Homework for Lecture 21 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file and/or .txt file) to

ShaloshBEkhad@gmail.com

by 8:00pm Monday, Dec. 1,, 2025.

Subject: hw21

with an attachment hw21FirstLast.pdf and/or hw21FirstLast.txt

1. By hand solve the system

$$\frac{dx}{dt} = x - y \quad , \frac{dy}{dt} = y - x \quad , \quad x(0) = 1 \quad , \quad y(0) = 1 \quad .$$

Plot, by hand, the phase-plane diagram.

2. Now use Maple with the command

```
S:=dsolve({diff(x(t),t)=x(t)-y(t),diff(y(t),t)=y(t)-x(t),x(0)=1,y(0)=0},{x(t),y(t)});
plot([subs(S,x(t)),subs(S,y(t)),t=0..10]);
```

did you get the same thing?

3. Use Maple to solve and then plot the phase-plane diagram for the system

$$\frac{dx}{dt} = a_{11}x + a_{12}y \quad , \frac{dy}{dt} = a_{21}x + a_{22}y \quad , \quad x(0) = 1 \quad , \quad y(0) = 1 \quad ,$$

for three randomly chosen matrices

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad .$$

4. Carefully read, and understand, the Maple code for the following procedures (type Help(ProcedureName); for instructions)

`Lotka, Volterra, VolterraM`

in the Maple package

`https://sites.math.rutgers.edu/~zeilberg/Bio25/DMB.txt` ,

For **each of them**, experiment with **three** randmom choices of parameters, and random initial conditions, using `Dis` (with $h = 0.01$), of *each* of the quantities in question.

Send me these nice plots.

Confirm the numerics by using `SEquP`.

1. By hand solve the system

$$\frac{dx}{dt} = x - y \quad , \frac{dy}{dt} = y - x \quad , \quad x(0) = 1 \quad , \quad y(0) = 1 \quad .$$

Plot, by hand, the phase-plane diagram.

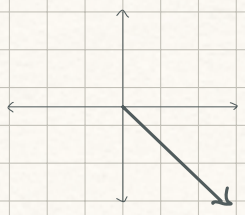
$0 = x - y$ $\qquad$ $0 = y - x$

$x = y$

$(0,0)$ is eq. point

$$J = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad J(0,0) = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

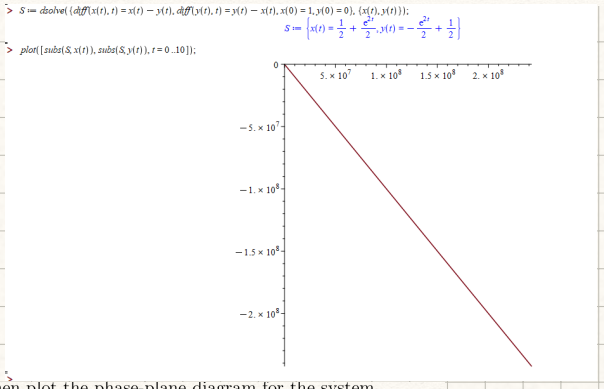
$$(1-\lambda)(1-\lambda) - 1 = 0$$
$$1 - 2\lambda + \lambda^2 - 1 = 0$$
$$\lambda^2 - 2\lambda = 0$$
$$\lambda(\lambda - 2) = 0$$
$$\lambda = 0, 2$$



2. Now use Maple with the command

```
S:=dsolve({diff(x(t),t)=x(t)-y(t),diff(y(t),t)=y(t)-x(t),x(0)=1,y(0)=0},{x(t),y(t)});
plot([subs(S,x(t)),subs(S,y(t)),t=0..10]);
```

did you get the same thing?

