1. $\frac{dy}{dt} = \frac{y^3}{t+1}$ $y(0)=1$

$\frac{dy}{y^3} = \frac{dt}{t+1}$

$\int y^{-3}\,dy = \int \frac{dt}{t+1}$

$-\frac{1}{2y^2} = \ln|t+1| + C$

$y(0)=1$, $C = -\frac{1}{2}$.

$-\frac{1}{2y^2} = \ln(t+1) - \frac{1}{2}$

$\frac{1}{y^2} = 1 - 2\ln(t+1)$

$y^2 = \frac{1}{1-2\ln(t+1)}$

$y(t) = \frac{1}{\sqrt{1-2\ln|t+1|}}$

2. $y''(t) - 3y'(t) + 2y(t) = 0$. $y(0)=2$ $y'(0)=3$.

$r^2 - 3r + 2 = 0$.

$(r-2)(r-1) = 0$

$r_1 = 2$ $r_2 = 1$.

$y(t) = e^{2t}C_1 + e^{t}C_2$.

plug in $y(0)=2$: $2 = C_1 + C_2$. $C_1 = 2 - C_2$.

$y'(t) = 2e^{2t}C_1 + e^{t}C_2$

plug in $y'(0)=3$.

$3 = 2C_1 + C_2 \rightarrow 3 = 2(2-C_2) + C_2$.

$3 = 4 - 2C_2 + C_2$.

$3 - 4 = -C_2$.

$-1 = C_2$.

$C_1 = 2 - 1 = 1$

$y(t) = e^{2t} + e^{t}$.

Find

3. Eigenvalues and Eigenvectors of the matrix $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$

Eigenvalue $= \begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix}$.

$(3-\lambda)(3-\lambda)+16=0$

$(3-\lambda)^2+16=0$.

$(3-\lambda)^2=-16$.

$3-\lambda=\pm 4i$.

$\lambda = 3 \pm 4i$.

Find Eigenvectors:

For $\lambda_1 = 3+4i$

$$\begin{bmatrix} 3-(3+4i) & -4 \\ 4 & 3-(3+4i) \end{bmatrix} = \begin{bmatrix} -4i & -4 \\ 4 & 4i \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-4ix-4y=0$.

$-4ix=4y$.

$x=iy$

$\begin{bmatrix} i \\ 1 \end{bmatrix}$

For $\lambda_2 = 3-4i$.

$$\begin{bmatrix} 3-(3-4i) & -4 \\ 4 & 3-(3-4i) \end{bmatrix} = \begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-ix=y$

$4ix-4y=0$.

$4ix=4y$

$ix=y$.

$x=-iy$.

$\begin{bmatrix} -i \\ 1 \end{bmatrix}$