ) ] :
plot( [N1curve, N2curve], color = [red, blue], legend = ["N1(t)", "N2(t)"], axes = boxed, size = [200, 200]);
```




```
SEquP(F, [N1, N2]);
```

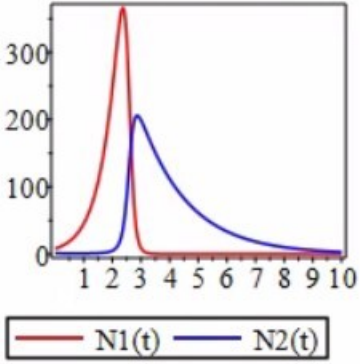
∅

```
F2 := evalf(Volterra(1.2, 0.2, 1.0, 0.1, x, y)) :
X := Dis(F2, [x, y], [5.0, 2.0], 0.01, 10) :
N1curve := [ seq( [ X[i][1], X[i][2][1] ], i = 1 ..nops(X) ) ] :
N2curve := [ seq( [ X[i][1], X[i][2][2] ], i = 1 ..nops(X) ) ] :
plot([N1curve, N2curve], color = [red, blue], legend = ["N1(t)", "N2(t)"], axes = boxed, size = [200, 200]);
```




```
SEquP(F2, [N1, N2]);
```

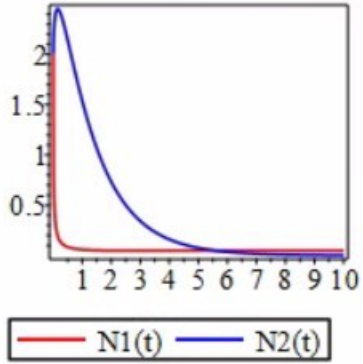
```
F3 := evalf(Volterra(1.8, 0.05, 0.6, 0.02, x, y)) :
X := Dis(F3, [x, y], [8.0, 1.0], 0.01, 10) :
N1curve := [ seq( [ X[i][1], X[i][2][1] ], i = 1 ..nops(X) ) ] :
N2curve := [ seq( [ X[i][1], X[i][2][2] ], i = 1 ..nops(X) ) ] :
plot([N1curve, N2curve], color = [red, blue], legend = ["N1(t)", "N2(t)"], axes = boxed, size = [200, 200]);
```




```
SEquP(F3, [N1, N2]);
```

∅

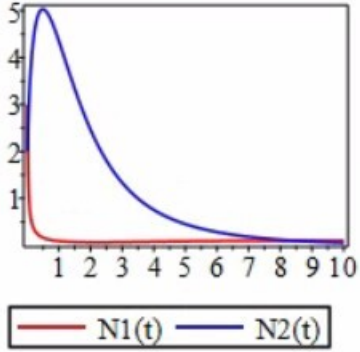
```
F1M := evalf(VolterraM(1.5, 0.1, 1.0, 0.05, 5, x, y)) :
X := Dis(F1M, [x, y], [2.0, 2.0], 0.01, 10) :
N1curve := [ seq( [ X[i][1], X[i][2][1] ], i = 1 ..nops(X) ) ] :
N2curve := [ seq( [ X[i][1], X[i][2][2] ], i = 1 ..nops(X) ) ] :
plot( [N1curve, N2curve], color = [red, blue], legend = ["N1(t)", "N2(t)"], axes = boxed, size = [200, 200]);
```




```
SEquP(F1M, [N1, N2]);
```

∅

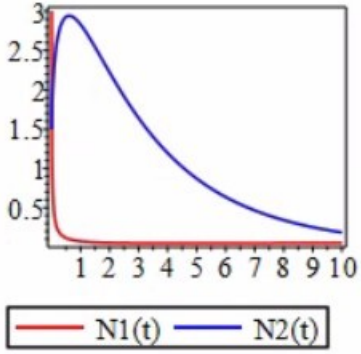
```
F2M := evalf(VolterraM(1.2, 0.2, 1.0, 0.1, 6, x, y)) :
X := Dis(F2M, [x, y], [3.0, 2.0], 0.01, 10) :
N1curve := [ seq( [ X[i][1], X[i][2][1] ], i = 1 ..nops(X) ) ] :
N2curve := [ seq( [ X[i][1], X[i][2][2] ], i = 1 ..nops(X) ) ] :
plot( [N1curve, N2curve], color = [red, blue], legend = ["N1(t)", "N2(t)"], axes = boxed, size = [200, 200]);
```




```
SEquP(F2M, [N1, N2]);
```

∅

```
F3M := evalf(VolterraM(1.0, 0.05, 0.6, 0.05, 6, x, y)) :
X := Dis(F3M, [x, y], [3.0, 1.5], 0.01, 10) :
N1curve := [ seq( [ X[i][1], X[i][2][1] ], i = 1 ..nops(X) ) ] :
N2curve := [ seq( [ X[i][1], X[i][2][2] ], i = 1 ..nops(X) ) ] :
plot( [N1curve, N2curve], color = [red, blue], legend = ["N1(t)", "N2(t)"], axes = boxed, size = [200, 200]);
```




```
SEquP(F3M, [N1, N2]);
```

∅