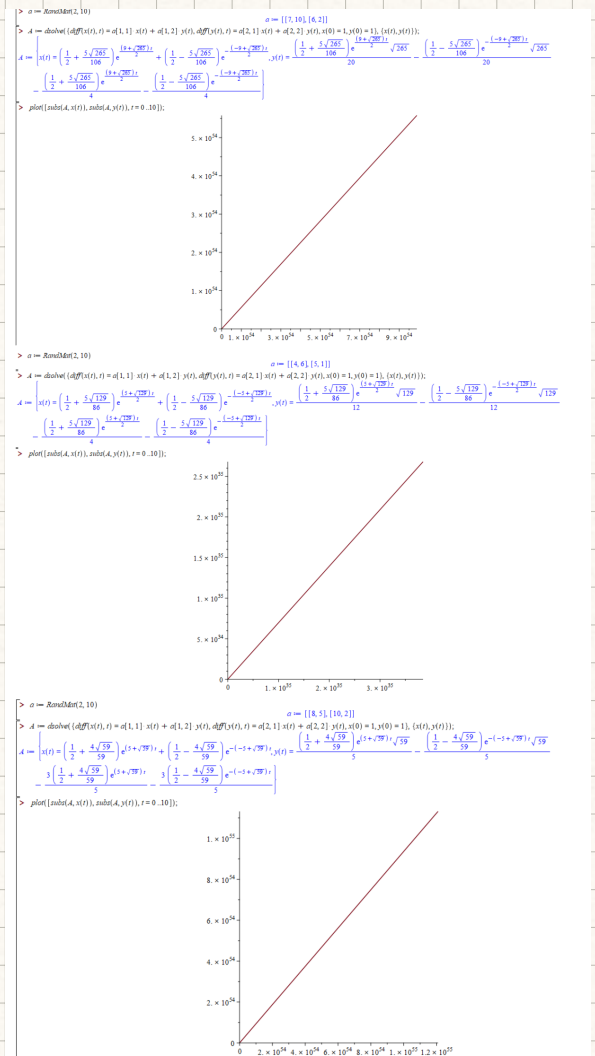


3. Use Maple to solve and then plot the phase-plane diagram for the system

$$\frac{dx}{dt} = a_{11}x + a_{12}y \quad , \frac{dy}{dt} = a_{21}x + a_{22}y \quad , \quad x(0) = 1 \quad , \quad y(0) = 1 \quad ,$$

for three randomly chosen matrices

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad .$$



4. Carefully read, and understand, the Maple code for the following procedures (type Help(ProcedureName); for instructions)

