1. Solve the following differential equation, subject to the given initial condition

$$\frac{dy}{dt} = \frac{y^3}{(t+1)} \quad , \quad y(0) = 1 \quad .$$

$\int y^{-3}\,dy = \int \frac{dt}{(t+1)}$

$-\frac{1}{2y^2} = \ln|t+1| \Rightarrow y(0)=1$

$-\frac{1}{2(1)^2} = \ln|0+1| + C$

$-1/2 = C$

$\frac{1}{y^2} = 1 - 2\ln|t+1|$

$y(t) = \frac{1}{\sqrt{1-2\ln|t+1|}}$

2. Solve the folloing differential equation, subject to the given initial conditions

$$y''(t) - 3y'(t) + 2y(t) = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 3 \quad .$$

$z^2 - 3z + 2 = 0$

$(z-2)(z-1) = 0$

$\left.\begin{array}{l} z_1 = 2 \\ z_2 = 1 \end{array}\right\}$ $y(t) = C_1e^{2t} + C_2e^{t} \longrightarrow 2 = C_1e^0 + C_2e^0$

$y'(t) = 2C_1e^{2t} + C_2e^{t}$

$2 = C_1 + C_2$

$C_1 = 2 - C_2$

$\longrightarrow 3 = 2C_1e^0 + C_2e^0$

$3 = 2C_1 + C_2$

$3 = 2(2-C_2) + C_2$

$3 = 4 - 2C_2 + C_2$

$-1 = -C_2$

$C_2 = 1 \ , \ C_1 = 1$

$y(t) = e^{2t} + e^{t}$

3. Find all the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

$A - \lambda I : \begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} = (3-\lambda)(3-\lambda) - (-16)$

$9 - 6\lambda + \lambda^2 + 16 = 0$

$\lambda^2 - 6\lambda + 25 = 0$ $\quad \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{-(-6) \pm \sqrt{-6^2 - 4(1)(25)}}{2(1)} = 0$

$\frac{6 \pm \sqrt{36-100}}{2} = 0$

$\frac{6 \pm \sqrt{-64}}{2} = 0 \Rightarrow 3-4i \ , \ 3+4i$

for $\lambda = 3-4i$: $\begin{bmatrix} 3-(3-4i) & -4 \\ 4 & 3-(3-4i) \end{bmatrix} = \begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \left.\begin{array}{l} 4ia - 4b = 0 \\ 4ia = 4b \\ ia = b \end{array}\right\} \begin{bmatrix} 1 \\ i \end{bmatrix}$

for $\lambda = 3+4i$: $\begin{bmatrix} 3-(3+4i) & -4 \\ 4 & 3-(3+4i) \end{bmatrix} = \begin{bmatrix} -4i & -4 \\ 4 & 4i \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \left.\begin{array}{l} -4ia - 4b = 0 \\ -4ia = 4b \\ -ia = b \end{array}\right\} \begin{bmatrix} -1 \\ i \end{bmatrix}$