`Lotka, Volterra, VolterraM`

in the Maple package

`https://sites.math.rutgers.edu/~zeilberg/Bio25/DMB.txt` ,

For **each of them**, experiment with **three** randmom choices of parameters, and random initial conditions, using `Dis` (with $h = 0.01$), of *each* of the quantities in question.

```
> E := Dis(Volterra(5, 1, 1, 3, x, y), [x, y], [1, 1], 0.01, 1.5)
E := [[0.01, [1., 1.]], [0.02, [1.04, 1.02]], [0.03, [1.081392, 1.041624]], [0.04, [1.124197561, 1.064999876]], [0.05, [1.168434736, 1.090267985]], [0.06, [1.214117403, 1.117582515]], [0.07, [1.261254509, 1.147112981]], [0.08, [1.309849220, 1.179045894]], [0.09, [1.359897958, 1.213586605]], [0.10, [1.411389317, 1.250961357]], [0.11, [1.464302848, 1.291419548]], [0.12, [1.518607697, 1.335236233]], [0.13, [1.574261082, 1.382714872]], [0.14, [1.631206594, 1.434190349]], [0.15, [1.689372316, 1.490032269]], [0.16, [1.748668739, 1.550648524]], [0.17, [1.808986470, 1.616489157]], [0.18, [1.870193724, 1.688050475]], [0.19, [1.932133596, 1.765879412]], [0.20, [1.994621127, 1.850578066]], [0.21, [2.057440162, 1.942808348]], [0.22, [2.120340051, 2.043296623]], [0.23, [2.183032217, 2.152838167]], [0.24, [2.245186677, 2.272301237]], [0.25, [2.306428606, 2.402630439]], [0.26, [2.366335080, 2.544849002]], [0.27, [2.424432179, 2.700059476]], [0.28, [2.480192677, 2.869442213]], [0.29, [2.533034615, 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[0.84, [0.04541204869, 14.86466938]], [0.85, [0.04093230022, 14.73627374]], [0.86, [0.03694701942, 14.60700669]], [0.87, [0.03339751679, 14.47712718]], [0.88, [0.03023239165, 14.34686091]], [0.89, [0.02740661205, 14.21640450]], [0.90, [0.02488070782, 14.08592916]], [0.91, [0.02262006433, 13.95558391]], [0.92, [0.02059430549, 13.82549836]], [0.93, [0.01877675539, 13.69578518]], [0.94, [0.01714396908, 13.56654220]], [0.95, [0.01567532373, 13.43785431]], [0.96, [0.01435266276, 13.30979505]], [0.97, [0.01315998590, 13.18242803]], [0.98, [0.01208317953, 13.05580817]], [0.99, [0.01110978177, 12.92998276]], [1.00, [0.01022877799, 12.80499241]], [1.01, [0.009430422645, 12.68087187]], [1.02, [0.008706083965, 12.55765073]], [1.03, [0.008048108546, 12.43535406]], [1.04, [0.007449703180, 12.31400295]], [1.05, [0.006904831670, 12.19361499]], [1.06, [0.006408124664, 12.07420469]], [1.07, [0.005954800808, 11.95578383]], [1.08, [0.005540597736, 11.83836182]], [1.09, [0.005161711616, 11.72194595]], 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9.045305645]], [1.36, [0.001174988582, 8.955184875]], [1.37, [0.001128515611, 8.865948693]], [1.38, [0.001084887777, 8.777589367]], [1.39, [0.001043905172, 8.690099154]], [1.40, [0.001005384036, 8.603470311]], [1.41, [0.0009691553210, 8.517695102]], [1.42, [0.0009350633917, 8.432765800]], [1.43, [0.0009029648554, 8.348674697]], [1.44, [0.0008727274998, 8.265414107]], [1.45, [0.0008442293329, 8.182976370]], [1.46, [0.0008173577127, 8.101353855]], [1.47, [0.0007920085577, 8.020538967]], [1.48, [0.0007680856306, 7.940524147]], [1.49, [0.0007454998871, 7.861301876]], [1.50, [0.0007241688849, 7.782864675]], [1.51, [0.0007040162448, 7.705205111]]]
```

```
> E := Dis(Volterra(2, 3, 3, 2, x, y), [x, y], [1, 1], 0.01, 1.5)
E := [[0.01, [1., 1.]], [0.02, [0.99, 0.99]], [0.03, [0.980397, 0.979902]], [0.04, [0.9711841506, 0.9697187996]], [0.05, [0.9623545677, 0.9594627462]], [0.06, [0.9539013584, 0.9491457309]], [0.07, [0.9458176435, 0.9387791870]], [0.08, [0.9380965788, 0.9283740898]], [0.09, [0.9307313737, 0.9179409582]], [0.10, [0.9237153077, 0.9074898585]], [0.11, [0.9170417457, 0.8970304082]], [0.12, [0.9107041507, 0.8865717826]], [0.13, [0.9046960956, 0.8761227212]], [0.14, [0.8990112734, 0.8656915357]], [0.15, [0.8936435054, 0.8552861186]], [0.16, [0.8885867490, 0.8449139527]], [0.17, [0.8838351037, 0.8345821210]], [0.18, [0.8793828165, 0.8242973169]], [0.19, [0.8752242859, 0.8140658553]], [0.20, [0.8713540654, 0.8038936837]], [0.21, [0.8677668658, 0.7937863938]], [0.22, [0.8644575572, 0.7837492326]], [0.23, [0.8614211699, 0.7737871145]], [0.24, [0.8586528953, 0.7639046331]], [0.25, [0.8561480854, 0.7541060726]], [0.26, [0.8539022530, 0.7443954198]], [0.27, [0.8519110703, 0.7347763757]], [0.28, [0.8501703678, 0.7252523670]], [0.29, [0.8486761330, 0.7158265574]], [0.30, [0.8474245083, 0.7065018590]], [0.31, [0.8464117888, 0.6972809430]], [0.32, [0.8456344203, 0.6881662509]], [0.33, [0.8450889966, 0.6791600048]], [0.34, [0.8447722571, 0.6702642176]], [0.35, [0.8446810837, 0.6614807034]], [0.36, [0.8448124983, 0.6528110870]], [0.37, [0.8451636593, 0.6442568137]], [0.38, [0.8457318591, 0.6358191582]], [0.39, [0.8465145207, 0.6274992339]], [0.40, [0.8475091947, 0.6192980012]], [0.41, [0.8487135561, 0.6112162762]], [0.42, [0.8501254010, 0.6032547387]], [0.43, [0.8517426437, 0.5954139400]], [0.44, [0.8535633133, 0.5876943107]], [0.45, [0.8555855505, 0.5800961675]], [0.46, [0.8578076045, 0.5726197205]], [0.47, [0.8602278301, 0.5652650799]], [0.48, [0.8628446841, 0.5580322626]], [0.49, [0.8656567227, 0.5509211981]], [0.50, [0.8686625980, 0.5439317350]], [0.51, [0.8718610554, 0.5370636461]], [0.52, [0.8752509302, 0.5303166342]], [0.53, [0.8788311450, 0.5236903377]], [0.54, 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```

```
> E := Dis(VolterraM(2, 3, 3, 2, 3, x, y), [x, y], [1, 1], 0.01, 1.5)
E := [[0.01, [1., 1.]], [0.02, [0.980000000, 1.00]], [0.03, [0.9605960000, 0.9994000000]], [0.04, [0.9417798839, 0.9982185893]], [0.05, [0.9235429225, 0.9964750972]], [0.06, [0.9058758400, 0.9941894700]], [0.07, [0.8887688798, 0.9913821525]], [0.08, [0.8722118680, 0.9880739760]], [0.09, [0.8561942744, 0.9842860522]], [0.10, [0.8407052710, 0.9800396731]], [0.11, [0.8257337873, 0.9753562185]], [0.12, [0.8112685627, 0.9702570694]], [0.13, [0.7972981954, 0.9647635290]], [0.14, [0.7838111886, 0.9588967497]], [0.15, [0.7707959926, 0.9526776672]], [0.16, [0.7582410440, 0.9461269410]], [0.17, [0.7461348017, 0.9392649012]], [0.18, [0.7344657794, 0.9321115011]], [0.19, [0.7232225752, 0.9246862761]], [0.20, [0.7123938981, 0.9170083075]], [0.21, [0.7019685917, 0.9090961920]], [0.22, [0.6919356553, 0.9009680154]], [0.23, [0.6822842621, 0.8926413317]], [0.24, [0.6730037752, 0.8841331457]], [0.25, [0.6640837616, 0.8754598996]], [0.26, [0.6555140033, 0.8666374637]], [0.27, [0.6472845075, 0.8576811296]], [0.28, [0.6393855141, 0.8486056069]], [0.29, [0.6318075020, 0.8394250227]], [0.30, [0.6245411940, 0.8301529228]], [0.31, [0.6175775599, 0.8208022760]], [0.32, [0.6109078187, 0.8113854797]], [0.33, [0.6045234394, 0.8019143673]], [0.34, [0.5984161414, 0.7924002172]], [0.35, [0.5925778930, 0.7828537631]], [0.36, [0.5870009103, 0.7732852052]], [0.37, [0.5816776542, 0.7637042226]], [0.38, [0.5766008280, 0.7541199863]], [0.39, [0.5717633731, 0.7445411730]], [0.40, [0.5671584658, 0.7349759790]], [0.41, [0.5627795124, 0.7254321351]], [0.42, [0.5586201446, 0.7159169213]], [0.43, [0.5546742144, 0.7064371821]], [0.44, [0.5509357891, 0.6969993413]], [0.45, [0.5473991459, 0.6876094176]], [0.46, [0.5440587664, 0.6782730393]], [0.47, [0.5409093305, 0.6689954599]], [0.48, [0.5379457115, 0.6597815727]], [0.49, [0.5351629698, 0.6506359255]], [0.50, [0.5325563476, 0.6415627353]], [0.51, [0.5301212627, 0.6325659024]], [0.52, [0.5278533034, 0.6236490244]], [0.53, [0.5257482225, 0.6148154096]], [0.54, [0.5238019317, 0.6060680906]], [0.55, [0.5220104966, 0.5974098370]], [0.56, [0.5203701307, 0.5888431681]], [0.57, [0.5188771907, 0.5803703650]], [0.58, [0.5175281708, 0.5719934823]], [0.59, [0.5163196979, 0.5637143600]], [0.60, [0.5152485268, 0.5555346340]], [0.61, [0.5143115348, 0.5474557471]], [0.62, [0.5135057177, 0.5394789589]], [0.63, [0.5128281849, 0.5316053560]], [0.64, [0.5122761548, 0.5238358616]], [0.65, [0.5118469507, 0.5161712444]], [0.66, [0.5115379964, 0.5086121274]], [0.67, [0.5113468122, 0.5011589965]], [0.68, [0.5112710111, 0.4938122083]], [0.69, [0.5113082949, 0.4865719981]], [0.70, [0.5114564501, 0.4794384872]], [0.71, [0.5117133449, 0.4724116898]], [0.72, [0.5120769253, 0.4654915201]], [0.73, [0.5125452120, 0.4586777985]], [0.74, [0.5131162970, 0.4519702578]], [0.75, [0.5137883404, 0.4453685492]], [0.76, [0.5145595676, 0.4388722477]], [0.77, [0.5154282661, 0.4324808577]], [0.78, [0.5163927826, 0.4261938178]], [0.79, [0.5174515209, 0.4200105056]], [0.80, [0.5186029384, 0.4139302426]], [0.81, [0.5198455439, 0.4079522985]], [0.82, [0.5211778954, 0.4020758950]], [0.83, [0.5225985972, 0.3963002103]], [0.84, [0.5241062982, 0.3906243820]], [0.85, [0.5256996890, 0.3850475115]], [0.86, [0.5273775005, 0.3795686669]], [0.87, [0.5291385010, 0.3741868861]], [0.88, [0.5309814949, 0.3689011801]], [0.89, [0.5329053203, 0.3637105357]], [0.90, [0.5349088475, 0.3586139180]], [0.91, [0.5369909770, 0.3536102732]], [0.92, [0.5391506376, 0.3486985308]], [0.93, [0.5413867852, 0.3438776060]], [0.94, [0.5436984007, 0.3391464015]], [0.95, [0.5460844885, 0.3345038102]], [0.96, [0.5485440753, 0.3299487162]], [0.97, [0.5510762084, 0.3254799971]], [0.98, [0.5536799542, 0.3210965257]], [0.99, [0.5563543971, 0.3167971712]], [1.00, [0.5590986379, 0.3125808011]], [1.01, [0.5619117928, 0.3084462821]], [1.02, [0.5647929919, 0.3043924817]], [1.03, [0.5677413783, 0.3004182694]], [1.04, [0.5707561066, 0.2965225178]], [1.05, [0.5738363423, 0.2927041034]], [1.06, [0.5769812601, 0.2889619079]], [1.07, [0.5801900434, 0.2852948189]], [1.08, [0.5834618830, 0.2817017307]], [1.09, [0.5867959763, 0.2781815455]], [1.10, [0.5901915263, 0.2747331734]], [1.11, [0.5936477407, 0.2713555339]], [1.12, [0.5971638311, 0.2680475559]], [1.13, [0.6007390121, 0.2648081784]], [1.14, [0.6043725006, 0.2616363511]], [1.15, [0.6080635149, 0.2585310351]], [1.16, [0.6118112741, 0.2554912027]], [1.17, [0.6156149973, 0.2525158385]], [1.18, [0.6194739029, 0.2496039394]], [1.19, [0.6233872080, 0.2467545150]], [1.20, [0.6273541278, 0.2439665878]], [1.21, [0.6313738749, 0.2412391936]], [1.22, [0.6354456590, 0.2385713815]], [1.23, [0.6395686858, 0.2359622146]], [1.24, [0.6437421572, 0.2334107695]], [1.25, [0.6479652701, 0.2309161370]], [1.26, [0.6522372165, 0.2284774220]], [1.27, [0.6565571827, 0.2260937436]], [1.28, [0.6609243489, 0.2237642354]], [1.29, [0.6653378890, 0.2214880452]], [1.30, [0.6697969700, 0.2192643355]], [1.31, [0.6743007520, 0.2170922830]], [1.32, [0.6788483873, 0.2149710792]], [1.33, [0.6834390206, 0.2128999299]], [1.34, [0.6880717885, 0.2108780556]], [1.35, [0.6927458192, 0.2089046911]], [1.36, [0.6974602324, 0.2069790859]], [1.37, [0.7022141389, 0.2051005037]], [1.38, [0.7070066405, 0.2032682228]], [1.39, [0.7118368299, 0.2014815356]], [1.40, [0.7167037905, 0.1997397488]], [1.41, [0.7216065960, 0.1980421834]], [1.42, [0.7265443107, 0.1963881743]], [1.43, [0.7315159893, 0.1947770704]], [1.44, [0.7365206765, 0.1932082345]], [1.45, [0.7415574072, 0.1916810433]], [1.46, [0.7466252066, 0.1901948869]], [1.47, [0.7517230898, 0.1887491692]], [1.48, [0.7568500623, 0.1873433074]], [1.49, [0.7620051196, 0.1859767320]], [1.50, [0.7671872473, 0.1846488867]], [1.51, [0.7723954214, 0.1833592282]]]
```

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> E := Dis(VolterraM(1, 5, 3, 7, 2, x, y), [x, y], [1, 1], 0.01, 1.5)
E := [[0.01, [1., 1.]], [0.02, [0.958571429, 0.99]], [0.03, [0.9193952016, 0.9792797143]], [0.04, [0.8823643465, 0.9679082243]], [0.05, [0.8473733662, 0.9559519318]], [0.06, [0.8143189158, 0.9434743379]], [0.07, [0.7831003474, 0.9305358878]], [0.08, [0.7536201360, 0.9171938707]], [0.09, [0.7257842013, 0.9035023700]], [0.10, [0.6995021379, 0.8895122538]], [0.11, [0.6746873684, 0.8752712007]], [0.12, [0.6512572308, 0.8608237532]], [0.13, [0.6291330099, 0.8462113945]], [0.14, [0.6082399234, 0.8314726431]], [0.15, [0.5885070715, 0.8166431609]], [0.16, [0.5698673562, 0.8017558716]], [0.17, [0.5522573780, 0.7868410855]], [0.18, [0.5356173146, 0.7719266288]], [0.19, [0.5198907873, 0.7570379753]], [0.20, [0.5050247182, 0.7421983774]], [0.21, [0.4909691820, 0.7274289966]], [0.22, [0.4776772546, 0.7127490311]], [0.23, [0.4651048621, 0.6981758402]], [0.24, [0.4532106296, 0.6837250646]], [0.25, [0.4419557342, 0.6694107420]], [0.26, [0.4313037602, 0.6552454180]], [0.27, [0.4212205602, 0.6412402518]], [0.28, [0.4116741201, 0.6274051158]], [0.29, [0.4026344308, 0.6137486913]], [0.30, [0.3940733652, 0.6002785577]], [0.31, [0.3859645609, 0.5870012768]], [0.32, [0.3782833097, 0.5739224723]], [0.33, [0.3710064521, 0.5610469039]], [0.34, [0.3641122787, 0.5483785372]], [0.35, [0.3575804368, 0.5359206083]], [0.36, [0.3513918423, 0.5236756846]], [0.37, [0.3455285979, 0.5116457214]], [0.38, [0.3399739153, 0.4998321144]], [0.39, [0.3347120429, 0.4882357486]], [0.40, [0.3297281982, 0.4768570438]], [0.41, [0.3250085042, 0.4656959968]], [0.42, [0.3205399305, 0.4547522201]], [0.43, [0.3163102378, 0.4440249784]], [0.44, [0.3123079262, 0.4335132219]], [0.45, [0.3085221872, 0.4232156175]], [0.46, [0.3049428588, 0.4131305772]], [0.47, [0.3015603833, 0.4032562843]], [0.48, [0.2983657688, 0.3935907182]], [0.49, [0.2953505521, 0.3841316766]], [0.50, [0.2925067655, 0.3748767964]], [0.51, [0.2898269043, 0.3658235725]], [0.52, [0.2873038982, 0.3569693756]], [0.53, [0.2849310832, 0.3483114682]], [0.54, [0.2827021762, 0.3398470195]], [0.55, [0.2806112512, 0.3315731187]], [0.56, [0.2786527168, 0.3234867881]], [0.57, [0.2768212956, 0.3155849939]], [0.58, [0.2751120048, 0.3078646570]], [0.59, [0.2735201380, 0.3003226626]], [0.60, [0.2720412485, 0.2929558686]], [0.61, [0.2706711335, 0.2857611141]], [0.62, [0.2694058194, 0.2787352264]], [0.63, [0.2682415480, 0.2718750274]], [0.64, [0.2671747638, 0.2651773402]], [0.65, [0.2662021019, 0.2586389939]], [0.66, [0.2653203771, 0.2522568290]], [0.67, [0.2645265728, 0.2460277016]], [0.68, [0.2638178320, 0.2399484879]], [0.69, [0.2631914475, 0.2340160871]], [0.70, [0.2626448536, 0.2282274252]], [0.71, [0.2621756181, 0.2225794576]], [0.72, [0.2617814346, 0.2170691720]], [0.73, [0.2614601156, 0.2116935904]], [0.74, [0.2612095861, 0.2064497713]], [0.75, [0.2610278769, 0.2013348114]], [0.76, [0.2609131193, 0.1963458471]], [0.77, [0.2608635392, 0.1914800558]], [0.78, [0.2608774523, 0.1867346574]], [0.79, [0.2609532594, 0.1821069149]], [0.80, [0.2610894414, 0.1775941354]], [0.81, [0.2612845557, 0.1731936704]], [0.82, [0.2615372316, 0.1689029169]], [0.83, [0.2618461671, 0.1647193174]], [0.84, [0.2622101249, 0.1606403603]], [0.85, [0.2626279295, 0.1566635801]], [0.86, [0.2630984637, 0.1527865573]], [0.87, [0.2636206661, 0.1490069188]], [0.88, [0.2641935278, 0.1453223373]], [0.89, [0.2648160903, 0.1417305316]], [0.90, [0.2654874427, 0.1382292662]], [0.91, [0.2662067196, 0.1348163509]], [0.92, [0.2669730987, 0.1314896408]], [0.93, [0.2677857990, 0.1282470355]], [0.94, [0.2686440785, 0.1250864791]], [0.95, [0.2695472327, 0.1220059595]], [0.96, [0.2704945928, 0.1190035081]], [0.97, [0.2714855237, 0.1160771990]], [0.98, [0.2725194230, 0.1132251486]], [0.99, [0.2735957192, 0.1104455151]], [1.00, [0.2747138702, 0.1077364980]], [1.01, [0.2758733624, 0.1050963373]], [1.02, [0.2770737090, 0.1025233128]], [1.03, [0.2783144492, 0.1000157437]], [1.04, [0.2795951468, 0.09757198792]], [1.05, [0.2809153892, 0.09519044137]], [1.06, [0.2822747866, 0.09286953733]], [1.07, [0.2836729708, 0.09060774579]], [1.08, [0.2851095945, 0.08840357279]], [1.09, [0.2865843302, 0.08625555975]], [1.10, [0.2880968695, 0.08416228280]], [1.11, [0.2896469225, 0.08212235212]], [1.12, [0.2912342169, 0.08013441129]], [1.13, [0.2928584973, 0.07819713660]], [1.14, [0.2945195245, 0.07630923642]], [1.15, [0.2962170750, 0.07446945053]], [1.16, [0.2979509403, 0.07267654947]], [1.17, [0.2997209263, 0.07092933392]], [1.18, [0.3015268529, 0.06922663401]], [1.19, [0.3033685534, 0.06756730877]], [1.20, [0.3052458742, 0.06595024544]], [1.21, [0.3071586737, 0.06437435889]], [1.22, [0.3091068227, 0.06283859097]], [1.23, [0.3110902033, 0.06134190998]], [1.24, [0.3131087089, 0.05988331002]], [1.25, [0.3151622437, 0.05846181044]], [1.26, [0.3172507223, 0.05707645524]], [1.27, [0.3193740693, 0.05572631251]], [1.28, [0.3215322190, 0.05441047391]], [1.29, [0.3237251152, 0.05312805410]], [1.30, [0.3259527107, 0.05187819019]], [1.31, [0.3282149672, 0.05066004121]], [1.32, [0.3305118547, 0.04947278765]], [1.33, [0.3328433517, 0.04831563088]], [1.34, [0.3352094446, 0.04718779268]], [1.35, [0.3376101274, 0.04608851478]], [1.36, [0.3400454017, 0.04501705833]], [1.37, [0.3425152766, 0.04397270345]], [1.38, [0.3450197680, 0.04295474880]], [1.39, [0.3475588986, 0.04196251109]], [1.40, [0.3501326980, 0.04099532464]], [1.41, [0.3527412021, 0.04005254097]], [1.42, [0.3553844531, 0.03913352837]], [1.43, [0.3580624994, 0.03823767147]], [1.44, [0.3607753952, 0.03736437085]], [1.45, [0.3635232006, 0.03651304263]], [1.46, [0.3663059813, 0.03568311811]], [1.47, [0.3691238083, 0.03487404336]], [1.48, [0.3719767581, 0.03408527885]], [1.49, [0.3748649124, 0.03331629911]], [1.50, [0.3777883578, 0.03256659237]], [1.51, [0.3807471859, 0.03183566019]]]
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> Dis(Lotka(1, 2, 2, 3, 1, 2, N1, N2), [N1, N2], [1, 1], 0.01, 1.5)
[[0.01, [1., 1.]], [0.02, [1., 1.]], [0.03, [1., 1.]], [0.04, [1., 1.]], [0.05, [1., 1.]], [0.06, [1., 1.]], [0.07, [1., 1.]], [0.08, [1., 1.]], [0.09, [1., 1.]], [0.10, [1., 1.]], [0.11, [1., 1.]], [0.12, [1., 1.]], [0.13, [1., 1.]], [0.14, [1., 1.]], [0.15, [1., 1.]], [0.16, [1., 1.]], [0.17, [1., 1.]], [0.18, [1., 1.]], [0.19, [1., 1.]], [0.20, [1., 1.]], [0.21, [1., 1.]], [0.22, [1., 1.]], [0.23, [1., 1.]], [0.24, [1., 1.]], [0.25, [1., 1.]], [0.26, [1., 1.]], [0.27, [1., 1.]], [0.28, [1., 1.]], [0.29, [1., 1.]], [0.30, [1., 1.]], [0.31, [1., 1.]], [0.32, [1., 1.]], [0.33, [1., 1.]], [0.34, [1., 1.]], [0.35, [1., 1.]], [0.36, [1., 1.]], [0.37, [1., 1.]], [0.38, [1., 1.]], [0.39, [1., 1.]], [0.40, [1., 1.]], [0.41, [1., 1.]], [0.42, [1., 1.]], [0.43, [1., 1.]], [0.44, [1., 1.]], [0.45, [1., 1.]], [0.46, [1., 1.]], [0.47, [1., 1.]], [0.48, [1., 1.]], [0.49, [1., 1.]], [0.50, [1., 1.]], [0.51, [1., 1.]], [0.52, [1., 1.]], [0.53, [1., 1.]], [0.54, [1., 1.]], [0.55, [1., 1.]], [0.56, [1., 1.]], [0.57, [1., 1.]], [0.58, [1., 1.]], [0.59, [1., 1.]], [0.60, [1., 1.]], [0.61, [1., 1.]], [0.62, [1., 1.]], [0.63, [1., 1.]], [0.64, [1., 1.]], [0.65, [1., 1.]], [0.66, [1., 1.]], [0.67, [1., 1.]], [0.68, [1., 1.]], [0.69, [1., 1.]], [0.70, [1., 1.]], [0.71, [1., 1.]], [0.72, [1., 1.]], [0.73, [1., 1.]], [0.74, [1., 1.]], [0.75, [1., 1.]], [0.76, [1., 1.]], [0.77, [1., 1.]], [0.78, [1., 1.]], [0.79, [1., 1.]], [0.80, [1., 1.]], [0.81, [1., 1.]], [0.82, [1., 1.]], [0.83, [1., 1.]], [0.84, [1., 1.]], [0.85, [1., 1.]], [0.86, [1., 1.]], [0.87, [1., 1.]], [0.88, [1., 1.]], [0.89, [1., 1.]], [0.90, [1., 1.]], [0.91, [1., 1.]], [0.92, [1., 1.]], [0.93, [1., 1.]], [0.94, [1., 1.]], [0.95, [1., 1.]], [0.96, [1., 1.]], [0.97, [1., 1.]], [0.98, [1., 1.]], [0.99, [1., 1.]], [1.00, [1., 1.]], [1.01, [1., 1.]], [1.02, [1., 1.]], [1.03, [1., 1.]], [1.04, [1., 1.]], [1.05, [1., 1.]], [1.06, [1., 1.]], [1.07, [1., 1.]], [1.08, [1., 1.]], [1.09, [1., 1.]], [1.10, [1., 1.]], [1.11, [1., 1.]], [1.12, [1., 1.]], [1.13, [1., 1.]], [1.14, [1., 1.]], [1.15, [1., 1.]], [1.16, [1., 1.]], [1.17, [1., 1.]], [1.18, [1., 1.]], [1.19, [1., 1.]], [1.20, [1., 1.]], [1.21, [1., 1.]], [1.22, [1., 1.]], [1.23, [1., 1.]], [1.24, [1., 1.]], [1.25, [1., 1.]], [1.26, [1., 1.]], [1.27, [1., 1.]], [1.28, [1., 1.]], [1.29, [1., 1.]], [1.30, [1., 1.]], [1.31, [1., 1.]], [1.32, [1., 1.]], [1.33, [1., 1.]], [1.34, [1., 1.]], [1.35, [1., 1.]], [1.36, [1., 1.]], [1.37, [1., 1.]], [1.38, [1., 1.]], [1.39, [1., 1.]], [1.40, [1., 1.]], [1.41, [1., 1.]], [1.42, [1., 1.]], [1.43, [1., 1.]], [1.44, [1., 1.]], [1.45, [1., 1.]], [1.46, [1., 1.]], [1.47, [1., 1.]], [1.48, [1., 1.]], [1.49, [1., 1.]], [1.50, [1., 1.]], [1.51, [1., 1.]]]
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> E := Dis(VolterraM(2, 4, 3, 6, 1, x, y), [x, y], [1, 1], 0.01, 1.5)
E := [[0.01, [1., 1.]], [0.02, [0.976666667, 0.98]], [0.03, [0.9547350744, 0.9601713333]], [0.04, [0.9341230090, 0.9405332858]], [0.05, [0.9147538986, 0.9211030250]], [0.06, [0.8965564243, 0.9018957600]], [0.07, [0.8794641572, 0.8829248916]], [0.08, [0.8634152179, 0.8642021529]], [0.09, [0.8483519579, 0.8457377412]], [0.10, [0.8342206629, 0.8275404417]], [0.11, [0.8209712757, 0.8096177418]], [0.12, [0.8085571387, 0.7919759386]], [0.13, [0.7969347540, 0.7746202384]], [0.14, [0.7860635608, 0.7575548491]], [0.15, [0.7759057284, 0.7407830662]], [0.16, [0.7664259643, 0.7243073524]], [0.17, [0.7575913360, 0.7081294114]], [0.18, [0.7493711056, 0.6922502562]], [0.19, [0.7417365773, 0.6766702719]], [0.20, [0.7346609547, 0.6613892746]], [0.21, [0.7281192097, 0.6464065652]], [0.22, [0.7220879605, 0.6317209786]], [0.23, [0.7165453585, 0.6173309303]], [0.24, [0.7114709837, 0.6032344585]], [0.25, [0.7068457477, 0.5894292628]], [0.26, [0.7026518036, 0.5759127406]], [0.27, [0.6988724628, 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